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HANDBOOK

FOR THE

ELECTRICAL LABORATORY  
AND TESTING ROOM.

---

By J. A. FLEMING,

M.A., D.Sc., F.R.S., M.R.I., &c.

VOL. II.









10 14/-

A HANDBOOK  
FOR THE  
ELECTRICAL LABORATORY  
AND  
TESTING ROOM.

BY  
J. A. FLEMING, M.A., D.Sc., F.R.S.

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VOLUME II.



LONDON:  
"THE ELECTRICIAN" PRINTING AND PUBLISHING COMPANY,  
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# TABLE OF CONTENTS.

## CHAPTER I.

	PAGE.
THE MEASUREMENT OF ELECTRIC QUANTITY AND ENERGY	1
§ 1. The Measurement of Electric Quantity ; Definitions ; The Coulomb and Ampere-hour ; Coulomb Meters and Ampere-hour Meters.—	
§ 2. The Ballistic Galvanometer ; Theory of the Ballistic Galvanometer ; Definition of Logarithmic Decrement.—§ 3. Practical Forms of Ballistic Galvanometer and their Use ; Determination of the Periodic Time and Logarithmic Decrement ; Correction for Damping.—§ 4. Various Forms of Coulomb Meter, Ampere-hour Meter and Voltmeter ; Electrolytic Meters : Edison, Long-Schattner, Bastian, and Wright ; Ampere-hour Meters : Chamberlain and Hookham, Ferranti and others.—§ 5. Measurement of Electric Energy by Watt-hour Meters ; Mengarini, Elihu Thomson, Aron, Evershed and others.—§ 6. Electric House Meters, Classification of and Notes on ; References to Patent Specifications ; Alternate-Current Meters : Ferranti, Mordey and Fricker, Schallenberger, Westinghouse ; Descriptions and Theory ; Meters for Polyphase Energy Measurement.—§ 7. The Testing of House Meters.—	
§ 8. Battery Testing ; Primary and Secondary Cell Testing ; References to the Literature of the Subject.	

## CHAPTER II.

THE MEASUREMENT OF CAPACITY AND INDUCTANCE .....	111
§ 1. Definition of Capacity.—§ 2. Standards of Electrical Capacity ; Capacity of Ellipsoid, Sphere, Disc and Concentric Cylinders ; Capacity of a Telegraph Wire or Aerial Wire ; Air Condensers, Muirhead's Standard Air Condenser ; Mica Condensers ; Glass Plate High Voltage Condenser.—§ 3. Determination of Capacity in Absolute Measure.—§ 4. The Measurement of Capacity by the Ballistic Galvanometer.—§ 5. Measurement of Capacity by Rapid Charge and Discharge through a Galvanometer ; Tuning Fork Commutators ; Rotatory Commutator.—§ 6. The Measurement of Capacity by Means of Wheatstone's Bridge and Commutator.—§ 7. Comparison Methods of Measuring Capacity ; De Sauty's Method.—§ 8. Comparison of Capacities by Methods of Mixture ; Kelvin's and Gott's	

CHAPTER II.—(*Continued*).

Methods ; Muirhead's Investigations.—§ 9. Rules for Adding Capacities.—§ 10. The Determination of Capacity by Alternating Currents ; Effect of the Form Factor and of Resonance on the Result.—§ 11. Measurement of Cable Capacity ; Coefficients of Capacity of Polycore Cables.—§ 12. The Measurement of Inductance ; Definitions ; Inductance of a Straight Wire and of Circular Conductors ; Inductance of an Endless Solenoid.—§ 13. Standards of Inductance.—§ 14. Measurement of Inductance ; Time-Constant of a Circuit.—§ 15. Lord Rayleigh's Method of Measuring Inductance.—§ 16. Theorem Concerning the Time-Constants of a Wheatstone's Bridge Circuit.—§ 17. Maxwell's Method of Measuring Inductance ; Sumpner's Improvement.—§ 18. Anderson's Method.—§ 19. Measurement of Inductance by the Secohmmeter ; Ayrton and Perry's Method.—§ 20. Comparison of Inductances by the Secohmmeter.—§ 21. Practical Methods and Appliances for the Measurement of Inductance.—§ 22. Determination of Inductance by Alternating Currents.—§ 23. Mutual Inductance.—§ 24. Standards of Mutual Inductance.—§ 25. Measurement of Mutual Inductance ; Carey Foster's Method.—§ 26. Maxwell's Method for the Comparison of Mutual Inductance.—§ 27. Measurement of Small Inductances by Hughes' Bridge ; Table of Dielectric Constants of Solids and Liquids.

## CHAPTER III.

## PHOTOMETRY ..... 229

§ 1. Photometry.—§ 2. Photometric Units and Standards of Light.—§ 3. Practical Light Standards ; Flame Standards ; The Sperm Candle ; Harcourt Pentane Lamp ; Hefner Lamp ; The Reichsanstalt Pattern of Hefner.—§ 4. Influence of the Surrounding Atmosphere on Flame Standards.—§ 5. Incandescence Standards ; Violle Platinum Standard ; Fleming-Ediswan Carbon Filament Glow Lamp Standard ; Electric Arc Standard.—§ 6. Photometric Processes and Photometers ; Ritchie-Wedge Gas Referees' Photoped or Foucault Photometer ; Dispersion Photometers ; Bunsen Disc, Theory of ; The Bunsen Photometer ; Lummer-Brodhun Photometer ; Fleming Total Reflection Photometer ; Fox-Talbot-Abney Variable Aperture Disc ; Flicker Photometers : Selected Ray or Spectro-Photometers ; Illumination Photometers, Preece-Trotter, Weber and others.—§ 7. The Measurement of Luminous Efficiency.—§ 8. Photometer Rooms.—§ 9. The Photometry of Electric Glow Lamps ; Life Curves ; Target Diagrams in Lamp Classification.—§ 10. The Determination of Mean Spherical Candle-power ; The Rousseau Diagram ; Photometry of Arc Lamps ; Matthews' Integrating Photometer.—§ 11. Heterochromatic Photometry ; Discrimination Photometers.—§ 12. Illumination

CHAPTER III.—(*Continued*).

Photometry.—§ 13. Determination of Curves of Illumination.—  
 § 14. Measurement of Coefficients of Absorption and Reflection  
 Brightness of Various Illuminated Surfaces; Sumpner's Investiga-  
 tions; List of Various Works on Photometry and References to  
 Original Papers on various parts of the Subject.

## CHAPTER IV.

## MAGNETIC AND IRON TESTING ..... 365

- § 1. Magnetic Definitions and Fundamental Relations between Mag-  
 netic Quantities.—§ 2. Magnetisation Curves, Single-valued and  
 Cyclical; Examples of Typical Magnetisation Curves for Iron and  
 Steel as used in Dynamo Construction.—§ 3. Standard Forms of  
 Magnetic Test-piece; Preparation of a Sample of Iron or Steel for  
 obtaining its Magnetisation Curve; Precautions to be taken; The  
 Ring and the Ellipsoid as typical forms; Demagnetisation Factors.  
 —§ 4. Magnetic Field or Flux Density Measurement in Air Spaces  
 by the Ballistic Galvanometer; Form of Ballistic Galvanometer  
 suitable for Magnetic Measurements.—§ 5. Measurement of Mag-  
 netic Field Strength by Change of Resistance of Bismuth; Hart-  
 mann and Braun's Spiral; Investigations of Fleming and Dewar,  
 Henderson and others.—§ 6. Measurement of Weak Fields by  
 Gauss' Method; Determination of the Terrestrial Magnetic Force  
 by Gauss' Arrangements.—§ 7. Determination of the Magnetisation  
 Curve of Iron by the Ballistic Galvanometer; Practical Details;  
 Tables giving Results for Various Samples; Cyclical Magnetisation  
 Curves and their Meaning.—§ 8. Permeability Curves; Examples  
 of Permeability Curves and Tables of Permeability for different  
 Samples of Iron and Steel.—§ 9. Bar and Yoke Method for obtain-  
 ing Magnetisation Curves; Yoke Permeameters of Hopkinson,  
 Ewing and others; Corrections for Joint Reluctance.—§ 10.  
 Traction Methods for Determining Magnetisation Curves; Bidwell's  
 Researches; Ewing's Traction Permeameter; Du Bois' Magnetic  
 Balance.—§ 11. Magnetic Bridge Methods; Ewing's Magnetic  
 Bridge.—§ 12. Commercial Forms of Permeameter: Lamb and  
 Walker, Drysdale and others.—§ 13. Measurement of Hysteresis;  
 Dissipation of Energy by Hysteresis; Alternating and Rotating  
 Hysteresis; Baily's Experiments.—§ 14. Delineation of a Stein-  
 metz Curve; Hysteretic Constants; Tables giving Results of  
 Various Investigations on Hysteresis; Loss in Watts per pound  
 per 100 cycles at various Flux Densities.—§ 15. Hysteresis  
 Measurement by the Wattmeter. § 16. Workshop Methods for  
 Measuring Hysteresis; Hysteresis Meters of Ewing and Holden.  
 —§ 17. Variation of Hysteresis with Temperature; Ageing of

CHAPTER IV.—(*Continued*).

Iron; Experiments of Mordey, Roget, Ford and others.—18. The Measurement of the Magnetic Susceptibility of Feebly Magnetic Substances; Measurement of the Susceptibility of Solids and of Liquid Gases.

## CHAPTER V.

## DYNAMO, MOTOR AND TRANSFORMER TESTING ..... 495

- §1. Definition of Term "Efficiency"; Efficiency and Total Loss Curves.—§2. Dynamo Testing; Measurements of Copper or Resistance Loss.—§3. Dynamo Efficiencies and Losses; Various Uses of the term "Efficiency."—§4. Practical Measurement of Dynamo Efficiency.—§5. Arrangements for the Absorption of Power in Dynamo Tests; Wire Rheostats; Determination of most Economical Size of Wire to Employ in Power Absorbing Rheostats; Construction of Inductive and Non-Inductive Power Absorbing Rheostats.—§6. Differential Methods for Testing Shunt-Wound Continuous-Current Dynamos and Motors; Hopkinson's Method; Modifications suggested by Lord Rayleigh, Kapp and others.—§7. Differential Efficiency; Measurement of Dynamos by the Series Method.—§8. Differential Efficiency Measurement by the Parallel Method of Kapp; Examples of such Measurements.—§9. Use of a Tested Shunt-Wound Continuous-Current Motor as a Means of Determining the Efficiency of another Dynamo.—§10. Efficiency Testing of Direct-Coupled Engines and Dynamos.—§11. Temperature Tests.—§12. Efficiency Testing by the Direct Measurement of Lost Power.—§13. Routin's Method for the Determination of Dynamo Efficiency.—§14. Separation of Hysteresis and Eddy Current Losses in Dynamo Armatures; Housman's Diagram.—§15. Continuous Current Motor Testing; Soame's Portable Brake.—§16. Motor Testing by the Brake Method.—§17. Motor Testing by the Cradle Method.—§18. Measurement of the Starting and Rotating Torques of Motors.—§19. Alternator Testing, Single-Phase or Polyphase; a Test of a Three-Phase Alternator.—§20. Differential Methods of Testing Alternators; Mordey's Methods.—§21. Alternating-Current Transformer Testing.—§22. Measurement of Secondary Drop of Transformers.—§23. Insulation Test of Transformers.—§24. Temperature Tests of Transformers.—§25. Ageing Tests of Transformers.—§26. Differential Methods of Transformer Testing and Efficiency Measurement by the Direct Measurement of the Internal Losses.—§27. Alternate-Current Motor Testing, Single-Phase.—§28. Polyphase Motor Testing; Two and Three-Phase; Measurement of Power-Factor; References to Papers.—§29. Temperature Tests of Dynamos and Motors.



VOL. II.

## HANDBOOK

FOR THE

# ELECTRICAL TESTING-ROOM AND LABORATORY.

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## CHAPTER I.

---

### THE MEASUREMENT OF ELECTRIC QUANTITY AND ENERGY.

§ 1. **The Measurement of Electric Quantity.**—If an electric current varying in magnitude from instant to instant flows in an electric circuit, the *time-integral of the current*, taken between any assigned time limits, is called the *electric quantity* which has passed through the circuit between the instants defining the limits of the time-integral. Conversely, the current at any instant is measured by the time-variation of the electric quantity passing through the circuit.

In mathematical symbols the above statements are expressed as follows:—Let  $i$  be the current in a circuit at any instant, then  $\int i dt$ , taken between any time limits, is the quantity of electricity,  $q$ , which has, between these time limits, passed through the circuit. Hence also  $i = \frac{dq}{dt}$ .

If the current is constant in value, then the quantity conveyed through the circuit in any time is simply measured by the product of the current and the time.

Electric quantity is practically measured in *coulombs*, and a coulomb is defined as an *ampere-second*, or is the quantity conveyed by a steady or uniform current of one ampere flowing for one second. The absolute unit of electric quantity in the C.G.S. system is equal to 10 coulombs. Another much used unit of quantity is the *ampere-hour*, and is the quantity conveyed by one ampere flowing uniformly for one hour. It is equal to 3,600 coulombs, or to 360 absolute C.G.S. electro-magnetic units of electric quantity.

Small quantities of electricity are generally reckoned in *micro-coulombs*, each of which is the one-millionth part of a coulomb. Larger quantities are reckoned in *coulombs*, *ampere-hours*, or *mega-coulombs*, as may be most convenient. This last unit is equal to one million coulombs.

Instruments for the measurement of electric quantity are called *coulomb meters* or *ampere-hour meters*, according to the magnitude of the quantity they are best fitted to measure. The ballistic galvanometer is adapted for the measurement of very small quantities of electricity, and might, therefore, be called a *micro-coulomb meter*.

If observations are taken of the current passing through a circuit, and if we suppose the current to vary in magnitude more or less gradually, but not to fluctuate too rapidly, the whole quantity passing through the circuit during any time can be approximately estimated by a graphical construction as follows:—

Take a horizontal line on which to mark off time, and starting from any instant assumed as the zero of time, set up ordinates to the base which are proportional in magnitude to the current in the circuit at the instant corresponding to the abscissæ of these ordinates. If, then, we join the tops of all these ordinates (*see Fig. 1*) by a curved or broken line, we obtain a closed area bounded by the two extreme ordinates

representing the currents at the beginning and end of the interval, the time-line or base line, and the broken line connecting the tops of all the current ordinates.

The numerical value of the area included by this boundary represents the electric quantity which has passed through the circuit, provided this area is reckoned in terms of a unit equal to a rectangular area, one side of which is the length taken as the unit of time and the other side the length taken as the unit of current. Thus, suppose the diagram is so drawn that one inch measured vertically is taken to represent one ampere and 0.1 inch measured horizontally is taken to represent one minute; then the area of the diagram in square inches will represent the electric quantity expressed in units, each of which is 10 ampere-minutes.

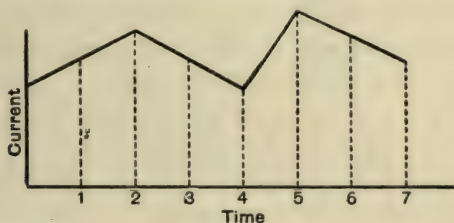


FIG. 1.

In some cases this diagram can be continuously drawn by an appropriate instrument, so that the ordinate of the curve represents at every instant exactly the value of the current then passing. In this case the area included between the base line, the extreme ordinates, and the curve can be obtained by the use of a planimeter, and its numerical value then translated, as above explained, into the proper value of the electric quantity represented by it.

**§ 2. The Ballistic Galvanometer.**—If a current is sent through any circuit for a very short time, so that, whilst the time integral of the current is suitably measured in micro-coulombs, the time itself during which the current lasts is a

small fraction of a second, the quantity thus passing may be measured by the employment of a *ballistic galvanometer*. The current creates round the coil a magnetic field having a brief duration. Suppose that in this field a small magnetic needle is suspended, so that the axis of the needle is initially in a direction at right angles to the direction of the field when the latter makes its appearance; then the needle will be subjected to an impulsive magnetic couple or torque of short duration. It will, therefore, be displaced through an angle, and it will then return to its initial position. This brief angular displacement is called a *throw*.

Again, the circuit may consist of a coil of wire which is freely suspended in a strong magnetic field, so that the direction of the axis of the coil is initially at right angles to the direction of the field. When a transitory current passes through the coil the latter will experience an impulsive torque and be displaced temporarily.

In both cases we have an arrangement which is called a *ballistic galvanometer*. In the first case it is a movable needle ballistic galvanometer, and in the latter case a movable coil ballistic galvanometer.

The mechanical forces acting on the coil or needle at the moment it begins to move and when in motion are then: (i.) An impulsive torque lasting for a very short time and due to the transitory current in the coil. (ii.) An opposing torque, brought into existence by the movement of the coil or needle, due to the torsion of the suspending wires of the movable coil or to the controlling magnetic field in the case of the needle. This opposing torque is proportional in magnitude at any instant to the angular displacement of the needle or coil. (iii.) We have a resistance due to air friction and to the action of eddy currents set up by the moving needle on fixed metallic masses near, or to eddy currents set up in the frame on which the coil is wound, which is proportional very nearly to the angular velocity of the needle or coil at any instant. The *throw* which the coil or needle experiences is

evidently due to the antagonistic actions of the impulsive driving torque and the restoring torque and frictional resistance at any moment.

The investigation of the action of the ballistic galvanometer requires us, therefore, to consider the motion of a heavy body oscillating round an axis and displaced by an impulsive torque; its motion being opposed by a counter torque, proportional at any moment to its actual displacement, and also by a resistance proportional to its instantaneous velocity.

Let  $I$  be the moment of inertia of the oscillating body with respect to the axis of suspension, let  $\theta$  be its angular displacement at any instant, and let  $\alpha\theta$  be the restoring torque and  $\beta\omega$  the frictional resistance, assumed to be at any instant proportional to the angular velocity  $\omega$ . Then, since  $\omega = \frac{d\theta}{dt}$ , we have the following equation of motion for the vibrating body:—

$$I \frac{d^2\theta}{dt^2} + \beta \frac{d\theta}{dt} + \alpha\theta = 0. \quad \dots \dots \dots (1)$$

This differential equation is of a type met with in many physical investigations.

For the sake of avoiding unnecessary fractions and square roots it is better to take  $\beta = 2kI$  and  $\alpha = h^2I$ , where  $I$  is, as above, the moment of inertia of the vibrating body and  $k$  and  $h$  are constants, so that the equation of motion may be written

$$\frac{d^2\theta}{dt^2} + 2k \frac{d\theta}{dt} + h^2\theta = 0. \quad \dots \dots \dots (2)$$

It is shown in treatises on differential equations that the solution of the above type of equation depends upon that of an auxiliary quadratic, viz.,

$$x^2 + 2kx + h^2 = 0. \quad \dots \dots \dots (3)$$

The roots of this quadratic are

$$-k \pm \sqrt{k^2 - h^2}.$$

They are, therefore, both *real* or both *imaginary* according as  $k$  is greater or less than  $h$ .

If  $k$  is greater than  $h$ , then  $\sqrt{k^2 - h^2}$  is a real quantity, and both roots of the quadratic are real; but if  $k$  is less than  $h$  then both roots of the quadratic are imaginary quantities.

Suppose we write  $m$  instead of  $\sqrt{k^2 - h^2}$ , so that  $m^2 = k^2 - h^2$ , and consider the case, first, in which  $k^2$  is greater than  $h^2$ , so that  $m^2$  is a positive quantity. The roots of (3) are, therefore,  $-k + m$  and  $-k - m$ , and the proper solution of (2) is then

$$\theta = A e^{(-k+m)t} + B e^{(-k-m)t},$$

or

$$\theta = e^{-kt} \{ A e^{mt} + B e^{-mt} \},$$

where  $A$  and  $B$  are constants determined by the circumstances of the motion. If the displacement  $\theta$  is reckoned as zero when the time  $t$  is zero, then  $A = -B$ , and

$$\theta = A e^{-kt} \{ e^{mt} - e^{-mt} \}. \quad \dots \dots \dots (4)$$

The graph of this last equation (4) is a curve, as shown in Fig. 2. The curve rises up quickly from the origin to a maximum, and then falls again asymptotically towards the axis. This indicates that, under the assumed

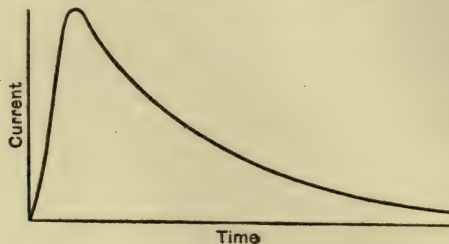


FIG. 2.

conditions, the suspended body, when acted upon by the impulsive torque, will experience a sudden angular displacement, which, however, soon subsides, but in such fashion that the body never makes an oscillation or passes to the other side of the zero position. This type of angular displacement is called a *dead-beat* displacement. It might be imitated by the behaviour of a pendulum hanging in a viscous fluid, such as treacle, or a short-circuited coil of very low resistance and inductance suspended in a strong magnetic field.

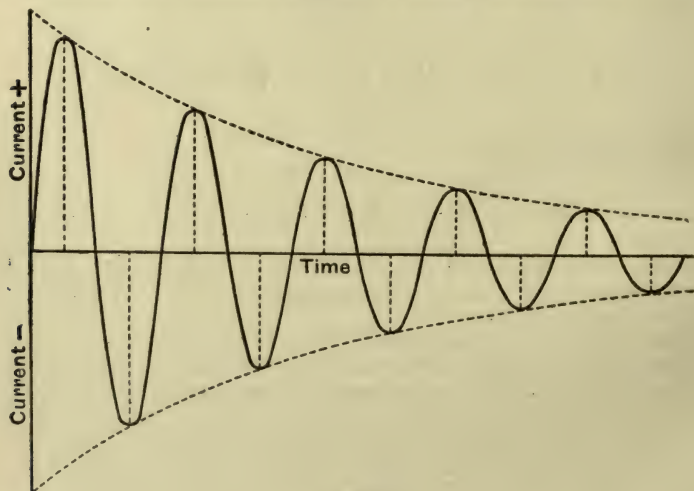


FIG. 3.

The solution shows us, therefore, that it is possible to so determine the constant factors of a ballistic galvanometer that when a sudden discharge is sent through the coil it experiences a sudden angular displacement in one

direction and then returns to its original position, but never passes to the opposite side of the zero position. A ballistic galvanometer so arranged is called a dead-beat galvanometer, and is useful for some purposes in telegraphy.

We have next to consider the alternative case—viz., when  $h^2$  is greater than  $k^2$ .

Let us write  $l$  for  $\sqrt{h^2 - k^2}$ , so that  $l$  is a positive quantity. It can be shown that the proper solution for the differential equation (2) is then

$$\theta = e^{-kt} \{ A \sin lt + B \cos lt \},$$

where  $A$  and  $B$  are again constants determined by the initial conditions of the motion. If we count the time  $t$  from the instant when  $\theta=0$ , the above equation reduces to

$$\theta = A e^{-kt} \sin lt. \quad \dots \dots \dots (5)$$

The graph of this last equation (5) is a curve of the form shown in Fig. 3.

The foregoing investigation indicates that the effect of the impulsive torque is to cause an angular displacement of the movable body, which gradually subsides by a series of decreascent swings or oscillations, the amplitude of each swing being smaller than the previous one. At the moments when the extreme excursions on either side are reached the body is instantaneously at rest. At these instants the angular velocity,  $d\theta/dt$ , is zero. From (5) we have

$$\frac{d\theta}{dt} = l A e^{-kt} \cos lt - k A e^{-kt} \sin lt. \quad \dots \dots (6)$$

Hence, when  $d\theta/dt=0$ , we have

$$\frac{l}{k} = \tan lt \text{ or } t = \frac{1}{l} \tan^{-1} \frac{l}{k}.$$

Since  $\tan lt = \tan (lt + \pi)$ , it follows that at intervals of time equal to  $\pi/l$  the angular velocity has the same value—viz., zero—and the oscillations are therefore isochronous and have a semi-periodic time equal to  $\pi/l$ . Accordingly, we see that, as the time  $t$  increases in arithmetic progression, the maximum excursions decrease in geometric progression. It is obvious that  $\theta=0$ —that is, the displacement is zero at instants corresponding to  $t=0$ ,  $t=\frac{\pi}{l}$ ,  $t=\frac{2\pi}{l}$ , &c., and also that, when

$t=\frac{\pi}{2l}$ ,  $t=\frac{3}{2}\frac{\pi}{l}$ ,  $t=\frac{5}{2}\frac{\pi}{l}$ , &c., we have maximum displacements.

If we call these successive elongations or maximum displacements  $\theta_1, \theta_2, \theta_3$ , &c., then, by equation (5), we have

$$\theta_1 = A\epsilon^{-\frac{k\pi}{2l}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\theta_2 = A\epsilon^{-\frac{3k\pi}{2l}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\theta_3 = A\epsilon^{-\frac{5k\pi}{2l}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence, the ratios  $\theta_1/\theta_2 = \theta_2/\theta_3$  &c., are identical. Accordingly,  $\epsilon^{k\pi/l}$  is the ratio of the two successive maximum displacements in opposite directions.

Corresponding to equal increments of time, each equal to  $\pi/l$ , the successive right and left excursions decrease in the ratio of  $\epsilon^{k\pi/l}$ . The successive amplitudes, therefore, form a decreasing geometric progression, of which the common ratio is  $\epsilon^{k\pi/l}$ , whilst the corresponding times form an arithmetic progression of which the difference is equal to  $\pi/l$ .

We shall denote this time interval  $\pi/l$  by the symbol  $\tau$ , and the logarithm of the ratio  $\theta_1/\theta_2$  to the Napierian base  $\epsilon$  by  $\lambda$ . Then  $\lambda = \frac{k\pi}{l}$  and  $\tau = \frac{\pi}{l}$ , hence  $\lambda = k\tau$  or  $k = \lambda/\tau$ .

The quantities  $\lambda$  and  $\tau$  are, therefore, easily observed. All we have to do is to observe the interval of time between two successive elongations or displacements in opposite directions of the vibrating body, and this interval of time is denoted by  $\tau$ . In the same manner we take the Napierian logarithm of the ratio of two successive elongations in opposite directions, and this is denoted by  $\lambda$ . We can then also express the value of  $h$  in terms of  $\tau$  and  $\lambda$ , for

$$h = \sqrt{l^2 + k^2} = \sqrt{\frac{\pi^2 + \lambda^2}{\tau^2}}.$$

We can also express the initial angular velocity with which the displaced body starts off from its zero position in terms of the same quantities. Call this initial angular velocity  $\Omega$ . Then, putting  $t=0$  in equation (6), we have  $\Omega = A\lambda$ .

Also, from equation (5) we have also

$$\theta = A e^{-kt} \sin lt.$$

In the above equation put  $\theta = \theta_1$ , where  $\theta_1$  is the first maximum displacement. Then, corresponding to  $\theta = \theta_1$ , we have the angular velocity  $d\theta/dt = 0$ , and, as already shown at that instant,  $\tan lt = l/k$ , and therefore

$$\sin lt = \frac{l}{\sqrt{k^2 + l^2}} = \frac{l}{h}.$$

Accordingly, since then  $t = \frac{1}{l} \tan^{-1} \frac{l}{k}$ , we can write the value of  $\theta_1$

$$\theta_1 = \frac{l}{h} A \cdot e^{-\frac{k}{l} \tan^{-1} \frac{l}{k}},$$

or

$$Al = h\theta_1 e^{\frac{k}{l} \tan^{-1} \frac{l}{k}}.$$

But  $k/l = \frac{\lambda}{\pi}$  and  $h = \sqrt{\frac{\pi^2 + \lambda^2}{\tau^2}}$ ; therefore, since the initial angular velocity  $\Omega$  is equal to  $Al$ , we have

$$\Omega = \theta_1 \sqrt{\frac{\pi^2 + \lambda^2}{\tau^2}} e^{\frac{\lambda}{\pi} \tan^{-1} \frac{\pi}{\lambda}} \quad (10)$$

The quantity  $\tau$  is called the *semi-periodic time* and  $\lambda$  is called the *Napierian logarithmic decrement*, and these are easily-observed quantities. Hence, we have all the fundamental quantities  $k$ ,  $h$  and  $\Omega$  determined in terms of  $\tau$ ,  $\lambda$  and known constants.

The semi-periodic time  $\tau$  and the logarithmic decrement  $\lambda$  are observed as follows:—Give the vibrating body an initial impulse and let it settle down into steady decrescent oscillations. Begin to count the time from the instant when the body reaches its greatest displacement in one direction, and take by a good chronometer the time, say, of 10 or 20 complete oscillations. Then this total time, divided by 20 or 40, as the case may be, gives the value of one semi-period—viz., the quantity denoted by  $\tau$  in the above equations.

In the next place, observe the angular excursions of the body right and left by means of a mirror and scale and make a diagram as in Fig. 4, in which the successive lines right and left are drawn to some scale to represent the successive angular deviations  $\theta_1$ ,  $\theta_2$ , &c. These lines are spaced horizontally at equal distances. The extremities of these horizontal

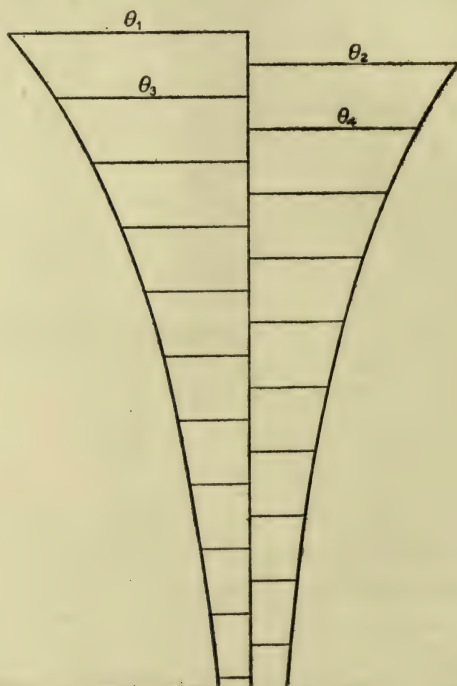


FIG. 4.

lines will delineate an exponential curve, since they decrease in magnitude in geometrical progression. If, then, we set off a similar diagram of lines (*see* Fig. 5), in which the horizontal lines represent the Napierian logarithms of  $\theta_1$ ,  $\theta_2$ , &c., or  $\log_e \theta_1$ ,  $\log_e \theta_2$ , &c., we find that the extremities of these last lines lie in a straight line. The difference of the Napierian

logarithms of two successive swings right and left, or  $\log_{\epsilon}\theta_1 - \log_{\epsilon}\theta_2$ , is the logarithmic decrement denoted by  $\lambda$ .

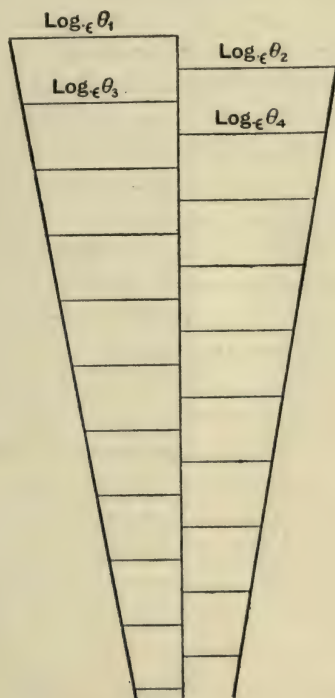


FIG. 5.

§ 3. **Practical Forms of Ballistic Galvanometer.**—As already mentioned, there are two forms of ballistic galvanometer, one in which the coil is fixed and the magnetic needle is the movable body, and the other in which the magnet is fixed and the coil carrying the current is the displaceable body. The movable needle ballistic galvanometer is a troublesome instrument with which to work. For certain purposes it is necessary to give the needle a considerable moment of inertia in order to bestow upon it a sufficiently

large periodic time. The suspended magnetic needle is, then, difficult to bring to rest, and, moreover, it is easily disturbed by electric currents in neighbouring wires. In addition to this, the magnetic moment of the needle is easily altered by the electric discharges sent through the coil, and the galvanometer, therefore, requires constant calibration, and there is no security that it is not altered in the very act of making a measurement. For these reasons the above type of instrument is never employed in electrical testing laboratories at the present time if its use can possibly be avoided.

The form of instrument nearly always now used is the movable coil galvanometer. In this instrument the constant magnetic field is provided by a well-aged steel permanent magnet of suitable form to give a uniform, constant and strong magnetic field. In this field is suspended a coil which is preferably in the form of a shuttle or narrow coil.

Good forms of movable coil ballistic galvanometer are those devised by Ayrton and Mather and by Crompton, of which views and descriptions are given in the first volume of this *HANDBOOK* (*see* Vol. I., Chap. I., pp. 123 and 124). The points to which attention should be paid in selecting a ballistic galvanometer are as follows:—Since it is necessary to be able to vary the resistance and sensibility of the galvanometer, unless a multiplicity of galvanometers can be purchased, it is desirable that the coil portion of the galvanometer should be capable of being readily changed. In the Crompton ballistic galvanometer the suspension is a bifilar wire suspension, and each galvanometer is provided with several coils, which can very quickly be exchanged by simply unhooking the coil in use from the suspension wires and hooking on another. In the Ayrton-Mather ballistic galvanometer this exchange of coil is effected by carrying the shuttle coil and suspension in a brass tube, which fits into the interpolar space of the field magnet and can easily be drawn out and replaced by another tube containing a coil of different resistance or a suspension of different sensibility.

If only one ballistic galvanometer can be purchased it should certainly be provided with a coil having a resistance of about 20 ohms or less and one having a resistance of 500 ohms or more.

In the next place, the suspension wires should be capable of being readily replaced if broken. Instrument makers are generally very indifferent to the question of the time taken to re-string or repair a broken suspension. It is very aggravating to have an experiment interrupted by a galvanometer suspension breaking and then to be compelled to spend a whole afternoon in repairing it. The instrument should be provided with a good concave mirror capable of forming a sharp image of a portion of the filament of an incandescent lamp upon a celluloid engraved scale placed one or two metres away from the galvanometer. The following preparations are then to be made :—

1. *To set up the Ballistic Galvanometer.*—The galvanometer must be placed upon a very steady stone support or table, and preferably on a support quite separate from the table which carries the keys and other apparatus. The position of the coil should be adjusted so that the axis is at right angles to that of the permanent magnetic field and the same small current sent through it successively in opposite directions must make equal right and left deflections. The lamp and scale are then arranged so that the scale is perpendicular to the line joining the centre of the scale and the mirror centre, and at such a distance that the image of the lamp filament is in sharp focus. The scale should be divided into centimetres and millimetres.

The terminals of the galvanometer are then to be connected to a reversing key, preferably of the block and plug form (see Fig. 6). It is convenient to have some means of rapidly bringing the vibrating coil to rest. This is achieved as follows:—A single dry cell of the Leclanché type has connected to one terminal a slip of vulcanised fibre about  $\frac{1}{2}$  in. wide and 4 in. long. This is well rubbed over with

plumbago and two brass clamping screws are fixed to it. This slip forms a very high resistance. In series with the resistance slip is a press key; and the cell, key and slip are connected to the terminals of the ballistic galvanometer. On pressing the key a small current is sent through the galvanometer, and by properly timing the contacts regulated impulses may be given to the galvanometer coil which bring it quickly to rest if swinging, or set it in motion if at rest. This cell, key and resistance enormously facilitate observational work by saving time. If the terminals of the resistance slip are short-circuited by touching them with the tips of the first and little finger of one hand the resistance will be so much lowered that the cell will send a steady current through

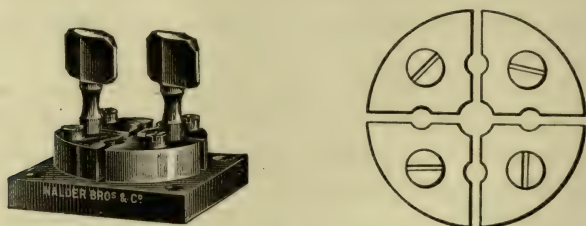


FIG. 6.

the galvanometer, causing a deflection of a few centimetres. This deflection should be the same in both directions when the connections with the galvanometer are reversed.

In addition, it is well to provide a condenser of about one-third of a microfarad capacity and a couple of dry cells and double contact key, so arranged that when the key is up the condenser is charged at the cell terminals, and when the key is down the condenser charge is sent through the galvanometer.

If the cells have a voltage of 1.5, then the third of a microfarad charged with 3 volts will hold about 1 microcoulomb. This quantity sent through the galvanometer should cause a throw of at least 4 or 5 centimetres

reckoned on the scale. If this quantity is sent first one way through the galvanometer and then the other way, it should give equal "throws" right and left. A convenient arrangement is to mount the two dry cells, keys and resistance slip on a board with the reversing plug, so that all operations can be quickly performed.

Having, then, arranged the galvanometer so that we can discharge through it a microcoulomb and obtain a convenient scale deflection or "throw," and then bring the coil quickly to rest, the next step is

2. *To Obtain the Time Period of Oscillation of the Coil.*—We proceed as follows:—Discharge through the galvanometer a quantity of electricity, say of 1 microcoulomb, and set the coil swinging freely. Then, with a good stop watch, note the instant when the scale image reaches its greatest elongation in one direction and is momentarily at rest. Count 20 complete swings and note the time in minutes and seconds occupied by these swings. Then one-twentieth of this time is the complete period of the galvanometer and one-fortieth is the semi-period denoted by the symbol  $\tau$  in the investigation above.

For useful work this time period should not be less than four or five seconds, and preferably as great as 10 or 15 seconds. There is much magnetic work which can only be accurately carried out with a ballistic galvanometer having a very long time period. Having determined the periodic time by taking the mean of a number of observations as above described, the third step is

3. *To Measure the Logarithmic Decrement of the Galvanometer.*—To do this well requires the co-operation of two and, if possible, three observers. The galvanometer coil is given an impulse to set it swinging, and the initial impulse may be such that the first "throw" of the scale spot of light extends to about 15 or 20 centimetres scale deflection on a scale placed at 100 centimetres from the galvanometer. In the course of a swing or two the observers begin to note

the elongations of the scale reading of the "spot" or line upon the scale. One observer confines his attention to the right-hand deflections and the other to the left-hand throws, and the third notes down the values they call out. The best way to take the readings is for the observer to follow the motion of the vibrating line of light or image of the filament, and at the moment when it is at rest at its greatest elongation to touch the scale lightly with a pencil or pointer and then read off at leisure between two swings the value of the excursion. Thus one observer will be calling out the gradually decreasing right-hand throws and the other the diminishing left-hand throws, and the third man will write these numbers down. When this is done the values are to be plotted off as equi-spaced ordinates right and left of a vertical line (*see* Fig. 4), and another diagram prepared from this last one in which the ordinates are the logarithms of the scale readings to the Napierian base. These Napierian values are obtained by multiplying the equivalent common logarithms to base 10 by the modulus  $M=2.303$ . The tops of these logarithmic ordinates should be on a straight line (*see* Fig 5). The logarithmic decrement  $\lambda$  is then the difference of the Napierian logarithms of two successive swings right and left.

If  $x_1$  is the scale value of the first elongation or "throw" to the right-hand and  $x_2$  is the first to the left, then, if also  $x_n$  is the scale value of the  $n$ th throw, the logarithmic decrement  $\lambda$  is found by taking the value

$$\lambda = \frac{1}{n-1} \{\log_e x_1 - \log_e x_n\}.$$

The *correcting factor for damping* is then the quantity  $1 + M \frac{\lambda}{2}$ , where  $\lambda$  is the logarithmic decrement taken for logarithms to the base 10 and  $M$  is the constant 2.303.

This factor is employed to correct the observed value of the first throw or swing of the ballistic galvanometer to obtain the value which would be observed if the logarithmic decrement were zero—in other words, if there were no

retardation or resistance to the motion of the coil by air friction or magnetic reactions.

The reason for using the factor  $1 + \frac{\lambda}{2}$ , where  $\lambda$  is the Napierian logarithmic decrement is as follows:—If the frictional resistances reduce the initial throw from  $x_1$  to  $x_2$  in the course of one-half complete oscillation, so that

$$\log_e x_1 - \log_e x_2 = \lambda,$$

then we may reasonably infer, since  $x_1$  and  $x_2$  are not very different, that, if all frictional resistances had been absent, the initial throw (call it  $x_o$ ) would have been greater than  $x_1$ , which is the actually observed first throw, in such degree that

$$\log_e x_o - \log_e x_1 = \frac{\lambda}{2}.$$

Accordingly,

$$\frac{x_o}{x_1} = e^{\frac{\lambda}{2}}.$$

But, by the exponential theorem,

$$x_o = x_1 \left( 1 + \frac{\lambda}{2} + \frac{1}{1} \frac{\lambda^2}{2} + \frac{1}{1 \cdot 2 \cdot 3} \frac{\lambda^3}{8} +, \&c. \right)$$

Hence, if  $\lambda$  is a small quantity such that  $\lambda^2$  and higher powers may be neglected in comparison with  $\lambda$ , we have

$$x_o = x_1 \left( 1 + \frac{\lambda}{2} \right);$$

or, if we are using common logarithms,

$$x_o = x_1 \left( 1 + M_2^{\lambda} \right).$$

The same result can be reached by considering the equation (9) (*see* above) for the initial angular velocity  $\Omega$ . If  $\lambda$  is a small quantity, then  $\pi/\lambda$  is very large, and  $\tan^{-1} \pi/\lambda = \frac{\pi}{2}$ . Hence the expression for  $\Omega$ , or the initial angular velocity, becomes

$$\Omega = \frac{\pi}{\tau} \theta_1 \left( 1 + \frac{\lambda}{2} \right),$$

or  $\left( 1 + \frac{\lambda}{2} \right)$  is a correcting factor by which  $\theta_1$  has to be

multiplied to obtain the throw  $\theta_0$  if frictional resistance were absent.

In actual practice the quantity  $M\lambda$ , where  $\lambda$  is the ordinary logarithmic decrement and  $M=2.303$ , should not exceed 0.05 or 0.06, so that  $\left(1 + M\frac{\lambda}{2}\right)$  has a value of about 1.025 or 1.03.

When this is the case the foregoing investigation applies, and we may correct sufficiently nearly the observed initial throw  $\theta$  for frictional resistance to obtain the frictionless throw by multiplying by  $\left(1 + M\frac{\lambda}{2}\right)$ .

The above constants having been observed, we can then proceed to obtain the *ballistic constant* of the galvanometer. This constant depends, however, in value upon the mode in which the galvanometer is being used. If it is to be employed to receive a condenser discharge, the galvanometer coil remaining upon open circuit during and after discharge, we can proceed as follows:—

4. *To Obtain the Ballistic Constant of the Galvanometer.*—The ballistic constant is defined as the number or constant by which we must multiply the throw of the galvanometer to obtain the quantity of electricity discharged through it. Hence it is determined if we know the throws corresponding to the passage of certain ascertained quantities. It must not, however, be assumed too readily that the ballistic “constant” is the same for all deflections or throws.

To determine the galvanometer ballistic constant on open circuit and its variation the following operations have to be performed:—Provide a condenser, the capacity of which can be varied, or several condensers of known capacity, which can be combined so as to make a variable total capacity. Charge these condensers with a known voltage for a given time and discharge this charge through the galvanometer. The capacity and voltage must be so varied that the throws extend from small value up to the largest the scale will conveniently receive. Thus, suppose the observer provided

with a graded condenser having capacities available of 5·0, 2·0, 1·0, 0·5, 0·25 microfarad and a single large Clark or Weston cell. Also a double contact condenser key, which enables him to charge the condenser at the cell terminals and discharge it through the galvanometer. Suppose the 1·0 microfarad to be charged at the Clark cell at 15°C. Then the quantity of electricity put into the condenser is 1·434 microcoulombs. Let this charge be sent through the galvanometer and produce a scale deflection or throw of  $\theta_1$  centimetres. Then, if  $\lambda$  is the observed ordinary logarithmic decrement and  $C$  is the ballistic constant, we have

$$1\cdot434 = C\theta \left(1 + 2\cdot303\frac{\lambda}{2}\right),$$

from which the value of  $C$  can be obtained. Next, let the same observation be repeated, using the 2·0 microfarad capacity, and in like manner, if we obtain a deflection or throw  $\theta_2$ , we have

$$2\cdot868 = C\theta_2 \left(1 + 2\cdot303\frac{\lambda}{2}\right).$$

After obtaining thus a series of increasing “throws” corresponding to various increasing quantities of electricity, we can set out a curve showing the variation of  $C$  with  $\theta$ , or else of the quantity  $Q$  discharged through the galvanometer and the deflection  $\theta$  produced by it. This curve will not always be a straight line, but from it we can obtain at once the value in micro-coulombs of any unknown quantity of electricity discharged through the galvanometer.

It must be particularly noticed, however, that the ballistic constant so obtained applies only to the galvanometer when its throw is taking place with the coil circuit open—that is, not metallically continuous. We cannot apply the result so obtained to determine the quantity of electricity discharged through the galvanometer when its terminals are connected to a closed coil, into which a certain magnetic flux is suddenly inserted. In this latter case also the “damping,” or retardation of the coil during its swing is quite different and much

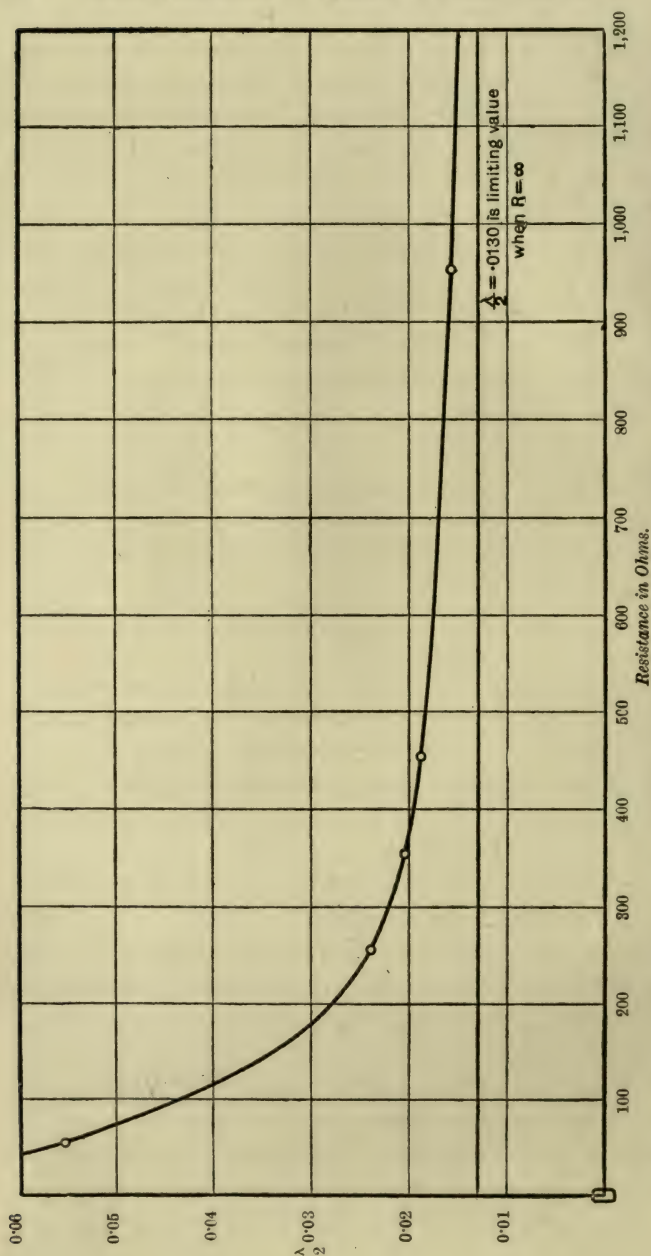


FIG. 7.—Typical Curve Showing the Increase in the Value of the Half Logarithmic Decrement  $\frac{\lambda}{2}$  with Decrease of Resistance in the Circuit of a Movable Coil Galvanometer.

greater than that which exists when the coil is swinging on open circuit. The curve in Fig. 7 shows the manner in which the logarithmic decrement of a movable coil galvanometer of a certain type was found to increase as the external resistance of its circuit decreased.

In dealing with the ballistic galvanometer as employed for the measurements of magnetic flux this matter will be again discussed.

The ballistic constant can be approximately determined by another method as follows:—If the duration of the discharge is so short that the coil has not time to move from its zero position before the discharge is finished owing to its inertia, then the coil is subjected to an angular impulse or impulsive torque, which is measured by the product  $I\Omega$ , where  $I$  is the moment of inertia of the coil and  $\Omega$  is the initial angular velocity the value of which has been given in terms of other quantities in equation (10), p. 9.

Again, the kinetic-energy with which the coil leaves its position of rest is expressed by  $\frac{1}{2}I\Omega^2$ . Suppose  $\mu$  is the torsional elasticity of the suspension and  $\theta$  is the throw of the coil, then  $\mu$  is the torque required to produce an angular deflection of unity, or otherwise  $\mu$  may be defined as the ratio between the torque and the twist when the twist is small. Hence the restoring torque brought into play when the coil is twisted through an angle  $\theta$  is  $\mu\theta$ , and the potential energy of the coil when at the extremity of the swing is  $\frac{1}{2}\mu\theta^2$  if  $\theta$  is assumed to be small. Hence, if there is no retardation by air or magnetic friction, we have,

$$\frac{1}{2}I\Omega^2 = \frac{1}{2}\mu\theta^2,$$

or

$$\sqrt{\frac{I}{\mu}} = \frac{\theta}{\Omega}.$$

But if  $T$  is the complete periodic time of the coil when swinging freely in small oscillations, then

$$\sqrt{\frac{I}{\mu}} = \frac{2\pi}{T}.$$

Hence, 
$$\frac{\theta}{\Omega} = \frac{2\pi}{T},$$

or 
$$\Omega = \frac{T}{2\pi} \theta,$$

and 
$$I = \mu \frac{4\pi^2}{T^2}.$$

Again, let us suppose a small, steady current to be sent through the coil by applying to it a voltage,  $V$ , through a resistance,  $R$ , which includes that of the coil itself. Then, if  $G$  is the magnetic moment of the coil when unit current flows through it, and if the angular deflection of the coil produced by this steady current is  $\phi$ , and if the permanent magnetic field of the galvanometer is  $B$ , we have

$$\frac{V}{R} = \frac{\mu}{GB} \tan \phi.$$

For the opposing torques are then  $\frac{V}{R} GB \cos \phi$  and  $\mu \theta$ , which, when  $\theta$  is small, can be taken to be equal to  $\mu \sin \theta$ .

Furthermore, suppose a quantity of electricity,  $Q$ , to be discharged through the galvanometer ballistically, then  $Q$  is the time-integral of the discharge current, and, since  $I\Omega$  is the angular momentum with which the coil leaves its position of rest, this last must be equal to the time-integral of the torque acting on the coil at the various instants during the discharge. Now, if  $i$  is the discharge current at any instant and  $B$  is the field-magnet flux density of the galvanometer or interpolar field, the torque acting on the coil at any instant is equal to  $GBi$ . Hence the time-integral of  $GBi$  or  $\int GBi dt = \int GB dq = GBQ$ , must be equal to the initial angular momentum  $I\Omega$ . Accordingly, we have

$$I\Omega = GBQ,$$

or 
$$Q = \frac{I}{GB} \Omega.$$

Then, since, as shown above,  $\Omega = \frac{T}{2\pi} \theta$  and  $I = \mu \frac{4\pi^2}{T^2}$ , we have

$$Q = \frac{T}{2\pi} \frac{\mu}{GB} \theta.$$

But, again, as already shown, we have  $\frac{V}{R} = \frac{\mu}{GB} \tan \phi$ .

Hence, 
$$\frac{\mu}{GB} = \frac{V}{R \tan \phi}.$$

Therefore, 
$$Q = \left( \frac{T}{2\pi} \cdot \frac{V}{R \tan \phi} \right) \theta. \quad \dots \dots (11)$$

The ballistic constant  $C$  is, therefore, equal to

$$\frac{T}{2\pi} \frac{V}{R \tan \phi},$$

where  $T$  is the complete periodic time of the coil, and  $R$  is the resistance, including that of the coil, through which a steady voltage,  $V$ , creates such a current that the coil is deflected through an angle  $\phi$ .

It is useful to compare the values of the ballistic constant  $C$  determined in the above manner with the ratio  $Q/\theta$  determined from the throw  $\theta$  produced by the ballistic discharge of a quantity of electricity,  $Q$ .

Hence, if  $\lambda$  is the ordinary logarithmic decrement of a ballistic galvanometer, we may determine by it the quantity,  $Q$ , of a discharge from the relation

$$Q = \frac{T}{2\pi} \frac{V}{R \sin \phi} \theta_1 \left( 1 + M \frac{\lambda}{2} \right), \quad \dots \dots (12)$$

or, from 
$$Q = C \theta_1 \left( 1 + M \frac{\lambda}{2} \right), \quad \dots \dots (13)$$

where  $C$  is the ballistic constant determined by sending condenser discharges of known quantity through the coil, and where  $T$ ,  $V$ ,  $R$  and  $\phi$  are determined by experiments on the time period of the coil and the effect of a steady voltage,  $V$ , in causing deflection  $\phi$  of the coil of the galvanometer.

The ballistic galvanometer, so calibrated and used, becomes available as a microcoulomb meter or measurer of small electric quantity.

**§ 4. Coulombmeters, Voltameters and Ampere-hour Meters.**—We have to consider in the next place the appliances and processes for measuring larger quantities of

electricity. The instruments for making these measurements are generically called *coulombmeters* or *ampere-hour meters* and may be classified as follows:—

Coulombmeters may be either

- (i.) Voltameters or electro-chemical ampere-hour meters,
- (ii.) Mechanical ampere-hour meters.

Mechanical ampere-hour meters may be either

- (a) Graphic ammeters,
- (β) Intermittently recording ampere-hour meters; or,
- (γ) Continuously recording instruments.

The term “voltmeter” is applied generally to any instrument in which the passage of a current of electricity through an electrolyte causes a chemical change which can be measured, and by which the time-integral of the current is measured. By Faraday’s first Law of Electrolysis the amount of chemical decomposition produced in any electrolyte by the passage of a current through it is exactly proportional to the time-integral of the current. The chemical decomposition produced may be estimated by the weight or volume of either or both ions into which the electrolyte is divided. In practice the electrolyte chosen for laboratory purposes is either (i.) a 10 per cent. aqueous solution of sulphuric acid; (ii.) a saturated aqueous solution of cupric sulphate, with 5 per cent. by volume of sulphuric acid added to it; or (iii.) a neutral saturated aqueous solution of nitrate of silver.

The instrument or vessel containing the electrolyte is called the *voltameter*, and the conductors by which the current is passed into it and through it are called the *electrodes*. In the case of the dilute sulphuric acid or so-called water voltmeter the electrodes are slips of platinum foil, in the case of the copper-sulphate voltmeter they are both plates of pure copper, and in the case of the silver nitrate voltmeter one (the anode) is a plate of silver, and the other (the cathode) is a platinum bowl or plate.

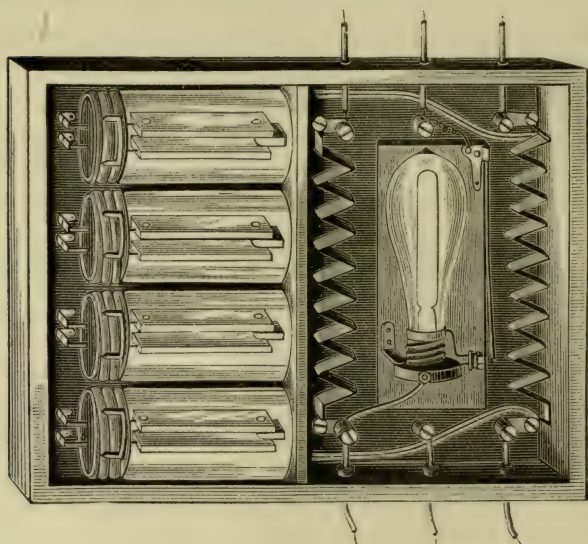
The instructions for using the copper and silver voltameters for the measurement of electric quantity have been given in full in the first volume of this HANDBOOK, and need not, therefore, be repeated here. (*See* Vol. I., Chap. I., § 6, and Chap. III., §§ 2 and 3.)

In employing either of the above metallic salt voltameters attention must be paid to the current density as explained in the instructions, or else the deposited metal will be non-adherent to the cathode and the accurate determination of its weight rendered very difficult.

A commercial form of metallic salt voltmeter, which has been extensively employed in the case of continuous currents, is the Edison electrolytic house meter (*see* Fig. 8). In its standard form it consisted of a glass bottle or vessel, or of two glass vessels, containing a 10 per cent. solution of pure zinc sulphate having a specific gravity of 1.11 at 15°C., and is only used once. In this solution is placed a pair of amalgamated plates cast from an alloy of zinc and mercury. These are fixed at a certain distance apart by means of an ebonite screw and distance piece. In series with this voltmeter cell is a small copper wire resistance called the *compensating coil*. The voltmeter cell and compensating coil are attached to the ends of a strip of German silver, called the shunt. The current to be measured passes mainly through the shunt, but a fraction (about one-thousandth) is sent through the voltmeter cell. The function of the compensating coil is to keep the shunt ratio constant, and it is so adjusted that any rise in temperature increases the resistance of the copper coil by the same amount by which it reduces that of the zinc sulphate voltmeter cell.

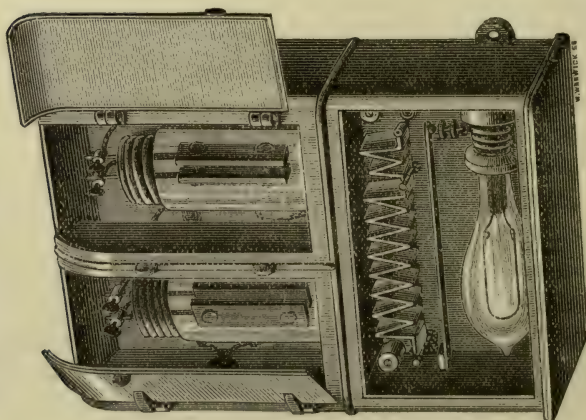
The amalgamated zinc plates are first weighed and then placed in the cell, and after the passage of a certain quantity of electricity they are weighed again. One plate is found to have gained and the other to have lost in weight, and from the gain in weight of the cathode the total quantity of the current which has passed can be estimated. In order to

check mistakes a duplicate voltameter and shunt are provided in each meter box.



Edison Electrolytic Three-Wire Meter.

FIG. 8.



Edison Electrolytic Two-Wire Meter.

The passage of one ampere hour between zinc plates immersed in zinc sulphate removes from one plate and deposits

on the other 1,224 milligrammes of zinc. In the Edison meter the resistance of the German silver shunt and that of the compensating coil and bottle circuits are so proportioned that for every ampere-hour of quantity passing through the meter one milligramme of zinc is transferred in the electrolytic cell of a 10-ampere meter, half a milligramme in a 20-ampere meter, and a quarter of a milligramme in a 40-ampere meter. These sizes of meter are called, respectively, the No. 1, No. 2 and No. 4 sizes.

The following table gives the resistances of the compensating coils, shunts and electrolytic cells in the various sizes:—

*Edison Electrolytic Meters.*

Size.	Amp.-hours passing per Milligramme of Zinc transferred.	Meter Number.	Ampere Capacity of Shunt.	Resistance of German Silver Shunt.	Resistance of Compensating Coil at 70°F.	Resistance of Cell at 70°F.
1	1	1	10	0·04 ohms.	46·46 ohms.	2·5 ohms.
2	2	2	20	0·02   "	"	"
4	4	4	40	0·01   "	"	"
8	8	8	80	0·005   "	"	"

As the zinc sulphate solution freezes at 27°F., an arrangement called a *thermostat* is provided in each meter. This consists of a slip of compound metal (steel and brass) which bends, when heated or cooled, by the unequal expansion of the two metals of which it is made. The bending of the slip is made to close a circuit and light up an incandescent lamp inside the meter, if the temperature falls below freezing-point. The contact points are adjusted as to distance by a set screw, and a difference of temperature of 10°F. will move the contact point on the slip about  $\frac{1}{48}$ th of an inch. Each meter box is provided with a duplicate bottle, coil and shunt, so that mistakes in weighing plates may be detected by the difference between the readings taken from the two pairs of plates.

In the use of this meter care has to be taken that the current density does not exceed the proper value, or else the deposit of zinc will be non-adherent.

The rules as to load are as follows :—

Meters may be worked at	Hours per day.
Full capacity .....	3 hours.
$\frac{3}{4}$ of full capacity .....	4    "
$\frac{1}{2}$ "    "    .....	5    "
$\frac{1}{4}$ "    "    .....	8    "
$\frac{1}{8}$ "    "    .....	Continuously.

The metallic salt voltameter has been introduced in two other forms as a commercial ampere-hour meter in the

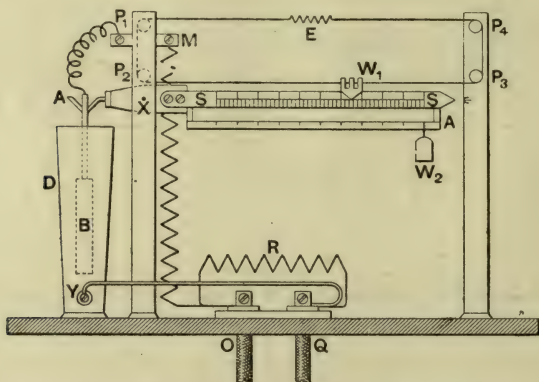


FIG. 9.—Long-Schnattner Standard Meter. D, Electrolytic Cell. B, Copper Anode. W<sub>1</sub>, W<sub>2</sub>, Sliding Weights. A, Balance Arm.

Long-Schattner standard meter and prepayment meter (*see* Figs. 9 and 10). In these instruments the current of which the quantity or time-integral is to be determined flows through a solution of sulphate of copper of specific gravity 1·080 with 1 per cent. by volume of free sulphuric acid added, contained in a copper vessel, in which is suspended a copper plate or cylinder. This latter is suspended from a steel-yard arm, along which, in the standard meter, a weight can

slide so as to determine the weight of the copper plate without removing it from the liquid. In the prepayment form this arm is provided with a mercury circuit-closing switch, which opens the main circuit when the steel yard-arm tilts up. In the prepayment meter the arm also carries a pocket into which a coin (say a shilling) can be dropped. The weight of the coin depresses the arm and closes the main circuit. The current then flows through the copper-sulphate cell in such a direction as to take copper off the suspended plate. When copper has been removed equal in weight to that of the coin the arm is again in equilibrium, and the smallest further passage of current will cause it to tilt up and insert a resistance into the circuit which dims the lights. This resistance can only be removed again so as to light up

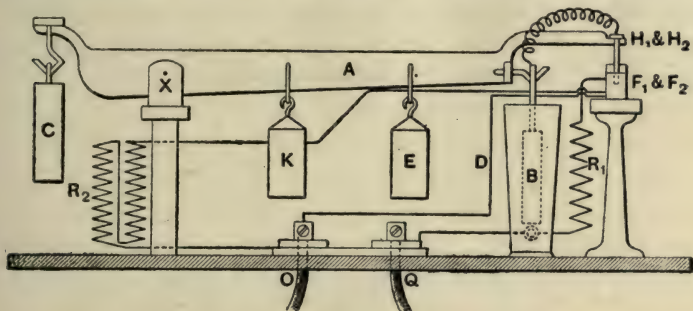


FIG. 10.—Long-Schnattner Prepayment Meter. D, Electrolytic Cell. B, Copper Plate or Anode. A, Balance Arm. K and E, Pockets to hold Coins,  $F_1, F_2$ , Switch.  $R_2$ , Resistance inserted in Circuit.

the lamps fully or supply the right voltage by dropping in another coin, when the process is repeated. The working parts of the meter are included in a sealed metallic case having an aperture for introducing the coins. It is applied as a house meter to enable a consumer to prepay for a given electric quantity, and then interrupt his light service when that quantity has been taken.

If the electrolytic cell takes only a fraction of the whole current passing, then it has to be coupled in series with

a copper compensating coil, and the two in parallel with a platinoid shunt, as in the case of the Edison meter. Metallic salt voltmeters of the above type should be tested as follows, before employing them in practice as house quantity meters:—

The meter to be tested should be joined up in series with a tested ammeter having the necessary range of readings and known currents passed through the voltmeter and the times of flow observed. Observations should be made with constant unidirectional currents of different values to ascertain if the meter readings obtained by weighing the plates are correct, and if the meter records the same reading for the same electric quantity, however the factors of time and current may be varied. Attention should be directed to the question of the range of current over which the meter will give useful readings, and within which the metallic deposit is adherent; also to the time period during which the meter can remain in action without re-weighing or adjustment. If it is a shunted voltmeter, an investigation should be made to ascertain how its readings are affected by variations of temperature within atmospheric ranges, and if it can be put out of action by the electrolyte freezing. The percentage accuracy of the meter must be ascertained for various currents and times. Another point which should receive attention is the drop in voltage which takes place through the meter for various currents, and the meter should be examined to see if any back E.M.F. due to polarisation exists.

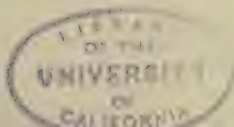
Theoretically, there should be no back E.M.F. due to polarisation in a zinc-sulphate of zinc-zinc cell, but, as a matter of fact, in practice the Edison meter is found to show about 0.0085 of a volt back E.M.F. The error thus produced is negligible at 10 or 20 amperes, but at low loads of 1 ampere it introduces an error of 8.5 per cent. as the total drop in the meter was only 0.01 volt at this load. (*See J. R. Dick, "Electricity Supply Meters of the Electrolytic Type," Proc. Inst. Elec. Eng., Vol. XXXI, p. 97.*)

No meter is useful as a constant voltage house-quantity meter which will not record the time-integral of currents as small as that of one 5 c.p. lamp used at the voltage of the house, when the time of usage is at least 6 hours; in other words, will record at least 0.1 of a kilowatt hour.

In another form of voltameter the measurement of current is made to depend upon the electrolysis of dilute sulphuric acid, yielding oxygen and hydrogen gases, which are collected and measured at known temperature and pressure. This instrument is sometimes called the *water voltameter*. The passage of a current through it should yield as the liberated ions nothing but pure oxygen and hydrogen gases, in the ratio, by volume, of 1 to 2. As a matter of fact, some secondary products are formed, such as ozone, which make the quantity of oxygen less than its theoretical amount. In addition, oxygen gas is slightly more soluble in water or dilute acids than hydrogen. Hence, instead of collecting both gases mixed together or both separately, it is better to collect and evaluate only the hydrogen, and the instrument might thus be called a *hydrogen voltameter*.

One form of the instrument which may be used in the laboratory is called Hoffmann's voltameter. It consists of a glass tube of **H**-form (*see* Fig. 11), the two limbs having glass taps at the top and having indiarubber corks through which pass platinum wires at the bottom. These wires have platinum plates welded to them. The whole apparatus is first filled with a 10 per cent. aqueous solution of sulphuric acid by raising the reservoir and opening the taps. The limb in which the hydrogen is to be collected should be surrounded with a wider glass tube filled with water in which is placed a thermometer. Before filling the instrument, a known volume space should be marked off in the hydrogen limb. This is done by inverting the instrument and placing in that limb a known weight of pure mercury at a known temperature.

It is convenient to mark off a volume, say, of 50 or 100 cubic centimetres. We proceed then as follows:—The



density of pure mercury at  $0^{\circ}\text{C.}$  is 13.596, and its absolute expansion, or increase in volume by increase of temperature, can be calculated by the following formula adopted by the Bureau International des Poids et Mesures, viz. :—

$$v_t = v_0 (1 + 0.000181792t + 0.00000000175t^2),$$

where  $v_0$  is the volume at  $0^{\circ}\text{C.}$  and  $v_t$  the volume at  $t^{\circ}\text{C.}$  Hence, if  $W$  be the weight in grammes of a mass of pure mercury at

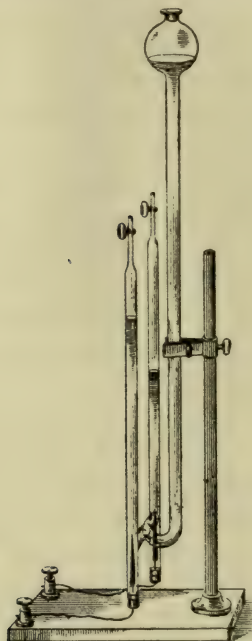


FIG. 11.—Hoffmann's Water Voltameter.

$t^{\circ}\text{C.}$  the volume  $v_t$  of that mercury is given by the equation,  

$$v_t = \frac{W}{13.596} (1 + 0.0001818t),$$
 sufficiently nearly for the present purposes.

Hence, knowing the temperature ( $t^{\circ}\text{C.}$ ) of the room in which it is weighed, we can determine the volume of a known weight of mercury at  $t^{\circ}\text{C.}$

This mercury is then poured into one limb of the voltmeter when held in an inverted position and a scratch made with a diamond on the glass, marking the upper surface of the mercury. The glass tube should be of sufficient diameter to make any error due to the convex meniscus of the mercury surface negligible. Otherwise the glass tube may be graduated right away down in cubic centimeters and tenths in the fashion of a burette or gas analysis tube.

In any case the exact cubic contents or volume of one limb between the top and certain marks on it must be known.

The voltameter having been filled with dilute acid, a current is passed through it by means of the platinum plates so that the plate under the graduated limb is the cathode. Gas will then accumulate in both tubes and force back the electrolyte into the reservoir. The exact time taken to depress the liquid in the cathode limb down to the known volume mark must be taken in minutes and seconds. The temperature of this hydrogen gas is then taken by the thermometer, and its pressure by noting the height of the barometer and also by measuring the difference of level between the surface of the electrolyte in the reservoir and that in the cathode limb.

Let  $t^\circ$  be the centigrade temperature,  $H$  the height of the barometer in millimetres,  $h$  the difference of level also in millimetres,  $d$  the density of the dilute acid or electrolyte taken at the temperature of the wire, and  $D$  the density of mercury at the same temperature. The values of  $d$  and  $D$  must be ascertained by experiment or taken from tables. The density  $D$  of mercury at  $t^\circ\text{C.}$  can be calculated from the formula

$$D = \frac{13.596}{1 + 0.0001818t}$$

The density  $d$  of the electrolyte at  $t^\circ\text{C}$  must be obtained by the use of the specific gravity bottle or by weighing a known volume of it. The pressure  $p$  under which the collected volume of hydrogen exists is then equal to the weight of  $H + \frac{d}{D}h$  millimetres of mercury, and its temperature is  $t^\circ\text{C.}$

Let the apparent collected volume be  $V$  cubic centimetres—that is, the volume of the cathode limb down to the scratch on the tube.

We have then to calculate the volume,  $V_{o, 760}$ , of this hydrogen gas at  $0^\circ\text{C}$ ., and 760mm. pressure, and ascertain its weight,  $W$ . We have

$$V_{o, 760} = V_{t, p} \cdot \left\{ \frac{273}{273 + t} \cdot \frac{H + \frac{d}{D}h}{760} \right\}.$$

Then, since the weight of 1,000 cubic centimetres of hydrogen gas at  $0^\circ\text{C}$ . and 760 is 0.08957 of a gramme, we have

$$w = 0.00008957 V_{o, 760},$$

where  $w$  is the weight in grammes of a volume  $V_{o, 760}$  reckoned in cubic centimetres.

Now the electro-chemical equivalent of hydrogen in grammes per coulomb is 0.00001038. In other words, this is the weight in grammes of the hydrogen liberated at the cathode for every coulomb of electric quantity passing through the voltameter. Hence the quantity, in coulombs,  $Q$ , which has passed through the voltameter to effect this evolution of the observed volume of gas of which the weight is  $w$  is

$$Q = \frac{w}{0.00001038}.$$

Combining all the corrections into one formula, we have

$$\begin{aligned} Q &= \frac{0.00008957}{0.00001038} \frac{273}{273 + t} \frac{H + \frac{d}{D}h}{760} V_{t, p} \\ &= 3.1 \frac{H + \frac{d}{D}h}{273 + t} V_{t, p}, \end{aligned}$$

where  $Q$  is the time-integral of the current in coulombs, and  $V_{t, p}$  the observed volume in cubic centimeters of the hydrogen evolved at  $t^\circ\text{C}$ . and under a pressure  $p = H + \frac{d}{D}h$  millimeters.

A practical and commercial form of non-shunted water voltmeter employed as a house ampere-hour meter is known as Bastian's electrolytic meter (see Fig. 12). In this house meter

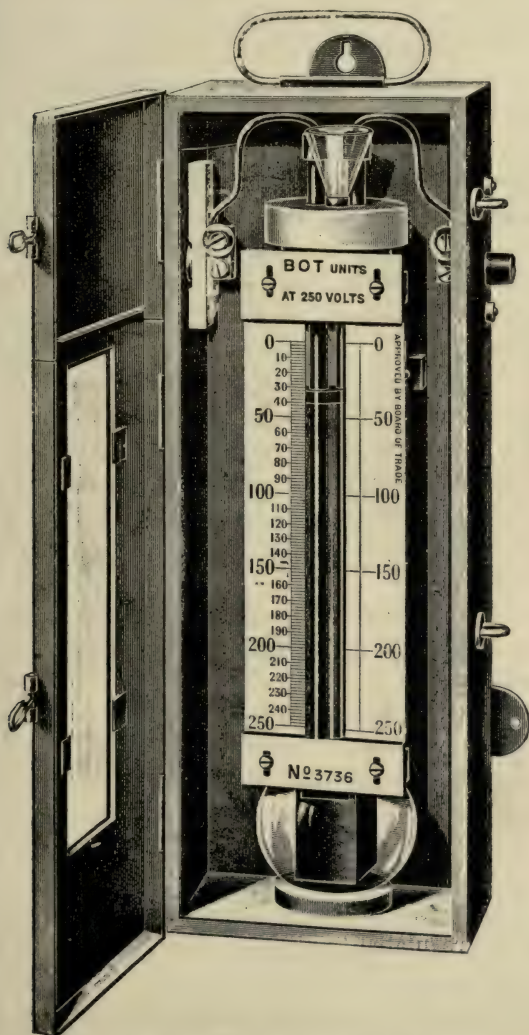


FIG. 12.—Bastian Electrolytic House Meter.

the current to be time-integrated passes through a vessel in the form of a large test tube, closed at the bottom, having two platinum electrodes carried on an ebonite support placed at the bottom and having two insulated wires led down the tube to the electrodes. The tube is filled with dilute sulphuric acid 1 : 10, and on the top is placed a little paraffin oil to prevent evaporation. The current to be measured passes through the electrolyte and decomposes it. The oxygen and hydrogen gas escapes and the volume of the electrolyte is diminished. This diminution of volume is proportional to the time-integral of the current. Hence the electric quantity is measured not by the quantity of the ions which have appeared but by the volume of the electrolyte which has disappeared. The gases escape through the layer of paraffin oil. There are no complication of shunts or compensating coils or weighing of electrodes. The decrease in volume of the electrolyte is read off at once on a graduated scale placed by the side of the tube. This graduation is primarily a scale of ampere-hours, but if the voltage of supply is constant it becomes, or can be made, a scale of Board of Trade units. The tube requires occasionally to be filled up with water to keep the acid at proper density.

A 5-ampere meter, working under full load for two hours, evolves something less than a fourth of a cubic foot of mixed oxygen and hydrogen. The meter is, of course, only adapted for continuous currents, but it can be used either with regular or but slightly variable currents, or with sudden rushes of large currents as in the case of motors frequently started, provided that these currents are unidirectional.

The meter has a back electromotive force of nearly 2 volts, hence it cannot well be used for taking the time-integral of a current flowing under a very small voltage unless the impressed voltage can be regulated to allow for the unavoidable drop in voltage in the meter.

Another ingenious form of electrolytic meter of the shunted voltameter type which deserves mention is that

invented by Mr. A. Wright (*see* Fig. 13). In this meter the electrolyte is a solution of mercurous nitrate which is electrolysed in a completely closed glass tube of particular

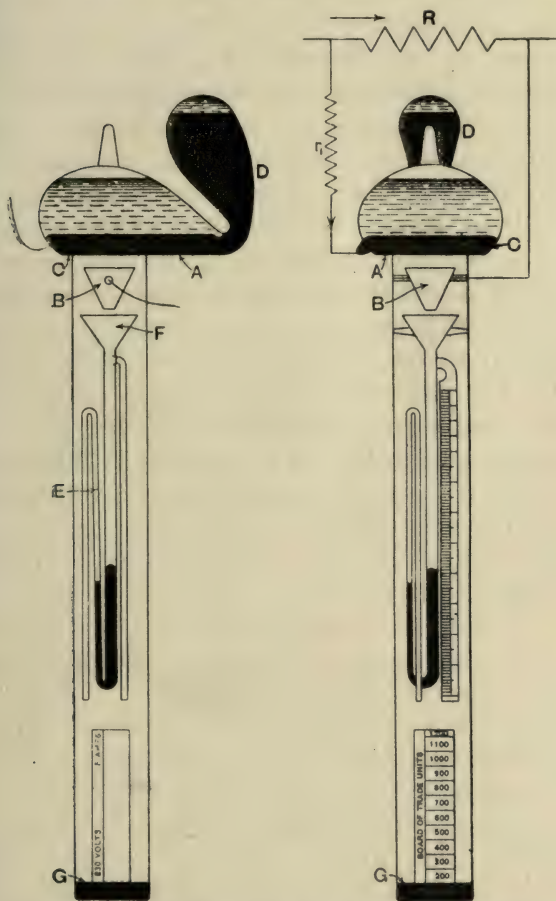


FIG. 13.—Wright's Electrolytic Ampere-Hour House Meter

form between a mercury anode, C, and a platinum or carbon cathode, B. The current is metered by measuring the volume of the mercury delivered at the cathode, which

drops from it into a collecting funnel tube, F, and the meter is re-set for use by tilting it so as to convey this mercury back to the anode vessel. For additional information on the Wright mercury meter, the reader may refer to a Paper by Mr. J. R. Dick, "Electricity Supply Meters of the Electrolytic Type" (see *Proc. Inst. Elec. Eng.*, Vol. XXXI, p. 95, 1901).

Considering, in the next place, the mechanical ampere-hour meters, one class of these may be called the *graphical ammeters*, such as the recorders of Holden-Pitkin, Mengarini, Elliott, Richard and others. In these instruments there is a paper-covered drum driven by clockwork which revolves in 24 hours, or any other period, and against this drum an ink pen presses which is attached to the needle of an ammeter. When no current is passing the ammeter pen draws a straight line as the paper on the drum revolves. When a current passes the pen is displaced by a distance proportional to the current, and a curve is drawn on the paper the ordinate of which is proportional to the current, and its abscissa to the time. Hence, if the area enclosed by the curve between any two time limits is taken by a planimeter in terms of a rectangular unit of area, one side of which is the length representing an ampere and one side the distance representing one hour, the area of the curve in terms of this unit area will be the quantity of electricity which has passed through the meter in ampere-hours.

In testing such a graphical ammeter it is necessary to make an independent test of the ammeter part to see if it records current correctly, and of the clock part to see if it records duration or time correctly. The integration of the area may be performed by an Amsler's planimeter, or, in default of this, may be roughly done by cutting out the paper along the line of the curves and weighing it and weighing also a rectangle of the same paper one side of which has a length equal to that taken to represent one ampere and an adjacent side the length taken to represent one hour. The ratio of the areas may be roughly considered as the ratio of the weights.

A graphical meter of this description is very useful in tests on secondary batteries, as it may be set to record automatically the ampere-hours taken out of a battery, and at the same time it gives the actual discharge current during every instant. Hence the capacity can be reckoned within the desired limits as to current discharge.

In the next place, there are a series of ampere-hour meters which may be described as *intermittent self-integrating ampere-hour meters*, also called "feeler" meters. In these instruments there is a part which is an ammeter or galvanometer, and a part which consists of a clock and counter. The instrument when in use performs the following operations:—The clock part takes, so to speak, a reading of the ammeter at small equidistant intervals of time, and adds up all these readings on the counter. Thus, if  $A$  is the current at any moment passing through the ammeter and  $\delta T$  is a small constant interval of time, the instrument records on the counter a reading proportional to the value of  $\Sigma(A) = \Sigma(A\delta T)$  between any required limits of time. This sum is approximately equal to the value of the true time-integral  $\int A dt$  between the same limits, provided the current has made no violent fluctuations between two of the registered current readings.

This class of intermittent self-integrating ammeter is very well represented by the Johnson and Phillips ampere-hour meter\* (see Figs. 17 and 18, p. 62-63), and by the ampere-hour meters of Cauderay, Swan, Kelvin and others. These intermittent self-integrating meters are not suitable for use in recording current quantity on circuits in which very large or sudden variations of current are liable to occur lasting only very short periods of time, as, for instance, in theatre stage electric lighting, or on motor circuits. In these cases a

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\* For a detailed description and illustration of the Johnson and Phillips meter see p. 257, "Electric Lamps and Electric Lighting," by J. A. Fleming, 2nd edition. ("The Electrician" Publishing Co.) Also see *The Electrician* August 18, 1899, Vol. XLIII., p. 605.

considerable electric quantity may escape registration, unless the time intervals between the moments of registration are very small.

The next class of ampere-hour meters are the *continuously self-integrating ampere-hour motor meters*, which are represented by the meters of Chamberlain and Hookham, Ferranti, Perry and others. The general principle on which these meters act may be described as follows:—Some conductor which is free to rotate, which may be either a copper disc, a thin circular layer of mercury, or a copper bell, is connected with a delicate counting mechanism, which records its rotations. This movable part is made to rotate in a magnetic field under the action of the current to be time-integrated. The movable part is, in fact, the armature of a small magneto machine or of a small series dynamo. The driving torque may, therefore, vary simply as the current to be integrated or as its square.

A retarding force, due to fluid friction or eddy currents, is then arranged to act upon the rotating part and this retarding or opposing torque must vary either as the speed or as the square of the speed, according as the driving torque varies as the current or as the square of the current. Then, under the operation of these opposing torques, a condition is reached in which the angular velocity  $\frac{dN}{dt}$  of the rotating parts is constant,  $N$  being the meter reading at any time,  $t$ , and  $dN$  the increase in the reading in a small time  $dt$ .

If, then, the retarding torque  $R$  varies as the speed, when a state of constant speed is reached under the condition that a steady current of  $A$  amperes flows through the meter, we have  $R \propto \frac{dN}{dt}$ , and if the driving torque  $F$  varies as the current we have  $F \propto A$ ; but when the speed is constant  $R=F$ , therefore,

$$A \propto \frac{dN}{dt}.$$

Hence,

$$dN \propto A dt.$$

If, therefore, we integrate between the time limits  $T_1$  and  $T_2$ , we have

$$N_2 - N_1 \propto \int_{T_1}^{T_2} A dt \propto Q,$$

where  $Q$  is the time-integral of the current or the quantity of electricity which has passed between the times  $T_1$  and  $T_2$  and  $N_1$  and  $N_2$  are the meter counter readings at those instants. Hence the total number of revolutions of the movable part in any time is proportional to the time-integral of the current which has passed through the meter in that interval.

If the law of resistance is that the retarding torque varies as the square of the speed or

$$R \propto \left( \frac{dN}{dt} \right)^2,$$

then the meter must be so constructed that  $F \propto A^2$ , and then when the speed is uniform we have

$$A^2 \propto \left( \frac{dN}{dt} \right)^2,$$

or  $N_2 - N_1 \propto \int_{T_1}^{T_2} A dt$ , as before.

In the Ferranti mercury meter (*see* Fig. 14), the magnetic field is created by an electromagnet excited by the current to be integrated. The core SP of the magnet is worked on a low part of the magnetisation curve, so that the magnetic field is nearly proportional to the current. In the field of this magnet a flat disc of mercury revolves, which is also traversed by the current to be integrated. The driving couple is, therefore, proportional to the square of the current passing through the meter, and the retarding resistance is the frictional resistance to the rotation of the mercury in its enclosing chamber. This last is proportional at low speeds to the square of the angular velocity. In order to overcome the steady, constant friction of the counting mechanism, the electromagnet is made of steel and has a small permanent

magnetisation given to it to enable the mercury to be set in recording action by a less current passing through the magnet coils than would otherwise be the case. This permanent magnetisation is so adjusted as to overcome the friction of the counting train. The theory of this meter may, therefore, be presented in the following form.

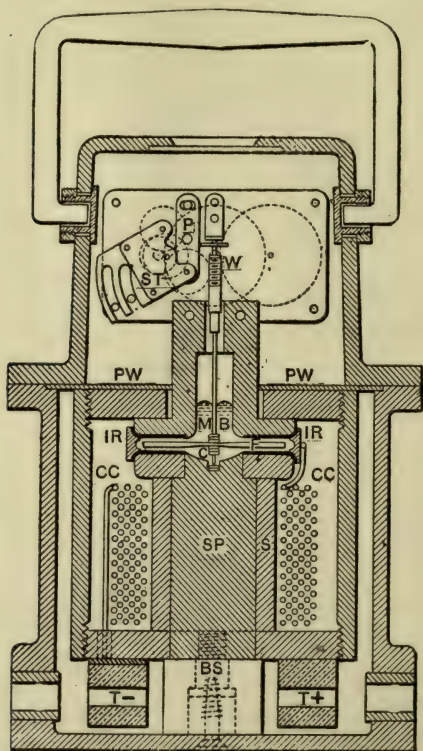


FIG. 14.—The Ferranti Mercury Meter.

Let  $M_A$  be the magnetisation due to the meter current  $A$ , and let  $M_p$  be the permanent magnetisation of the core. Let  $w$  be the angular velocity of the mercury,  $qw^2$  the resistance due to fluid friction,  $p$  the resistance due to pivot friction,

and  $k$  a constant. Then  $kA(M_A + M_p)$  is the driving torque, and when the motion is steady

$$kA(M_A + M_p) = p + qw^2.$$

The magnetisation  $M_A$  is adjusted so as to be proportional to  $A$  or  $M_A = bA$ , and the permanent magnetisation  $M_p$  is constant and equal, say, to  $c$ , so that  $M_p = c$ . Therefore

$$kA(bA + c) = ptqw^2$$

$$\therefore kbA^2 = qw^2 + p - kcA.$$

If, then, the residual magnetism is adjusted so that the driving torque  $kcA$  exactly overcomes the friction of the train, we have then  $p = kcA$  and

$$A^2 = \frac{q}{kb} w^2.$$

Hence  $A$  varies as  $w$ , and the time-integral of  $A$  varies as the number of revolutions of the meter.

As the pivot friction will generally be reduced in value as the speed of the mechanism increases, it is obvious that  $p$  can only be made equal to  $kcA$  for some particular value of  $A$ . Since, however,  $p$  is, or can be made small compared with the velocity friction  $qw^2$ , it is sufficient to adjust the permanent magnetisation to give a sufficient initial torque to start and keep the meter disc rotating at the right speed when the current  $A$  has a small value, say, of 0.1 ampere. This, however, will have the result that the meter will record too much at low loads or too little at high loads if its quantity consumption is reckoned from its number of rotations by the use of a constant multiplier. The effect of the permanent magnetisation is the same as if the current passing through the meter were increased in value by a small constant amount.

For if, using the same notation as before, we have the driving torque represented by

$$kA(M_A + M_p),$$

or by

$$kA(bA + c) = kbA^2 + kcA,$$

it is clear that the same torque would be produced, without any permanent field, by a current  $A+a$ , provided that  $a=c/b$ , for then

$$kA(bA+c)=k(A+a)bA.$$

We may regard the permanent field as a device for adding a small constant amount to the current to be measured to enable it to overcome the starting friction of the train. Hence the meter constant cannot be chosen so as to make the meter right at all loads. In some cases a separate shunt winding in the field electromagnet is used to produce the constant magnetisation, but this has the disadvantage that it involves a steady waste of energy, and, moreover, can only be applied when the meter is used on constant potential circuits.

For further information on the details of construction of ampere-hour house meters the reader is referred to the following Papers:—

C. H. WORDINGHAM. "Meters for Electrical Energy." *Proc. Inst. Civil Engineers*, 1892, Vol. CVIII., Part II.

Gives a good account of the meters known and used up to 1892.

G. W. D. RICKS. "Some Tests on the Variation of the Constants of Electricity Supply Meters with Temperature and with Current." *Proc. Inst. Elec. Engineers*, London, Vol. XXV., 1896, p. 57; also *The Electrician*, Vol. XXXIX., p. 573, 1897.

The author gives copious details of experimental tests made on certain forms of meter, *e.g.*, Thomson, Ferranti, &c.

T. P. WILMSHURST. "Electricity Meters." *The Electrician*, Vol. XXXIX., p. 408, 1897.

In this Paper the author discusses fully the necessary qualities of a good house electric meter, and criticises existing meters.

A. H. GIBBINGS. "Registration of Small Currents used for Electric Lighting and other Purposes." *Proc. Inst. Elec. Engineers*, Vol. XXVII., 1898, p. 547; or *The Electrician*, Vol. XLI., p. 112, 1898.

The author gives a good description and diagrams of the Bastian electrolytic meter, and a classified list of 19 different forms of electrolytic meter, chiefly unshunted forms, omitting, however, those of Edison, Sprague, Lane Fox and some others.

J. R. DICK. "Electricity Supply Meters of the Electrolytic Type." *Journal Inst. Elec. Eng.*, Vol. XXXI., 1901, p. 95.

The author deals with electrolytic meters of the shunted type, and gives a good description and diagrams of the Wright mercury meter.

For a description of the details of the various existing forms of ampere-hour meter, the reader is referred to the above Papers, also to the Author's treatise on "Electric Lamps and Electric Lighting," 2nd edition, 1900 ("The Electrician" Publishing Co., London).

The methods adopted in testing ampere-hour house meters, and the necessary qualities of a good meter for use in connection with electric supply are discussed in § 7 of this chapter.

Alternating-current ampere-hour or alternating-quantity meters are also described in § 6 of this chapter.

**§ 5. The Measurement of Electric Energy. Joule Meters or Watt-hour Meters.**—We have, in the next place, to discuss methods and means for the measurement of electric energy. The *electric energy* passing into a circuit in any period of time is measured by the *time integral of the electric power* given to that circuit, taken between the stated time limits. Conversely, the power given to the circuit is the time-differential of the energy. If  $P$  is the power, reckoned in watts, which is being taken up in any circuit at any instant and considered as constant during a small time interval  $dt$ , and if  $dE$  is the electric energy given to the circuit in that time, then we have

$$P = \frac{dE}{dt},$$

and, therefore,

$$E = \int_{T_1}^{T_2} P dt,$$

where  $E$  is the total energy imparted to the circuit between the times  $t = T_1$  and  $t = T_2$ .

Electric energy may be measured in terms of the absolute C.G.S. unit, called an *erg*, but, since this unit is a very small one for most purposes, it is more convenient to employ a larger

unit the *joule* =  $10^7$  ergs as the practical unit. Also, since a joule is equal to the energy conveyed by a power of one watt acting for one second, we may reckon electric energy in *watt-seconds*. A unit of energy which is 3,600 times greater than this last is called a *watt-hour*. A thousand watt-hours, or a *kilowatt-hour*, is also called a *Board of Trade Unit* (B.T.U.), and has received official sanction as a practical unit for the commercial supply of electric energy.

The numerical relations of these various units of energy are very easily traced. Appliances for measuring electric energy are called either *ergmeters*, *joulemeters*, or *watt-hour meters*, according to the magnitude of the energy with which they are best fitted to deal. The term electric meter is applied indifferently to ampere-hour or watt-hour meters. Instruments and processes for measuring electric energy may be classified under three heads:—

(*a*) Ammeter-voltmeter methods, or by use of ampere-hour meters combined with a voltmeter.

(*β*) Intermittent self-integrating watt-hour meters.

(*γ*) Continuously recording watt-hour meters.

The first (*a*) method can only be employed on power-absorbing circuits of which the power-factor is either unity or else is known and constant. That is to say, it can be employed always in the case of continuous currents, or in the case of alternating currents if the power-absorbing circuit is non-inductive, or has a known constant power-factor; but it cannot be employed on inductive circuits such as alternating-current motors or transformer circuits.

In the cases in which the above method is applicable, the energy measurement is made as follows:—The current entering the circuit is measured by an ammeter, the duration of the current is noted, and the fall in potential down the circuit is taken with a voltmeter. The product of current (in amperes) and potential drop (P.D.) (in volts) gives us the power absorption in watts. This power measurement is made at regular

or irregular but frequent intervals of time, and the results set out graphically in an *energy diagram*.

Take a horizontal line (see Fig. 15) on which are marked off distances representing to scale the times of observation measured from a selected zero. At these places set up ordinates representing the power absorption of the circuit (in watts) at that instant. Complete the curve defined by the tops of these ordinates. The area included by this curve, the time line and the extreme or limiting ordinates, is taken in terms of a rectangular unit of area, one side of which represents to scale the length taken to represent the unit of

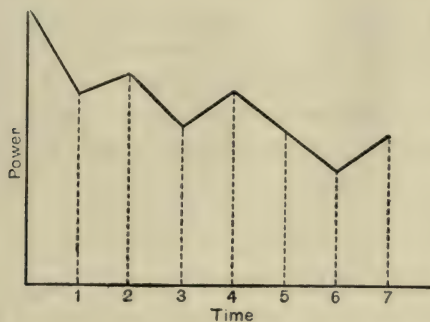


FIG. 15.

time and an adjacent side the length taken to represent the unit of power. The integration or area measurement effected in these terms gives us the energy taken up by the circuit. The greater the variation in the power taken up the shorter must be the time intervals at which the power measurements are made. The method, therefore, is only suitable in the case of power absorption which is constant or slowly varying. If the current is an alternating current, then a suitable ammeter and voltmeter (see Vol. I., Chaps. III. and IV., of this **HANDBOOK**) must be employed, and then the diagram area gives us the energy supplied to the circuit in joules or watt-hours, as the case may be.

If the circuit has a constant inductance, and if the power-factor is known, then the ordinates in the diagram must represent to scale the products of the current, voltage and power-factor of the power-absorbing circuit.

Instead of measuring the current and the time separately, we can employ an ampere-hour meter of any suitable kind, and take as the ordinates of the diagram the product of the reading of the ampere-hour meter at intervals and the average voltage of the circuit during that interval. If the voltage is constant, then the product of the ampere-hour meter reading and the voltmeter reading gives the energy taken up in that time.

The second class ( $\beta$ ) of energy meters are the intermittent self-registering watt-hour meters. These instruments consist of three parts—a wattmeter, a clock and a registering or counting part. The wattmeter is connected to the power-absorbing circuit. This wattmeter must be a deflectional instrument—that is, one in which the current flowing through the power-absorbing circuit passes through a fixed series coil in the wattmeter, and the ends of this circuit are attached to a voltmeter, shunt or high resistance coil suspended in this field of the series coil by an elastic or bifilar suspension, so that the electromagnetic forces tend to displace the shunt coil from its zero position. This deflection of the shunt coil is in some way or other measured, and at equal small time intervals is recorded by the clock and the deflections added up. Thus, if the deflection proportional to the power absorbed by the circuit is represented by  $P$ , and if  $\delta t$  is a small constant interval of time, the clock part adds up the deflections  $P$  or gives us the sum  $\Sigma(P)$  which is proportional to  $\delta t \Sigma(P)$  or to  $\Sigma(P\delta t)$ . Now,  $\Sigma(P\delta t)$  is nearly equal to the integral  $\int_{T_1}^{T_2} P dt$ —that is, to the time integral of the power taken between certain limits of time, and is very approximately the energy absorbed by the circuit, provided that no violent changes or very large variations occur on  $P$  between two successive measurements.

This class of energy meter, like the corresponding ammeter, is unsuited for energy measurements in those cases, such as motor supply or theatre lighting, in which large deliveries or changes occur in the power supply lasting for very brief periods. The intermittent recording energy meter is well represented by the meters of Cauderay, Brillié, Frager and Holden.\*

The intermittent self-integrating energy meters cannot, however, compare in general utility with the continuously recording or self-registering watt-hour meters which form the third class ( $\gamma$ ). In these last instruments there is a continual automatic time integration of the power supplied to the power-absorbing circuit. One class of continuously registering watt-hour meter is the graphic watt-hour meter of Mengarini. In this meter there is a wattmeter which consists of a series or thick coil, through which passes the whole of the current to the power-absorbing circuit. There is also a fine wire shunt coil, which is suspended by a metallic bifilar wire in the field of the large coil. This fine wire coil is attached to the terminals of the power-absorbing circuit. The movable shunt coil carries a long index pen, moving over a paper-covered drum driven by clockwork (*see* Fig. 16).† The angular displacement of the suspended coil, if small, is proportional to the power taken up in the power-absorbing circuit. Hence the displacement of the pen on the drum is proportional to the power in watts being absorbed. Accordingly, as the drum revolves, if the power varies, we have a curve drawn upon the paper the ordinate of which, measured from the zero line, is proportional to the power, and the abscissa of this ordinate is proportional to the time during which the power has been supplied. Hence the area included by this curve—the zero line and any two limiting

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\* See "A Watt-hour Meter," by Frank Holden. *Proc. Inst. Elec. Eng.*, May 2, 1901.

† The diagram in Fig. 16 represents the Mengarini voltmeter, but the wattmeter has exactly the same general form and appearance.

ordinates—is proportional to the energy supplied to the circuit between these times. This area must be integrated in terms of a rectangular unit of area, one side of which is the length taken to represent a watt, and the adjacent side, the length taken to represent an hour, and the curve area so taken gives us the energy supply in watt-hours.

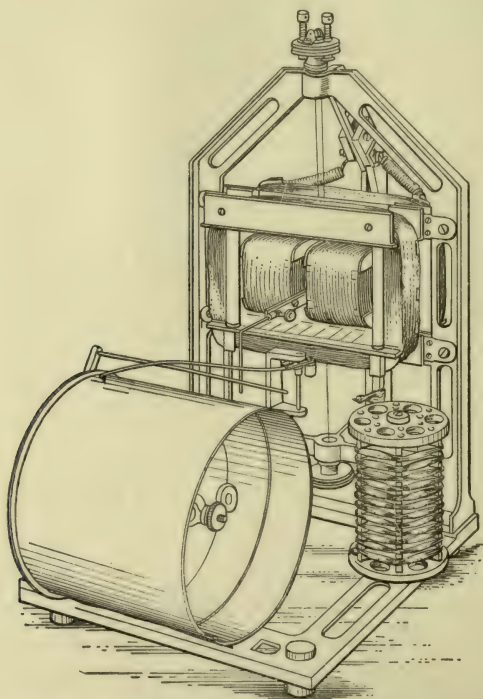


FIG. 16.—Mengarini's Self-recording Voltmeter.

This graphic meter has the advantage that it shows us at one and the same time the watts and the watt-hours, but it has the disadvantage of requiring a mechanical integration of an area. This last operation can, however, be very quickly performed with an Amsler's or other planimeter. In testing a graphic watt-hour meter separate tests must be applied to

the clock part to see that this registers *time* correctly, and to the watt-meter parts to see if this registers *electric power* correctly.

A graphic watt-hour meter of the above kind is very useful in conducting tests of secondary batteries for energy efficiency.

Passing on next to consider the self-integrating watt-hour meters, we find this class includes most of the ordinary house electric meters. No very scientific classification is possible. One of the simplest forms is represented by the motor meter or recording wattmeter of Elihu Thomson, and also by a similar form by Evershed. These consist of a small electric motor having an armature and field magnets without iron in them. The field coils, in the case of the Thomson meter, are compound-wound, and consist of the series coils, which carry the main current, and a shunt coil, called a *compensating coil*, which is, or should be, a constant voltage coil. The armature is a fine wire coil, which has in series with it a fixed resistance, and the two are a shunt across the ends of the power-absorbing circuit. The armature shaft carries a copper disc which revolves between the poles of a well-aged permanent magnet, and this shaft also drives a counter for recording its revolutions. The operation of the meter is as follows :—

When connected to a power-absorbing circuit the torque on the armature is proportional to the power  $P$  in watts being absorbed at that instant. The retarding force due to the eddy currents set up in the copper disc is proportional to the angular velocity. Let  $N$  be the number of revolutions which the armature has made from the time of starting, and let  $dt$  be a small interval of time. Then  $\frac{dN}{dt}$  is the mean angular velocity during that interval. The retarding torque  $R$  varies as the angular velocity, and when a state of constant speed is reached the driving torque  $F$  is equal to the retarding torque  $R$ . Hence, when the equilibrium is reached we have  $F = R$ .

But  $F \propto P$ , and  $R \propto \frac{dN}{dt}$ .

$$\therefore P \propto \frac{dN}{dt} \text{ or } dN \propto P dt,$$

and 
$$N_1 - N_2 \propto \int_{T_1}^{T_2} P dt \propto E.$$

In other words, the increase in the number of revolutions of the meter between the times  $T_1$  and  $T_2$  is a measure of the energy  $E$  which has passed into the circuit.

The object of the compensating coil or compound winding is to overcome the starting friction of the motor and counter mechanism friction, and, as in the case of the revolving mercury ampere-hour meters, the compound winding is equivalent to a small constant addition to the current passing into the power-absorbing circuit. The addition of this compound winding increases, however, the energy losses in the meter itself.

If the wattmeter part is properly constructed, and the armature coil and added resistance is sufficiently inductionless, this meter can be employed for energy measurements with alternating currents on inductive circuits. The readings of the meter, however, on absorbing circuits of small power-factor must not be accepted too hastily as correct, but means should be taken to verify the accuracy of the wattmeter pistons. To do this, the watt-hour meter must be set to read the energy taken up in a given time on a circuit of small power-factor, the power absorption of which has been determined by a correct wattmeter, as described in Vol. I., Chap. V., of this HANDBOOK.

Knowing the true power absorption and the time during which it is being steadily absorbed, we can calculate at once the real energy taken up and compare it with the readings of the watt-hour meter. Most motor meters, or self-recording wattmeters, are now sent out by their makers with the counting train so adjusted as to read directly in Board of Trade units, and in this case the test consists in ascertaining

whether the dials correctly record Board of Trade units when the power-factor of the power-absorbing circuit is unity, and also when it has a very low value—say, 0.02. A tested watt-hour meter of this description can be used with advantage to ascertain the energy efficiency of a secondary battery by measuring with it the energy put in and taken out. As constructed, the armature shunt-circuit has generally a resistance of about 1,000 ohms, so that on a 100-volt circuit the shunt resistance takes 0.1 ampere and a power of 10 watts. Nevertheless, when used as a house meter, this represents one Board of Trade unit in 100 hours of use, and it depends upon the mode of connection of one end of the shunt whether this internal power loss is paid for by the consumer or the supply company.

Another type of continuously recording watt-hour meter is the Aron meter, which involves the improved application of a principle suggested in 1882 by Profs. Ayrton and Perry. This meter contains a set of counting dials. The counting train is connected with a pair of pendulums by two wheel trains, but so that the counting mechanism is only driven forward by the difference between the rates of going of the pendulums. The counting mechanism thus records the difference in the rates of two clocks. One clock has an ordinary pendulum, the other has a pendulum having a coil of wire in the place of the bob, which is a shunt coil, and is connected through a resistance with the terminals of the power-absorbing circuit. It is thus traversed by a current proportional to the volt fall down the circuit. Under this coil bob is placed a coil of thick wire, through which passes the main current. The force acting on the pendulum is therefore partly due to gravity and partly to the electromagnetic pull between the shunt and series coils. This last force is proportional at any instant to the true power being taken up in the power-absorbing circuit. Let  $n_1$  be the oscillations per second of the ordinary bob, and  $n_2$  those of the electromagnetic bob. Then, if  $g$  is the

acceleration of gravity, we have  $n_1 \propto \sqrt{g}$  and  $n_2 \propto \sqrt{g + aP}$ , where  $P$  is the power being taken up in the circuit attached to the meter and  $a$  is some constant.

Hence, at any instant,  $n_2^2 - n_1^2 \propto P$ ,

or  $(n_2 + n_1)(n_2 - n_1) \propto P$ .

Therefore, if  $dt$  is a small interval of time, we have

$$(n_2 + n_1)(n_2 dt - n_1 dt) \propto P dt.$$

But  $n_2 dt$  is the number of oscillations made in the time  $dt$  by one pendulum and  $n_1 dt$  is that made by the other. If  $n_1$  does not differ greatly from  $n_2$ , which will be the case if the force due to the electromagnetic action is small compared with weight of the electromagnetic bob, then, instead of  $n_2 + n_1$  in the last equation, we may write  $2n_1$  which, being a constant, can be represented by the symbol  $C$ . Hence we have

$$C(n_2 dt - n_1 dt) \propto P dt,$$

and 
$$\int n_2 dt - \int n_1 dt \propto \int P dt \propto \int dE,$$

where  $dE$  is the increment in the energy supplied to the power-absorbing circuit. If we take the integrals between two time limits  $t = T_1$  and  $t = T_2$ , the first term on the left-hand side is simply the number of swings,  $N_2$ , made by the electromagnetic bob in the time  $T_2 - T_1$ , and the second term is the number of swings,  $N_1$ , made by the ordinary bob, and the term on the right-hand side is the energy supplied in that time to the power-absorbing circuit. Accordingly,

$$(N_2 - N_1) \propto E.$$

In other words, the energy delivery to the circuit is proportional to the total gain of one clock over the other, and, since the dials record this value the counting dials record the whole electric energy delivered.

The double pendulum energy meter must, however, be very well made if it is to record correctly. The equi-timing of the

pendulums and clocks when no current is passing must be very exact, otherwise the differential train will be driven one way or the other when no electric power is passing through it, and the meter may even record *negatively*. Also, as shown above, it is an essential condition that the rates of going of the pendulums should not be very different under any circumstances.

The double pendulum meter has, however, the advantage, like all wattmeter meters, that it can be used equally for continuous and for alternating currents.

For the general measurement of electric energy, when it can be conveniently measured in units, such as a watt-hour or kilowatt-hour, undoubtedly the most useful instrument is either a graphic wattmeter of the type of the Mengarini watt-hour meter or a continuously recording motor meter of the type of Elihu Thomson or Evershed. In testing such a watt-hour meter it is not sufficient to make tests merely at one load or on a circuit of unit power factor; it should be tested by passing through it currents from the smallest it will start with to the largest the series coil will bear, and should give the same readings when the same energy is passed through it, the factors of time and power being varied as much as possible. Also, it should be tested on circuits of various power factors, provided it can be used with both continuous and alternating currents.

**§ 6. Electric House Meters. Classification and References.**—In any system for the public supply of electric energy for the purposes of electric lighting and power a good electric meter is an essential element. The Electric Lighting Acts of 1882 and 1888 provide for the sale of electric energy in Great Britain by measurement, and regard it also as something which can be stolen as well as sold. Hence, there has been a prolific expenditure of ingenuity in the endeavour to provide an accurate, substantial and cheap electric meter, not liable to get out of order, and with which it is not easy for

the consumer to tamper. Electric meters may be classified in several different ways:

(i.) According to the kind of electric supply they are fitted to measure, *i.e.*, whether continuous current or alternating, and if the latter, whether monophase or polyphase.

(ii.) According to the method of their measurement, *i.e.*, whether graphic or self-integrating, and if the latter, whether intermittent or continuously integrating in their action.

(iii.) According to the principle of their action, *e.g.*, volta-metric or mechanical, and if the latter, whether ampere-hour or watt-hour meters.

It is impossible to give a final and complete classification of all meters, because new forms are constantly appearing and there is no finality in invention. Broadly, however, we may classify, on the above basis, the *Types* of (I.) ampere-hour house meters, and (II.) watt-hour house meters existing in 1903 as follows:

(i.) House meters may be of various *Classes*:—

(A) Available for continuous current use only.

(B) Available for alternating current, monophase or polyphase, only.

(C) Available for use with both alternating and continuous current.

(ii.) House meters may be of various *Genera*:

(a) Graphic meters, recording on a paper strip a curve whose area is proportional to quantity or energy, or gravimetric or volumetric, in which something is weighed or measured which is proportional to the quantity or energy which has passed.

(b) Intermittent self-integrating or “feeler” meters, which take a measurement of current or power passing through them at small equi-distant intervals of time and then add up all these readings.

(c) Continuously self-integrating or motor meters recording on dials the true time integral of the current or power up to the moment of observation.

(iii.) House meters may be of various *Species* :

(a) Electrolytic, depending on the electrolysis of a liquid or collection of ionic products.

(i.) Shunted voltmeters.

(ii.) Unshunted voltmeters.

( $\beta$ ) Mechanical meters, in which the mechanism for measuring is wholly mechanical and electro-mechanical.

The last kind are furthermore divisible into

(1) Ammeter or wattmeter meters, graphic or intermittently integrating.

(2) Motor meters, continuously recording.

(3) Clock or pendulum meters.

(4) Ballistic meters.

(5) Induction meters.

(6) Thermo-mechanical meters.

As it is impossible to give here the details of all the meters so far made which fall under the above categories, we shall merely mention a few forms, giving the inventor's name in each case and a brief mention of the details of construction, so as to enable the reader to obtain an idea of the type, class, genus and species, and then refer the student to some publication or patent specification in which a detailed account of the meter in question is given.

**ELECTROLYTIC METERS.**—These are exclusively ampere-hour meters, suitable for use only with continuous currents, but may be made either self-recording or require a gravimetric or volumetric measurement to be performed to ascertain the electric quantity which has passed.

The original idea of an electrolytic house meter in connection with public electric supply may be found described by St. George Lane Fox, see *British Patent Specification*, No. 3,988 of 1878, p. 4, line 25.

He proposed to use a voltmeter shunted by an electromagnet intermittently magnetised by a vibrating break, the ends of the magnet circuit being a shunt on the main supply circuit. The electrolytic decomposition was effected by the "extra" current of the electromagnet.

Lane Fox was followed by J. T. Sprague, *Brit. Pat. Spec.*, No. 4,762 of 1878.

He describes an electrolytic meter with copper plates in solution of sulphate of copper, and causes the current to deposit metal, first on one plate and then on the other, the reversals of direction being automatic, and a mechanical reckoning kept of the number of reversals, each corresponding to the passage of a known quantity of electricity.

J. T. Sprague, *Brit. Pat. Spec.*, No. 4,454 of 1881.

He describes a shunted voltmeter for use as an electrolytic quantity meter. He recognises that this shunt must have the same kind of temperature variation of resistance as the electrolyte, and he also describes an automatic thermo-regulator for heating up the electrolyte if the temperature falls too low.

J. T. Sprague, *Brit. Pat. Spec.*, No. 2,902 of 1882.

Gives further details of the shunted voltmeter with metallic salt electrolysed and methods of weighing the electrodes without removing them from the liquid.

T. A. Edison, *Brit. Pat. Spec.*, No. 4,391, of 1880.

Went at first over much the same ground as J. T. Sprague. In this specification he describes a pair of copper plates balanced on a scale and forming with other plates a copper sulphate voltmeter. The current was made to deposit, first on one side and then on the other, thus tipping over the balance from side to side. The number of reversals is recorded, and each corresponds to the passage of a known quantity of electricity.

T. A. Edison, *Brit. Pat. Spec.*, No. 1,016 of 1881.

Proposes to electrolyse water and explode the mixed gases again.

T. A. Edison, *Brit. Pat. Specs.*, Nos. 1,783 and 4,576 of 1881.

He describes in No. 1,783 the shunted voltmeter and in No. 4,576 the employment of amalgamated zinc electrodes in zinc sulphate solution. Also, he describes the electric lamp and thermostat for automatically keeping the electrolyte from freezing. This final form of meter is the one in practical use, and has been already described in detail in this chapter.

Between 1883 and 1897 a large number of electrolytic meters were devised, and a list of 19 names of inventors and a brief description of each meter is given in a Paper by Mr. A. H. Gibbings (see *Journal* of the Institution of Electrical Engineers, London, Vol. XXVII., 1898, p. 551).

In the above-mentioned Paper is also described the electrolytic meter of C. O. Bastian, *Brit. Pat. Spec.*, No. 5,034 of 1898 (see also *The Electrician*, Vol. XLI., p. 112).

This electrolytic meter is very much used. It has been already described in detail in this chapter. All that need be remarked here in addition, as an

illustration of its accuracy, is that in the case of a 10-ampere meter reading 300 B.O.T. units the error was found to be under 2 per cent., and in a 5-ampere meter about 1 per cent. There is no shunt coil and no temperature error, frictional error also is absent, and no current is too small to start the meter recording. This meter, however, requires filling up with water once every few months, according to the extent of its use. There is also a relatively large fall in voltage down the meter of nearly 2 volts, caused by the counter electromotive force. It has been criticised on the grounds of danger from the explosive gases evolved and non-portability unless the electrolyte is removed; but against this may be set its cheapness, accuracy for small currents and general simplicity.

To this same class belongs the electrolytic standard and prepayment meter of Long-Schattner already described (*see ante*, p. 28). For further details the reader may be referred to the following specifications:—

F. M. Long and E. Schattner, *Brit. Pat. Spec.*, No. 14,107 of 1898; also a subsidiary *Brit. Pat. Spec.*, No. 22,608 of 1898, for an improvement consisting in a device for preventing the current being completely cut off when the paid-for quantity is delivered.

The next large group of meters comprises the intermittent integrating meters, or “feeler” meters as they have been called, in which the values of the current in amperes or the power in watts passed through are added up at small equal intervals of time. These are principally represented by the meters of the following inventors: Hopkinson, Cauderay, Brillié, Frager, Richard Frères, Swan, Kelvin, and many others. A few details are here given of some which have been in practical use.

Jules Cauderay, *Brit. Pat. Specs.*, No. 1,756 of 1883, No. 7,515 of 1884, No. 13,704 of 1885 and No. 6,369 of 1885.

This meter consists either of an ammeter or a wattmeter having connected with its movable portion an index needle, which is moved through an angle proportional to the current or the power passing through the meter. The needle end rests on a metal cylinder, made to revolve uniformly by a clock portion. The cylinder carries a number of pins or studs arranged round its circumference. The cylinder is divided into parallel circles, and the first circle, corresponding to a unit deflection of the needle, carries one pin, and the second, corresponding to two units, carries two pins, and so on. The revolution of the cylinder causes a number of contacts to be made with the index needle which tilt it up and actuate a recording mechanism, the number

of contacts in a short fixed time depending upon the position of the needle and therefore upon the actual value of the power or current then passing. The index needle is able to make its way backwards and forwards through the collection of slowly-revolving pins, like a man working his way through a crowd of people. In the intervals of being gripped by the pins the needle is quite free to move.

L. Brillié, *Brit. Pat. Spec.*, No. 14,772 of 1888.

This meter is a watt-hour intermittent integrating meter, the record of the power being taken about 100 times in an hour. For a description of the meter see the *Engineer*, May 13, 1892. It is based upon the employment of a torsion electro-dynamometer, the torsion of the spring carrying the shunt coil being periodically measured. The Brillié meter is said to be distinguished by its very small starting current. It will start with as little as one-thousandth of its full load, and takes up very little power in the wattmeter part owing to the very high resistance of the shunt coil—viz., 10,000 ohms.

For further information on this meter the reader may refer to the remarks of Mr. G. P. Roux in a discussion at the Institution of Civil Engineers on a paper by Mr. J. Swinburne on "Electrical Measuring Instruments," *Proc. Inst. Civ. Eng.*, Vol. CX., Part IV., 1892.

Lord Kelvin, *Brit. Pat. Specs.*, Nos. 15,769 of 1889, 18,436 of 1891 and 24,471 of 1893.

Lord Kelvin has devised various forms of intermittent self-integrating ampere-hour or watt-hour meters. The electrical portion consists of a series and a shunt coil, which exert an electro-dynamic attraction upon each other. The attraction is made to alter the length of the path of a trailing wheel connected to a counting mechanism over a revolving cam, the time during which the wheel is in contact being regulated by the power passing through the meter. For details, see *The Electrician*, Vol. XXV., 1890, p. 711.

A. Frager, *Brit. Pat. Spec.* No. 8,958 of 1889.

This meter may be made either as an ampere-hour or preferably as a watt-hour meter, and is an intermittently integrating meter. It consists of an electric clock which drives round an axis which carries a steel plate, cut into a peculiar spiral shape, and is called the snail plate. This snail plate has a pawl on its underside, and when it is slightly depressed it engages with a tooth on a toothed crown wheel or circular rack connected with a counting mechanism. Adjacent is a deflectional wattmeter with fixed current or series coils and a suspended volt or shunt coil having a long index needle attached to it, the end of which projects over and rests on the snail. When the end of the needle so rests on the snail plate it depresses it and causes the counting mechanism to run. The snail plate is of such shape that the length of the travel of the end of the index needle upon it is proportional to the angular deflection of the index needle or to the power in watts passing through the meter. Hence, at each revolution of the snail the counter is carried on by an amount proportional to the power passing through the meter. The snail makes one revolution in from two to five minutes, according to the size of

meter. For fuller description see *Journal of the Inst. of Elec. Eng.*, Vol. XXX., p. 944, 1901; also *The Electrician*, 1890, Vol. XXV., p. 254; also see a Paper by Mr. Wordingham, on "Meters for Recording Electrical Energy," *Proc. Inst. Civ. Eng.*, Vol. CVIII., Part II., 1892.

Another principle which has been invoked in the construction of electric house meters is that of the ballistic wattmeter. An electro-dynamometer is constructed, the series coil of which is traversed by the current flowing into any circuit, and the shunt coil by a current proportional to the fall of potential down that circuit. If the shunt circuit is closed only for a moment the shunt coil receives an impulse proportional to  $Iidt$ , where  $I$  and  $i$  are the two currents. This impulse is proportional to the energy delivered to the circuit in the time  $dt$ . If a mechanism is provided for recording these "throws" of the shunt coil and adding them up, we have a form of ballistic energy meter. Of this form is the meter of Magunna (see *The Electrician*, Vol. XLIII., p. 769; also *L'Industrie Electrique* of 1899).

Another meter of the same type is that of F. Holden (*Brit. Pat. Spec.*, No. 26,646 of 1897; see also *Journal of the Inst. of Elec. Eng.*, Vol. XXX., p. 944, 1901).

This meter is an intermittently integrating watt-hour meter. Its peculiarity consists in the fact that its electrical parts form a ballistic wattmeter. The shunt circuit is closed by a clock, say once a minute, and when this happens the shunt coil receives an impulse proportional to the power then passing. The movable shunt coil has on its axis a cast-iron disc which revolves between the poles of a magnet, and the "throw" of the movable coil is resisted by the effects of magnetic hysteresis and eddy currents in the disc. The angular displacement of the shunt coil is made to take effect on a counting mechanism through the intervention of a pawl and circular rack. Hence the counting mechanism adds up these "throws," which are individually proportional to the power, and hence their sum to the energy delivered. This meter can be made to work with a very small starting load which, in the larger sizes, is under 2 per cent. of full load. It is one of the few examples of an intermittently integrating meter which has a high degree of accuracy at low loads.

Another excellent form of intermittent self-integrating ampere-hour meter is that of Johnson and Phillips (see *Brit. Pat. Spec.* of A. W. Stavely, T. H. Parsons, T. J. Murday,

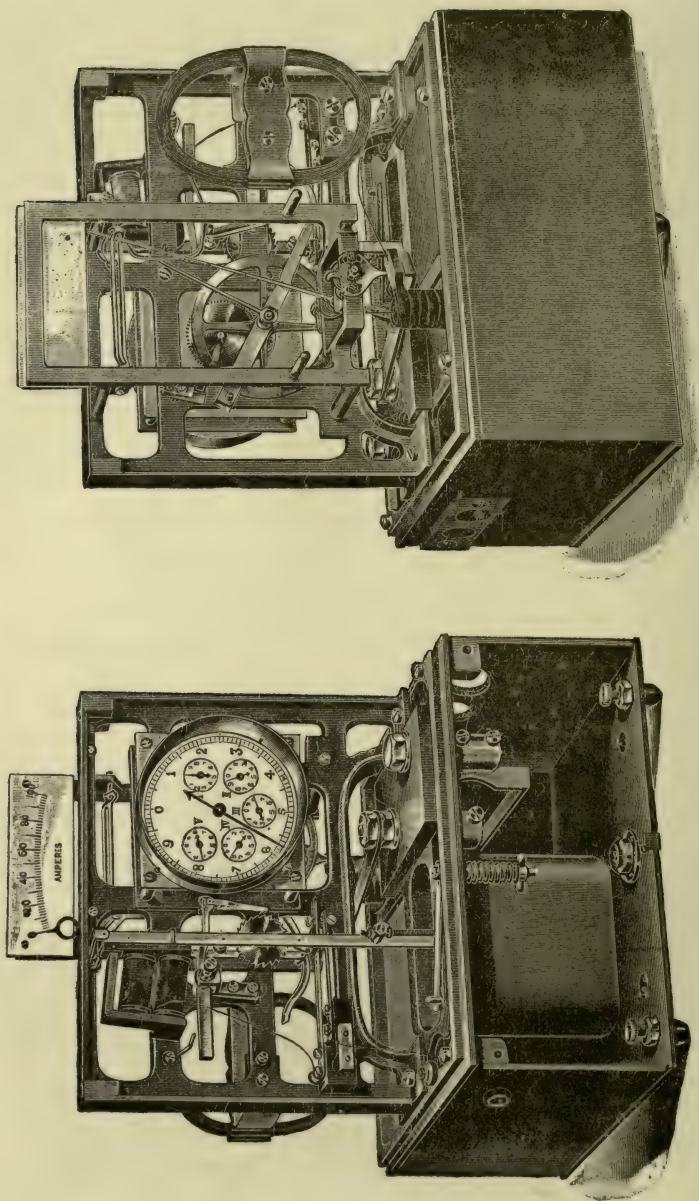


FIG. 17.—Johnson and Phillips Ampere-Hour House Meter.

No. 12,390 of 1896 ; also *The Electrician*, Vol. XLIII., p. 604, 1899).

This meter (see Fig. 17) consists of an ammeter and an electrically-driven and controlled clock. The clock drives a counting mechanism through the intervention of a gear, the ratio of which is controlled by the ammeter. The gear (see Fig. 18) consists of a crank on one of the clock axes which in its revolution depresses a lever and causes a pawl to shift round one of the wheels of the counter. The position of the shifting lever is controlled by the ammeter, so that according to the magnitude of the current so is that of the "feed" of the pawl at each stroke. The reading takes place every half minute. The meter will start with a current which is one-two hundredth of its full load.

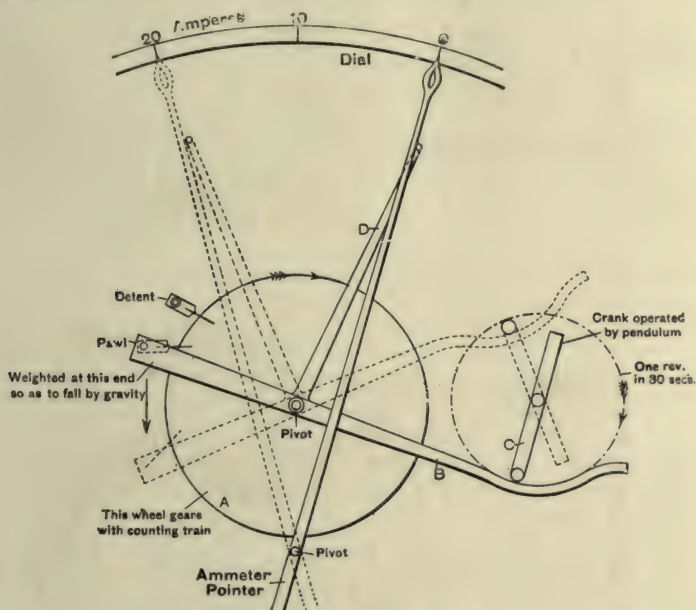


FIG. 18.

In the third place, there are the continuously self-integrating meters, which form the largest and most important group. We cannot attempt to put on record anything like a complete list of these inventions, and the reader must be referred to the indices of the Patent Office for further information. A few of the most useful forms are here mentioned.

As far as regards the continuously-recording current or power meters, the majority of forms of this type of meter may be said to be based upon principles enunciated by Profs. Ayrton and Perry in 1882 and described in their *Brit. Pat. Spec.*, No. 2,642 of June 5, 1882. In this specification the essential principles on which are based the motor meters of the type of Hookham, Ferranti, Perry, Thomson and others, are discussed, and also the differential clock meter construction out of which has been evolved the Aron meter. Motor meters were, however, known before June, 1882.

C. H. Faure. (See *Brit. Pat. Spec.*, No. 730, February 15, 1882; also *The Electrician*, Vol. X., p. 144, 1882.)

M. Faure described, in 1882, a rudimentary form of the motor meter in which a bent copper rod is made to revolve over a magnetic pole, its lower ends dipping into and being retarded by mercury in which they revolved.

Also another pioneer in this department of invention was John Monro (see *Brit. Pat. Spec.*, No. 1,626 of 1882).

He employed in an electric meter a copper disc rotating between the poles of an electromagnet. This disc was traversed by the current to be measured, the axle of the disc being geared to a suitable counting mechanism.

A similar type of motor meter was evolved a little later by T. A. Edison (see *Brit. Pat. Spec.*, No. 2,675 of 1883).

This inventor was early in the field with a motor meter based on the same principle as that of C. A. Faure. An electromagnet is constructed with one central pole and a concentric shell or cylinder as the other pole. On this central pole is balanced a copper cylinder or hat which rests and revolves on a pivot in a mercury cup. The lower edge of the cylinder is provided with fans or blades which dip into an annular mercury trough surrounding the central pole. The cylinder is connected with a counting mechanism. The current to be measured passes in at centre or top of cylinder and out at the lower edge, and causes it to revolve in the radial magnetic field. The eddy currents set up in the disc and the fluid friction resist this driving force, and were supposed to give a constant speed for constant currents. The meter was, in all probability, never made, or it would have revealed its defects; but it is typical of the class of meter in which the principal resistance arises from eddy currents set up in a copper disc.

None of these inventors went further than to produce a model more or less defective, but did not evolve a commercially useful motor meter.

Invention, however, is largely a matter of demand, and in 1882 there was not much demand for a motor meter. It was not until the agitation preceding the Electric Light Amendment Act of 1888 cleared the way for public electric supply that the inventor of a house meter had a market for his wares. One of the earliest inventors in the field with a really useful motor meter was S. Z. de Ferranti. See *Brit. Pat. Specs.* :—

No. 5,926 of 1883	{	This was for the original continuous-current mercury meter with solid electromagnet.
„ 701 „ 1887	{	This was for the improvements comprising the laminated electromagnet and constant-current shunt coil, for alternating currents.
„ 13,621 „ 1888	{	These two specifications refer to alternating-current meters.
„ 13,622 „ „	{	
„ 1,903 „ 1889	{	These specifications refer to various improvements in continuous-current and alternating-current motor, intermittent integrating, and induction meters.
„ 18,361 „ „	{	
„ 2,653 „ 1890	{	
„ 3,096 „ „	{	
„ 3,097 „ „	{	
„ 9,061 „ „	{	

Mr. Ferranti devised in 1883 his original mercury ampere-hour motor meter, which consisted of an electromagnet within which was a flat disc-like cavity containing mercury, the sides of which were stamped with grooves. The mercury was traversed radially by the current to be metered and perpendicularly by a magnetic field proportional in strength to that current. The friction of the mercury against the walls of its enclosing cavity constitutes the principal retarding force. The mercury is set in rotation by the electromagnetic torque, and its revolutions are counted by an aluminium vane immersed in it the shaft of which is connected with a counting mechanism. The theory has already been given above (*see* p. 42). For further information the reader is referred to Wordingham "On Meters for Electrical Energy," *Proc. Inst. Civ. Eng.*, Vol. CVIII., Part II., 1892; also G. W. D. Ricks "On Testing Supply Meters," *Proc. Brit. Assoc.*, Toronto, 1897; or *The Electrician*, Vol. XXXIX., p. 573.

The friction of the train is overcome in the case of the alternating-current meter with laminated electromagnet by the use of a constant-voltage shunt coil winding on the field magnet, and in the case of the continuous-current meter by the use of a steel core for the magnets having a small permanent magnetisation imparted to it. In this last case the core of the electromagnet is, of course, solid. The continuous-current meter is very extensively used, and has been the subject of numerous detail patents for improvements.

Litigation has taken place over the question of priority in the use of the constant shunt coil or compensating coil in the alternating-current meter for creating a small constant field to overcome the friction of the meter. For details of one of these actions see *The Electrician*, Vol. XLVIII., pp. 146-185, 1901.

Another very extensively used form of motor ampere-hour meter, which has undergone many improvements and been covered by many patents, is that of Chamberlain and Hookham, see *Brit. Pat. Specs.* of G. Hookham :—

No. 4,225 of 1887	{ This is the original or master patent, and the specification was amended by disclaimer in 1889 and 1895.
" 17,502 " "	
" 4,371 " 1889	
" 5,832 " "	
" 13,123 " 1890	
" 13,525 " "	
" 40 " 1891	
" 4,064 " "	
" 14,239 " 1892	
" 12,404 " 1895	
" 2,683 " 1896	This is the so-called "1897 pattern."
" 23,512 " "	

The original form of continuous-current Hookham meter was described in *The Electrician*, Vol. XXII., p. 369 ; a second form in *The Electrician*, Vol. XXXII., p. 220 ; and the third form in *The Electrician*, Vol. XXXVIII., p. 405. In addition to this, there is an alternating-current meter (*Brit. Pat. Spec.*, No. 13,525 of 1890), and a more recent two-rate form, reckoning on two sets of dials, so that two rates of charge are possible.

Mr. G. Hookham has described and made a number of forms of motor ampere-hour meter depending essentially upon the same principle. A description of the first form of this meter (generally known as the Chamberlain and Hookham meter) was given by Mr. Hookham in the *Journal of the Inst. of Elec. Eng.*, 1889, Vol. XVIII., p. 688.

The meter in the latest form consists of a copper disc or cylinder armature placed in a constant magnetic field passing at right angles to its plane. The disc is immersed in mercury or else a disc armature with mercury contacts is used. This armature is traversed by the currents to be metered which, in the case of the 1892 pattern of meter, passes radially through it. The armature shaft carries another copper disc, which also revolves a magnetic field, cutting the disc perpendicularly to its plane, and acts as a brake. The rotating portion has thus a driving torque applied to it which is proportional to the current to be integrated, and the retarding torque due to the eddy current is proportional to the speed. Hence the number of revolutions in any time is proportional to the time integral of the current.

The essential principle of the meter is the use of a relatively strong magnetic braking power so that the resistance to rotation due to eddy currents in the brake disc altogether swamps the resistance due to static friction of bearings and counting train, and provides an effective resistance exactly proportional to speed of rotation. Also another feature is the constant magnetic field. The pole pieces of the magnet are grooved to increase the

braking action by the increased production of eddy currents, and they are played out and brought as close to each other as possible to secure the strongest possible magnetic field. Mr. Hookham's master patent (No. 4,225 of 1887) has been twice amended by disclaimer, and in an action for infringement brought against the manufacturers of Prof. Perry's meter (see *The Electrician*, Vol. XXVII., p. 191) its validity was upheld. [See Hookham and Another v. Johnson and Another, *The Electrician*, Vol. XXXVIII., pp. 622, 665 and 839 (judgment) 1897.] The case was tried before Mr. Justice Wills, whose judgment reviews the history of motor meter making, and he decided that, although the principles of a motor meter with magnetic brake were disclosed by Profs. Ayrton and Perry in 1882, they did not bring the meter to such a point of practical perfection as to prevent any other inventor from being able to create a fresh monopoly.

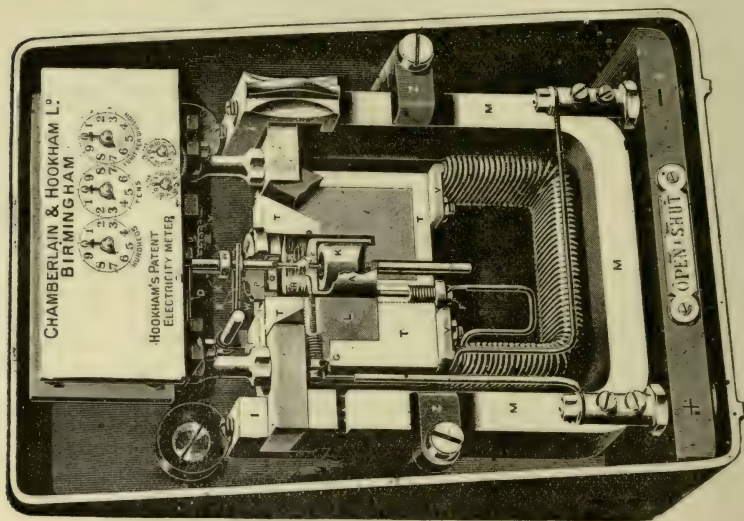
The Hookham meter has undergone many improvements since 1887. In succession to the early form appeared what was known as the 1892 pattern (see *The Electrician*, Vol. XXXII., p. 220). In this instrument the armature was a simple copper disc, and the brake armature was another similar disc on the same shaft. The magnetic field had two air gaps in which these discs revolved and the pole pieces were grooved. Later on still further improvements were introduced and resulted in the production of the 1897 pattern. (See *The Electrician*, Vol. XXXVIII., p. 405; also see Fig. 19.)

In this last form, the meter consists of a single horse-shoe permanent magnet, A, formed of Tungsten steel, giving a strong and constant field. Two air-gaps are made in this field in parallel to each other. In one of these the copper brake disc O revolves, and in the other the armature disc N. This latter is slit radially, and the magnetic field is so arranged that it perforates each half of the disc in opposite directions, and the current to the meter passes transversely across the disc and not radially. The braking action is strong enough to swamp friction, and hence the calibration curve is nearly a straight line (see Fig. 20). A 25-ampere meter has an internal resistance of only 0.003 ohm and a voltage drop of 0.075 volt. The driving force in this meter is six times that of a meter of the 1892 pattern of the same capacity.

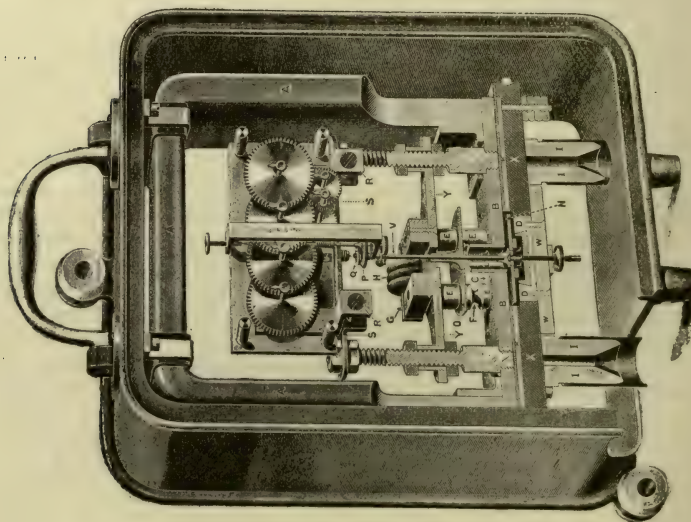
In a pattern called the 1901 form, a copper bell forms at the same time the armature disc and the brake disc.

It will be seen that the inventor has overcome the friction difficulty by making the driving force so strong that any outstanding pivot friction becomes negligible in comparison with the magnetic brake resistance. Mr. Hookham has also devised a two-rate meter which automatically changes the rate of charge when the current exceeds a certain value, and so acts as a maximum demand meter (see *The Electrician*, Vol. XLIII., p. 440, 1899).

Another form of motor meter very extensively used, which contains a copper brake wheel, is that of Elihu Thomson (see *Brit. Pat. Spec.*, No. 10,595 of 1890).



1901 Pattern.



1897 Pattern.

Fig. 19.—Chamberlain and Hookham's Meter.

The meter, as already explained, is a watt-hour meter, the motor part consisting of the series coils which carry the current, a shunt-compounding field coil for overcoming friction, and an armature, which, together with a fixed resistance, is a shunt across the ends of the power-absorbing resistance. The armature is drum-wound, and has a delicately made silver commutator and brushes. The braking action is applied by the magnetic field due to three permanent well-aged horse-shoe-shaped steel magnets, which embrace and nearly touch a copper disc on the armature shaft. This meter has already been fully described (*see* § 5, p. 51).

S. Evershed. (*See Brit. Pat. Spec.*, No. 9,127 of 1897 and No. 3,507 of 1898; also No. 13,616 of 1899 and No. 3,534 of 1900.) The first two specifications cover "magnetic support"

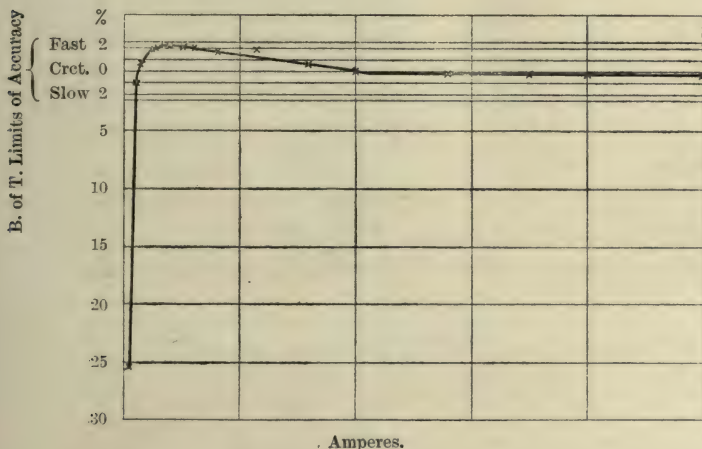


FIG. 20.

of bearings, and the last two the meter as described below. (Also see *Journal Inst. Elec. Eng.*, Vol. XXIX., p. 743, 1899-1900, "On a Frictionless Motor Meter.")

Mr. Evershed has here described a "frictionless" motor meter, in which great attention has been paid to the question of reducing to a minimum all sources of friction. The meter is a watt-hour meter which in general outline is not unlike the Thomson meter, but with important improvements. The motor consists of an ironless dynamo, the armature having a most carefully made elastic or spring-wire commutator pressed against a pair of collecting wheels on antifriction bearings. The retarding force is supplied by a copper bell, the edge of which runs between the poles of fixed permanent steel magnets. The armature coils are in a shunt circuit, and are embraced by the

field coils, which carry the main current. The suspension of the principal axle is by a "magnetic" support. The method of connecting the armature shaft to the counting train is exceedingly ingenious. The shaft carries a

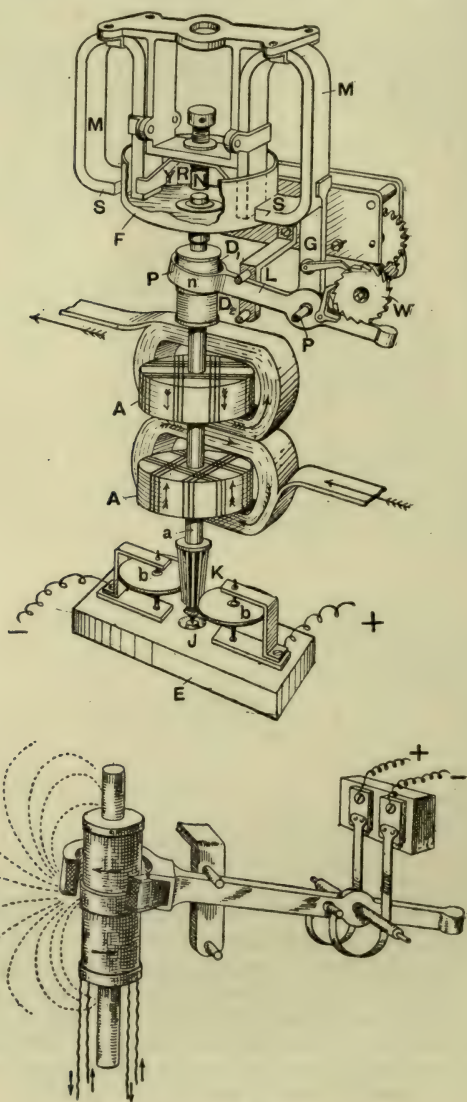


FIG. 21.—Evershed Meter.

solenoid with a "consequent pole" in the middle, and this solenoid is in series with the armature. It is therefore traversed by a current altered in direction every revolution. This coil is embraced loosely by a magnet with ring-shaped end (see Fig. 21), and this magnet acts as a lever to work the train. The ring-pole of the magnet is, of course, lifted and lowered once every revolution.

How perfectly the inventor has overcome friction may be seen from the fact that in most forms of known motor meters the frictional torque varies from 57 dyne-cm. to 1,400 dyne-cm., whereas in the frictionless meter it varies from 1.7 dyne-cm. when new to 3.5 dyne-cm. after long use.

The Paper of the inventor (*Journal Inst. Elec. Eng.*, Vol. XXIX., p. 743) and the discussion which followed are well worth careful study. The Paper is reproduced in *The Electrician*, Vol. XLV., pp. 283, 328, 438, 562 (1900).

The differential clock principle described by Profs. Ayrton and Perry in 1882, has been most fully developed by H. Aron. (See *Brit. Pat. Specs.*, No. 6,770 of 1884, No. 8,856 of 1887, No. 6,876 of 1888, No. 18,916 of 1890, Nos. 12,602 and 21,354 of 1891 and Nos. 11,822 and 11,823 of 1892; also see *The Electrician*.)

The principle of this meter has already been explained. It can be made as an ampere-hour meter or as a watt-hour meter. In its most widely used form as a house meter it is made up as a watt-hour meter. The meter case contains two clock motions, one having an ordinary gravity pendulum and the other a bob which consists of a fine wire coil which is part of the shunt or potential coil and is joined in series with a fixed resistance. Under this is the controlling coil which carries the main current. The two clock mechanisms are connected differentially to a counter, so that the counter is only moved by the difference in their rates. The electromagnetic pendulum is then slightly accelerated by the magnetic force between the shunt and series coil and is affected therefore by the power given to the circuit. Provided that the difference in the rates of the pendulums is not great, the movement of the counter records energy in watt-hours delivered to the circuit. The objection has been raised that this meter ticks, but its accuracy and its utility both for continuous and alternating currents cause it to be extensively used. It made in forms suitable for two and for three-wire circuits.

An interesting form of magnetic meter has been invented by Messrs. Mordey and Fricker (see Fig. 22). In this meter a coil (shown removed from its place in the diagram) carries the current to be measured, and within it is suspended a slate disc having little bars of soft iron embedded in it like the needles of a Kelvin compass card. This disc is suspended by a torsionless fibre, and is geared to the escape-ment of a clock mechanism. The normal position of the

soft iron rods is inclined to the plane of the coil. When the current flows it creates a force which tends to set the iron in the direction of the field of the coil. The disc is therefore set in oscillation, and drives the clock at a rate proportional to the current strength. Hence the reading of the clock can be made to record ampere-hours or quantity.

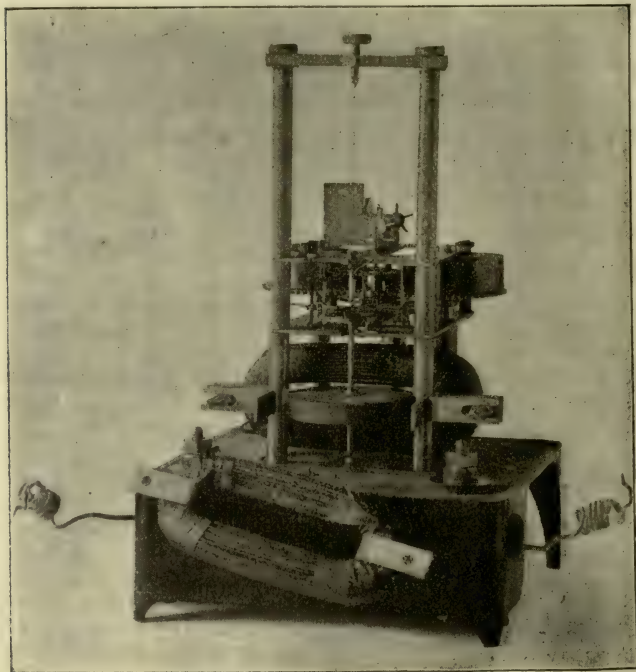


FIG. 22.—The Mordey and Fricker Meter.

There are, in the next place, a number of types of meters called induction meters, which are only suitable for use with alternating currents, and even in this case have generally to be adjusted for some particular frequency. These meters depend for their action on the production of either magnetic polarity in an iron disc or of eddy currents in a metal disc by an alternating current, and the action upon these induced

poles or eddy currents of another alternating current which is out of step with the first. They may be either ampere-hour or watt-hour meters adapted for the measurement of quantity or energy. As an instance of an induction ampere-hour meter we may mention that invented by Mr. O. B. Shallenberger. (*See* "Cantor Lectures" at the Society of Arts on "The Practical Measurements of Alternating Currents," 1893, by J. A. Fleming.)

The Shallenberger meter may be said to consist (*see* Fig. 23) of a small transformer, the primary coil of which is in series with the circuit in which the current to be metered is flowing. The core of this transformer consists of a little soft iron disc, which is capable of revolving on an axis. This axis is geared at the top with a counting mechanism, which records the number of revolutions of the disc, and at the bottom there is a vane or fan of thin aluminium, which serves to retard the rotation of the disc. The secondary circuit of this transformer consists of a small coil of copper, which is closed upon itself, and which is placed with its axis inclined at 45deg. to the axis of the primary coil. When the primary current flows through the primary coil it does two things—it magnetises the core and it induces a secondary current in the closed secondary circuit. It is not difficult to show that the phase of this secondary current must be about 90deg. behind the phase of the primary current, and also that the magnetism of the iron core, which is in a direction at right angles of the plane of the primary coils, also lags in phase behind the primary current by about 90deg. The magnetism of the core and the induced secondary current are, therefore, in step, and are in such directions that the axis of the disc is always being pulled round by the induced field of the secondary coil. If, then, there were no friction of any kind, the iron disc would be continually accelerated in speed, but since the air friction varies approximately as the square of the velocity, and since the mean driving force is proportional to the mean square of the current strength, it follows that the total number of revolutions which the disc makes in any given time is proportional to the total mean quantity or ampere-hours which have passed the primary circuit. The meters can therefore be calibrated by a constant in such a way that they read directly ampere-hours, and if the pressure between the mains is kept constant, they may be graduated to read in Board of Trade units.

These meters, being very simple to construct and very fairly accurate in performance, came into extensive use in the early days of alternating-current distribution. The velocity of the disc being at any time proportional to the mean current passing through the meter, we can, if the current is kept tolerably constant, employ the instrument as an ammeter. By moving the position of the secondary coil a little adjustment can be made in the meter for change of frequency, and the meter can be calibrated for the particular frequency for which it is intended to be used.

A meter of an analogous nature is the induction meter devised by Mr. A. Wright and Mr. Ferranti jointly, and

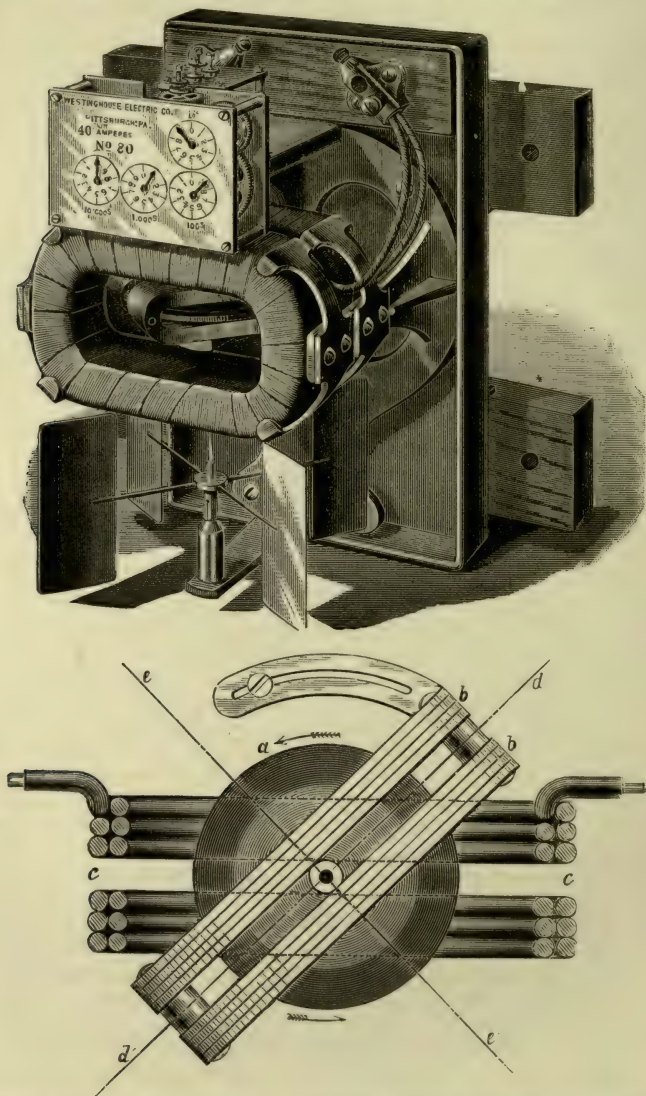


FIG. 23.—The Shallenberger Alternating-Current Ampere-Hour Meter.

based upon an earlier form by MM. Borel and Paccaud. (See *Brit. Pat. Specs.*, No. 1,383 of 1888, by Borel and Paccaud ; No. 1,480 of 1889, by A. Wright, Alderton and Philpott ; No. 9,061 of 1890, by S. Z. de Ferranti.)

The Wright-Ferranti induction meter is very similar in principle to that of Shallenberger. It is a self-integrating alternating-current ampere-hour meter. The movable part consists of a wheel with a light iron rim connected to a revolution-counter. This wheel is nearly embraced by two curved horns of

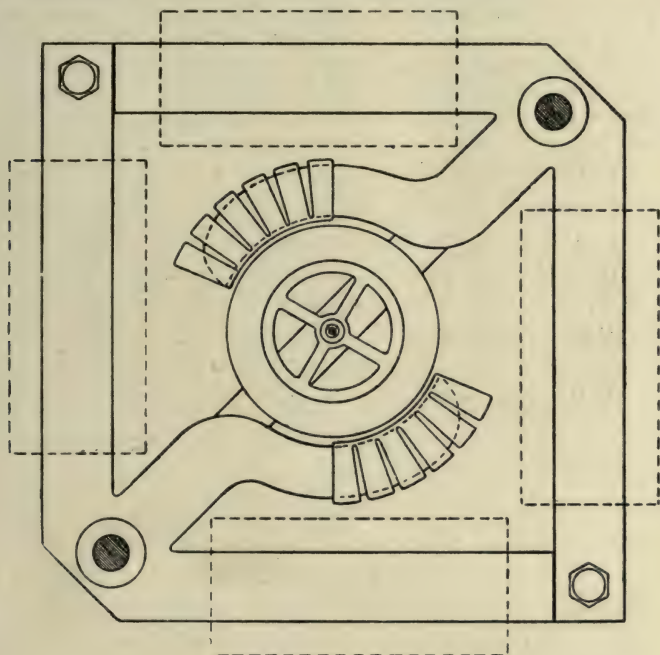


FIG. 24.

laminated iron (see Fig. 24). These horns spring from the corners of a rectangle of laminated iron. This laminated iron rectangle is wound over on all four sides with coils of wire. Through one pair of these coils the current to be metered passes ; the other pair of coils are traversed by a shunt current. The curved iron horns are embraced by certain closed copper bands which "throttle" them, and alternating magnetic poles are produced in these horns which progress along in proximity to the periphery of the iron disc or wheel. This procession of magnetic poles up the horns induces eddy currents in the disc, and it is thereby set in revolution by the magnetic repulsion.

The axis which carries the wheel has upon it four aluminium or mica vanes, and also is connected, as mentioned already, at the top of a counting mechanism. When an alternating current is passed through the meter it tends to drive the wheel round with a speed which is proportional to the mean-square strength of the current, and, therefore, it follows that the number of revolutions made by the wheel at a given time is proportional to the mean quantity in ampere-hours which has passed through the meter in that time.

A 20-ampere meter starts with a current of about  $\frac{1}{2}$  ampere, and reads up to about 20 amperes. Within these limits the greatest deviation in accuracy is only about 1·8 per cent. Both the Shallenberger meter and the Wright-Ferranti meter require to be calibrated for the particular frequency for which they are to be used, and they only give Board of Trade units provided that the pressure is kept constant between the supply mains. The meter really reads quantity directly, and power only on the assumption of the constancy of pressure. (For further details see *The Electrician*, Vol. XXV., p. 32, 1890.)

An example of an induction meter intended for use on single-phase, polyphase, inductive or non-inductive working circuits is that made by the Westinghouse Electric and Mfg. Co., under the patents of Messrs. Shallenberger, Davis and Conrad. It is a watt-hour meter recording correctly alternating current energy when delivered either to non-inductive or to inductive circuits. (See *Brit. Pat. Specs.* of O. B. Shallenberger, No. 148 of 1895, also H. P. Davis and F. Conrad, No. 10,705 of 1898, also catalogues of the Westinghouse Electric and Mfg. Co.) Its construction is as follows:—

In the form patented by Mr. Shallenberger the movable portion consists of a disc of aluminium,  $d d$  (see Fig. 25), the axis of which is geared to a revolution counter, not shown in the diagram. This disc runs between the poles of permanent magnets  $F F_1$  which create eddy currents in it, and therefore a retarding force. In proximity to the upper side of this disc is placed a highly inductive shunt coil  $A$ , wound over a laminated iron core,  $a a_1 a_1$ , the magnetic field due to which is nearly 90deg. in phase behind the impressed electromotive force of the circuit to be metered. Under the disc are one or two main or series coils,  $B B_1$ . These coils, when traversed by alternating currents, induce eddy currents in the disc. If the main current is in step with the potential difference of the power-absorbing circuit, which is the case when this latter is

non-inductive, then the field due to the main coil and that due to the current in the shunt coil differ in phase by  $90^\circ$ . Since the eddy currents induced in the disc are  $90^\circ$  in phase behind the inducing fields the eddy currents due to the main coil are in step with the field due to the shunt coil, and hence the disc is driven round by the force due to the shunt field on

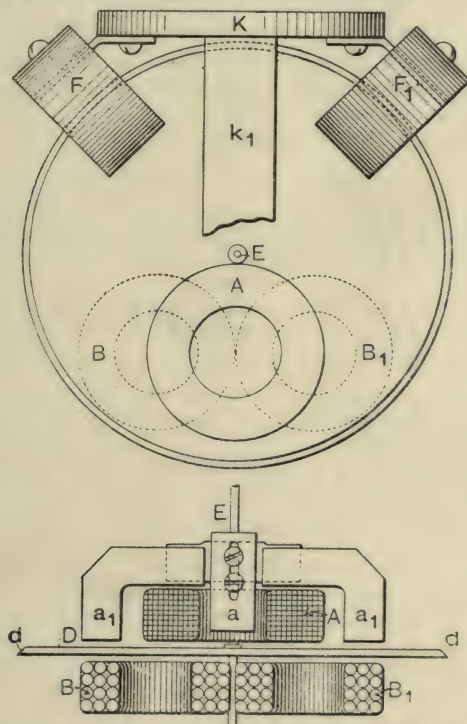


FIG. 25. —Westinghouse-Shallenberger Meter.

the eddy currents set up by the main coil. If, however, the main current lags in phase behind the potential difference of the power-absorbing circuit, then its field comes more completely into coincidence with that of the inductive shunt coil; and when the difference of phase of the current and potential difference is  $90^\circ$ , which is the case when the

power-absorbing circuit is inductive, then the fields due to the meter main coil and meter inductive shunt coil are in step, and hence the meter disc does not revolve. It is easy to adjust the meter so that the number of revolutions per second of the disc is made to be proportional to the true energy which has passed through the meter, and accordingly this number of revolutions in any given time is proportional to the true energy which has passed through the meter.

The theory of this Westinghouse-Shallenberger meter may be stated as follows:—Let  $e$  be the potential difference of the ends of the shunt coil of the meter, also let  $e = E \sin pt$  where  $p = 2\pi n$ , and  $n$  = the frequency. Let  $i_1$  be current through this inductive shunt coil, then

$$i_1 = \frac{E}{Im} \sin (pt - \phi) = I_1 \sin (pt - \phi),$$

where  $\phi$  is the phase difference of  $e$  and  $i_1$  and  $Im$  is the impedance of the shunt circuit. If  $\phi$  is nearly 90deg., we may write

$$i_1 = I_1 \cos pt.$$

Hence we may say that the field due to the shunt coil is proportional to  $AE \cos pt$ , where  $A$  is some constant.

Then, let  $i$  be the current in the series coil of the meter, and we have

$$i = I \sin (pt - \theta),$$

where  $\theta$  is the phase difference of the current and electromotive force in the power-absorbing circuit.

The field due to this coil is in step with the current in it, and the electromotive force inducing the eddy currents in the disc varies as the time-rate of change of this field, and is, therefore, proportional to  $\frac{di}{dt}$  or is expressed by

$$e' = pI \cos (pt - \theta).$$

Again, the eddy currents themselves, being produced in a practically non-inductive circuit—viz., the aluminium disc—are in step with the electromotive force producing them. Hence, we can say that the eddy currents  $i'$  set up in the disc are represented in magnitude by  $i' = A'I \cos (pt - \theta)$ , where  $A'$  is some constant, and the field due to the shunt coil, in which the disc moves, is proportional to the quantity  $AE \cos pt$ .

The mechanical torque acting on the disc at any instant is proportional to the product of the eddy currents created in the disc by the series coil of the meter, and the field in which they exist is due to the shunt coil. Hence, the mean mechanical torque on the disc is proportional to the mean value of the product  $i'i$ , or to the mean value of  $AA'EI \cos pt \cos (pt - \theta)$  throughout a period. Hence, calling this mean torque  $F$ , we have

$$F \propto \frac{1}{T} \int_0^T EI \cos pt (\cos pt - \theta),$$

or

$$F \propto \frac{1}{2} EI \cos \theta.$$

But this last is the expression for the mean power  $P$  given up to the power-absorbing circuit. Accordingly,

$$F \propto P.$$

Now, the retardation  $R$  acting on the disc, due to the action of the fixed permanent magnets, varies as its angular velocity  $\frac{dN}{dt}$ , where  $N$  is the number of rotations made by the meter up to that time, or is the meter reading.

Therefore, 
$$R \propto \frac{dN}{dt},$$

or 
$$dN \propto P dt = dE,$$

and 
$$F \propto P.$$

When the meter speed is constant we have  $F=R$ , and therefore

$$P \propto \frac{dN}{dt},$$

We see, therefore, that the increment in the meter reading in any time is proportional to the increment  $dE$  in the energy  $E$  given to the power-absorbing circuit in that time, having regard to the difference of phase of the current and voltage. In other words, the meter is correct on an inductive circuit.

In the form devised by Messrs. Davis and Conrad, the Westinghouse watt-hour meter consists of an aluminium disc geared to a counting train, and revolving between the poles of permanent magnets to create a retarding force. The disc is also embraced between the poles of a laminated iron electro-magnet wound with several distinct coils and having an air-gap in its circuit.

The general appearance of the single-phase meter is shown in Fig. 26, and the electro-magnet is shown diagrammatically in Fig. 27.

It will be seen that the electro-magnet consists of a laminated ring iron circuit with an air-gap at  $V$  and a magnetic shunt circuit across  $H$  (see Fig. 27).

In the gap  $H$  the above-mentioned aluminium disc revolves. The projection on the lower part of the iron ring carries the series coil of the meter, and the shunt coil is wound in two sections on the upper part of the iron ring. The shunt circuit is made highly inductive by adding an inductance coil of special form in series with it. Hence, there is a difference in phase of nearly 90deg. between the series coil

current and the shunt coil current when the working circuit is non-inductive; and if the latter is inductive, then the series coil currents and shunt currents come more into step

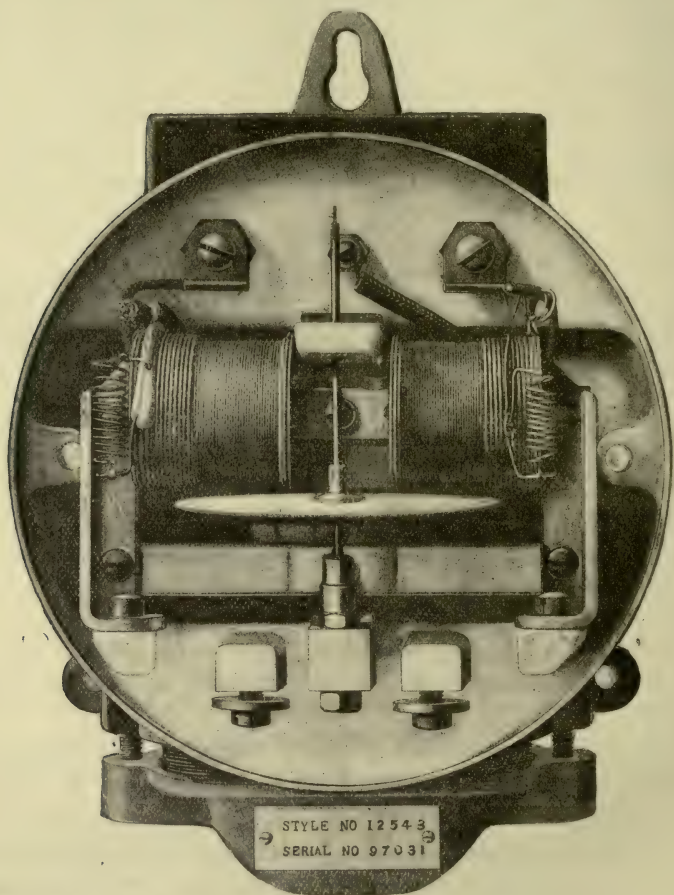


FIG. 26.—Westinghouse Davis-Conrad Meter.

with each other. The action of the meter is as follows:— Let us first suppose the working circuit of the meter to have a power factor of unity or to be inductionless, then, when the

series current is at its maximum, the shunt current is zero. The magnetic flux created in the iron core by the series coil passes round right and left in the iron ring and returns equally out of the two pendant poles across the gap H. As this series current dies away the shunt current increases, and causes the flux to issue more out of one pendant pole than the other.

Finally, when the series current is zero, the shunt current is a maximum, and the flux is annulled across the gap H, but exists across the gap V.

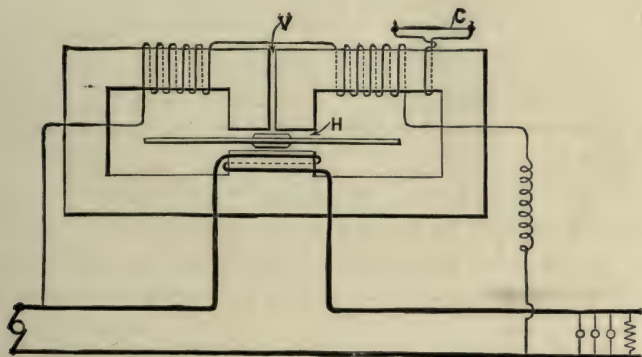


FIG. 27.

Then, as the series current rises again in value, the flux issues out of one of the pendant poles and gradually spreads across until it issues from both these poles equally. Accordingly, during each period, there is a shifting across of a stream of magnetic flux from one pendant pole to the other, and this lateral movement causes the stream of magnetic flux to sweep through the aluminium disc, thus creating eddy currents in it and repelling the disc round.

If the working circuit is inductive, then the series and shunt currents come more into step with each other, and when this is the case the lateral movement of the flux across the gap H would cease; in the limit, the flux passing across

from the series pole to one of the shunt poles but remaining fixed in position. The mathematical theory given above for the Shallenberger meter applies with slight modifications to the Davis-Conrad form of inductive meter. The driving torque acting on the disc is proportional to the sine of the angle of phase difference between the current and E.M.F. in the working or power-absorbing circuit. Hence, when this last circuit is highly inductive, the torque diminishes nearly to zero.

Other things being equal, the shunt current varies as the frequency. Hence, an adjustable resistance coil has to be added in series with the shunt coil to correct for any change in frequency. In the diagram (Fig. 27) will be seen a small short-circuited coil C. This is to produce a small permanent want of symmetry, and so compensate for running friction in the meter train.

Both the above-mentioned meters measure true energy, whether supplied to inductive or non-inductive circuits. They can be adapted for single, two-phase or three-phase energy measurements. The forms just described are the single-phase meters.

The polyphase meters can be made to read true energy whether the circuits are balanced or not, and no matter what may be the power-factor.

This Westinghouse meter recommends itself for alternating-current circuits by its accuracy and small internal losses and strong construction, and especially by its ability to measure true power delivered to inductive circuits. The 100-ampere meter starts with 0.25 ampere, and has a voltage drop in the main coil of 0.1 volt and a power loss in the shunt of 1.3 watts. (For further details the reader is referred to a Paper by Mr. Wilmshurst on "Electricity Meters," *The Electrician*, Vol. XXXIX., p. 410.)

Further details of the Westinghouse watt-hour alternating-current meters are as follows: Frictional error is eliminated by making the revolving point—viz., the aluminium disc—

very light. The disc and shaft weigh only 15 grammes, and are supported on a sapphire bearing. There is little or no temperature error, because the variation in conductivity of the disc affects alike the strength of the eddy current driving the disc and those due to the permanent magnets retarding the disc. The meter starts reading with a current equal to  $\frac{1}{2}$  per cent. of its full rated capacity. The disc revolves 50 times a minute when measuring its full rated energy, and the speed is proportional to the load.\* The number of revolutions of the disc in two minutes gives, therefore, the percentage of the full normal load of the meter which it is then carrying.

The internal meter losses are kept small. The shunt coil loss is 1.25 watts, and the series coil loss is reduced as much as possible consistently with maintaining an effective driving torque. A very similar type of alternating-current ampere-hour meter has been designed by Chamberlain and Hookham. In this meter a copper disc revolves between two flat series coils placed on its underneath side and an inductive shunt coil placed on its upper side (*see* Fig. 28).

The changes necessary to adapt the meters for the measurement of polyphase energy on two and three-wire circuits are very simple. In the single-phase Shallenberger watt-hour meter there are two series coils. These coils are so wound that they present opposite magnetic faces to the disc and are set one a little before, and one a little behind the shunt coil (*see* Fig. 25, where A is the shunt coil and B and B<sub>1</sub> the series coils). The series coils are never made to carry more than 5 amperes, hence in large size meters the house-current is reduced by means of a series transformer. The watt-meters for voltages exceeding 400 volts are made with 100-volt shunts and with a shunt potential-reducing transformer. In the single-phase meter the necessary lag of 90deg. in the phase of the shunt current behind the circuit electromotive force is secured by means of an inductance coil in series with the shunt. In the two-phase meter the shunt for one side is

connected across the adjacent circuit wires. In the case of an equally-balanced three-wire single-phase meter, the meter may be connected to the circuit through a transformer having

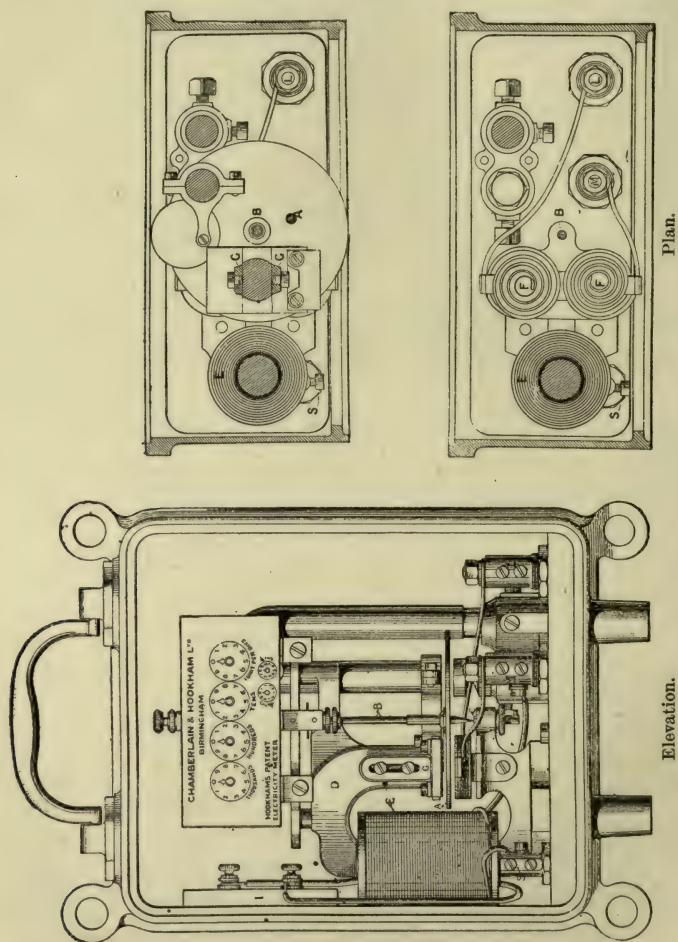


Fig. 28.—Chamberlain and Hookham Alternating-Current Meter.

two primary coils and one secondary coil. These primary coils are placed in the circuit of the outers of the three-wire circuit, and hence the current in the secondary of this

transformer is proportional to the sum of the currents in the two primary coils. The shunt circuit of the meter is connected between the middle wire and one of the outside wires, whichever will give the proper direction of rotation. This connection, of course, assumes that the two sides of the circuit are equally loaded.

The arrangement of the Shallenberger meter when employed with polyphase circuits can be best understood by considering the following diagrams, which are taken from Shallenberger's *Brit. Pat. Spec.*, No. 148, 1895. Let us consider, in the first case, two-phase currents supplied from a two-phase alternator to two separate power-absorbing

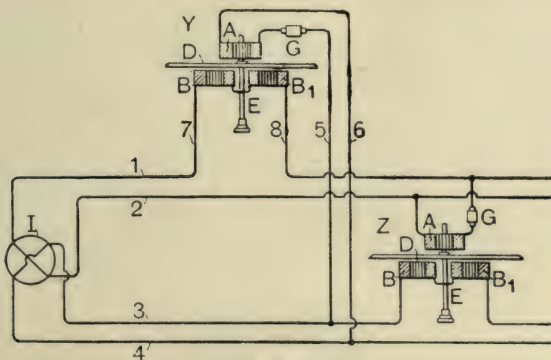


FIG. 29.

circuits. If the power absorbed on the two sides is different in amount, and if the two circuits have a separate return, then the power must be measured by employing two single-phase meters, one on each circuit (*see* Fig. 29). If, however, the power transmitted to each of the two circuits is identical, then a single meter may be modified so as to measure at one operation the whole power. This is done by inserting the two series coils underneath the revolving plate in one circuit and inserting the corresponding inductive shunt coil, above the revolving plate, across the other circuit. Since these two circuits normally differ in phase 90deg. as regards

their E.M.F.s, the necessary phase difference for the meter shunt coils is secured (*see* Fig. 30).

The meter may be also modified by employing two shunt coils and two separate series coils, as shown in Fig. 31.

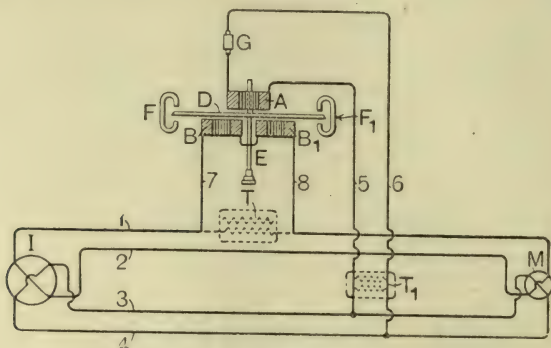


FIG. 30.

In this case the two series coils are inserted in the two separate two-phase circuits, and the shunt-coil co-operating with each series coil is joined across the circuit in quadrature

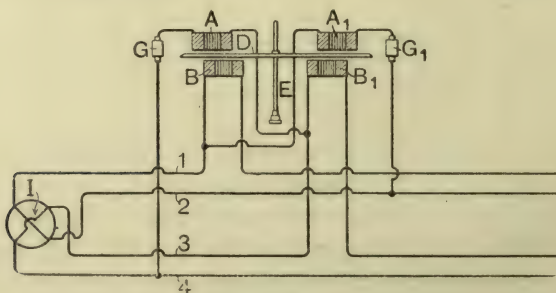


FIG. 31.

with that which contains the series coil ; hence the necessary phase difference for the respective associated shunt and series coils is obtained. Each pair of coils exerts its separate torque on the revolving disc, and, therefore, the total torque is the

sum of the two. The meter, therefore, reads the whole power transmitted to the power-absorbing circuit at one operation. If, however, the two two-phase circuits have a common return, then one single-phase meter may be used to measure

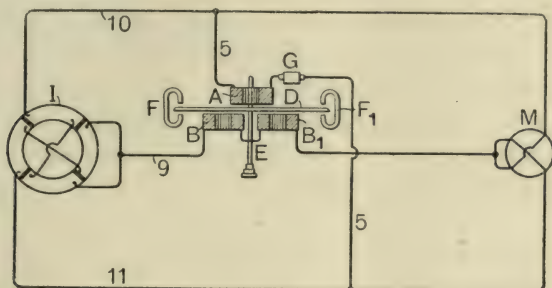


FIG. 32.

the whole power, as shown in Fig. 32, provided that the power is equally divided between the two sides.

It will be seen from the diagram in Fig. 32, that the two series coils are placed in the common return and the shunt coil is connected across the two outer mains. There is,

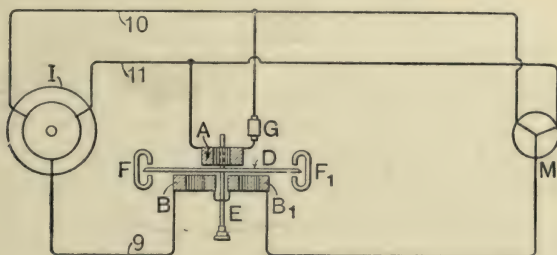


FIG. 33.

therefore, a difference of phase between the current in this shunt coil and the current in the two series coils of 90deg.

In the case of three-phase circuits, whether *star* or *delta*-connected, one single meter can be employed to measure the

whole power, provided that the power is equally divided between the three branches of the circuit. The method of connecting the coils is shown in Figs. 33 and 34.

In Fig. 34 the arrangement shown is that in which a transformer, *T*, is used to obtain the required phase relation of the E.M.F.s impressed upon the shunt circuit of the meter. As here shown, a rotating field is produced in the transformer by the connections 12, 13 and 14 to the three-phase circuit 9, 10, 11, and currents having the required phase relation are derived through the circuits 15 and 16 which are connected to the shunt coil of the meter.

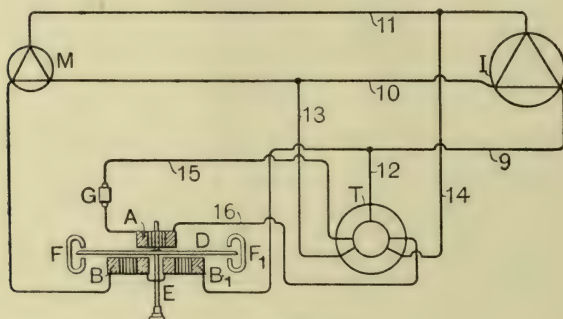


FIG. 34.

The transformer may be of any suitable form, and may contain a single coil or a separate primary and secondary, and should be connected to the transmission circuit in a manner suitable to obtain a rotating field. Two transformers may be used to produce the resultant E.M.F.s in the proper phase, if desired. In order to obtain the necessary inductance in the shunt circuit, Shallenberger employs a special form of inductance consisting of a magnetic circuit with a large air-gap, the proportion of air-gap to iron circuit being so arranged that the current through the inductive circuit is proportional to the E.M.F. at its terminals, but lags nearly 90deg. behind it.

§ 7. **The Testing of House Meters.**—The following is a list of the qualities into which inquiry should be made in pronouncing upon any particular house meter.\*

1. Accuracy. 2. Constancy. 3. Durability. 4. Starting current and range. 5. Drop in voltage through meter. 6. Back E.M.F., if any. 7. Accuracy of shunt ratio at all loads if shunted. 8. Noiselessness. 9. Impossibility of being tampered with by consumers. 10. Degree to which it is affected by stray magnetic fields. 11. Number of moving parts. 12. Internal consumption of power. 13. Defects in respect of clockwork, liquids likely to freeze, mercury contacts or friction at commutators. 14. Degree to which it is affected by vibration, damp, temperature, frequency (if alternating) or barometric pressure. 15. Whether available only for continuous or alternating currents or for both. 16. Facility for being checked in a short time. 17. Price. 18. Ease of transport. 19. Any necessity for exact levelling.

The above list of qualifications is not set down in order of importance, because circumstances must decide which, in the particular case, are the leading requirements.

In testing a meter it should first be carefully examined to see that it is in order, and then fastened up to a wall or placed on a steady shelf, and if necessary carefully levelled. In the case of some motor meters with revolving parts, the exact levelling is a matter of importance, and if this is the case then a spirit level or plumb line with levelling screws should form part of the instrument.

Having seen that the meter runs true and made a preliminary trial run with a current, a judgment can be formed as to a number of its qualities, such as noiselessness, liability to be affected by vibration, moisture, barometric changes and stray magnetic fields. This last quality can be tested by holding magnets near the meter and seeing how it is affected. The mode of sealing the meter case and its ease or difficulty of transport, its price and range, can be noted and criticised. Before beginning a long test the meter should be set to work for some time on varying loads to see generally how it performs, what is its range, and what liability there is to stop or fail in working.

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\* See Mr. T. P. Wilmshurst. Paper read before the Municipal Electrical Association at Manchester, 1897. *The Electrician*, Vol. XXXIX., p. 408, 1897.

Having in this way obtained a general idea of the value of the meter, it can be tried with different currents. If available both for alternating and continuous currents, it should be tested with both to see if its operation is equally good in the two cases. If it is available for alternating currents, it must be tried carefully with various frequencies extending at least over the range of variation likely to be found in practice.

These preliminary trials having decided many of the qualifications of the meter, the next step is to make a careful test for accuracy and for starting current. If the meter is a continuous-current meter, intended for a constant potential circuit, it should be connected up to a bunch of incandescent lamps, or to a power-absorbing resistance, and set to work. If the meter is an ampere-hour meter, then in series with the meter must be put a calibrated ammeter, or, better still, a low-resistance strip for potentiometer measurement (*see* § 5, Chap. I., Vol. I. of this HANDBOOK), and a potentiometer connected up to it for making current measurements (*see* § 10 Chap. III., Vol. I. of this HANDBOOK). Even if the meter is an ampere-hour meter it is well to connect a correct voltmeter across the terminals of the power-absorbing circuit to enable the constancy of the P.D. to be determined. If the meter is an energy meter or watt-hour meter, then it is essential that this P.D. shall be read at the same time as the current, otherwise a properly calibrated wattmeter may be used in connection with the power-absorbing circuit. Also a good clock or watch must be at hand to give time intervals. An important matter in connection with meter testing is the use of a good chronometer. A common watch with seconds hand is not generally sufficiently accurate. Messrs. Chamberlain and Hookham have introduced a convenient form of stop watch for this purpose.

The true value of the power or current supplied to the circuit is then obtained from the readings of the ammeter and voltmeter, or from the potentiometer or wattmeter. The power readings may be quickly taken with a potentiometer

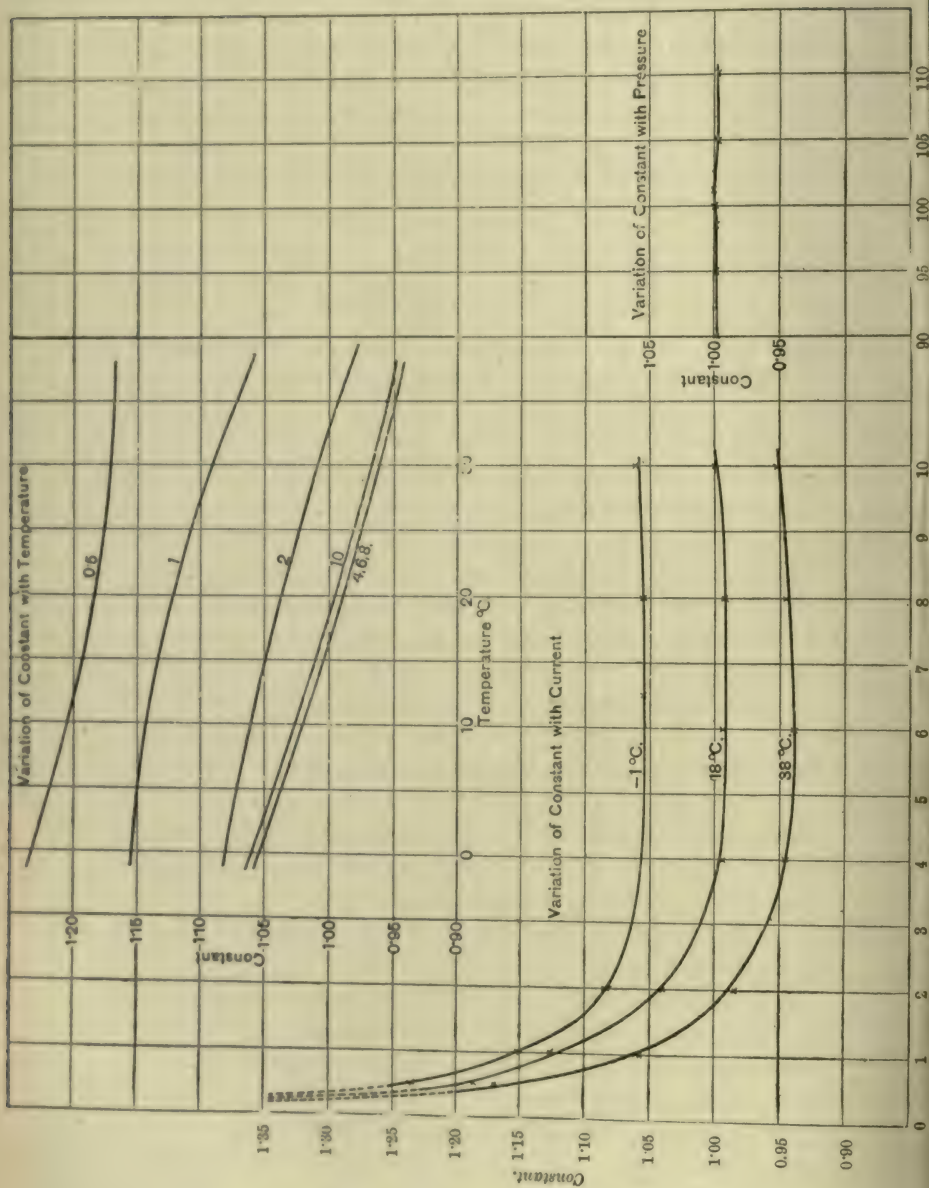
in the manner shown in Chap. V., § 2, of this HANDBOOK. The current or power delivery should be kept constant for a certain time, observed by the chronometer, and the product of time in hours or fractions of an hour, and the value of the current or power in amperes or watts, will give the quantity or energy delivered to the circuit. The meter readings by the dials should be taken at the beginning and end of that interval of time.

We have, then, the true observed value of the quantity or energy in ampere-hours or watt-hours and the meter reading equivalent to it. The *constant* of the meter is the number by which the reading of the difference between the two meter readings must be multiplied to give the true value of the quantity or energy passed in that interval.

If this measurement is carried out for different meter loads it is then possible to set out a curve called a meter constant curve, showing the variation of the meter constant for different meter currents. This curve would, of course, in the case of a perfect meter, be a straight line parallel to the axes of meter load. As a matter of fact, it never is a perfectly straight line throughout the whole of its course. The mode of delineating the variation of meter constant curves can be seen from the inspection of the curve here given (see Fig. 35) for a 10-ampere Thomson meter. These curves are taken from a Paper by Mr. G. W. D. Ricks (see *The Electrician*, Vol. XXXIX., pp. 573 and 601), "On the Variation of the Constants of Electricity Supply Meters with Temperature and with Current."

It will be seen that the meter constant is affected by temperature considerably.

It is customary also to represent the errors of the meter at various current loads by a curve the ordinates of which are proportional to the error expressed as a percentage at that load. Thus, if a 10-ampere meter is being tested, and it reads 5 per cent. too much when passing 5 amperes, this meter would be said to be 5 per cent. *fast* at half-load.



The curves given in Fig. 36) are such curves taken from a Paper by Mr. T. P. Wilmshurst, "On Electricity Meters" (see *The Electrician*, Vol. XXXIX., p. 408). From

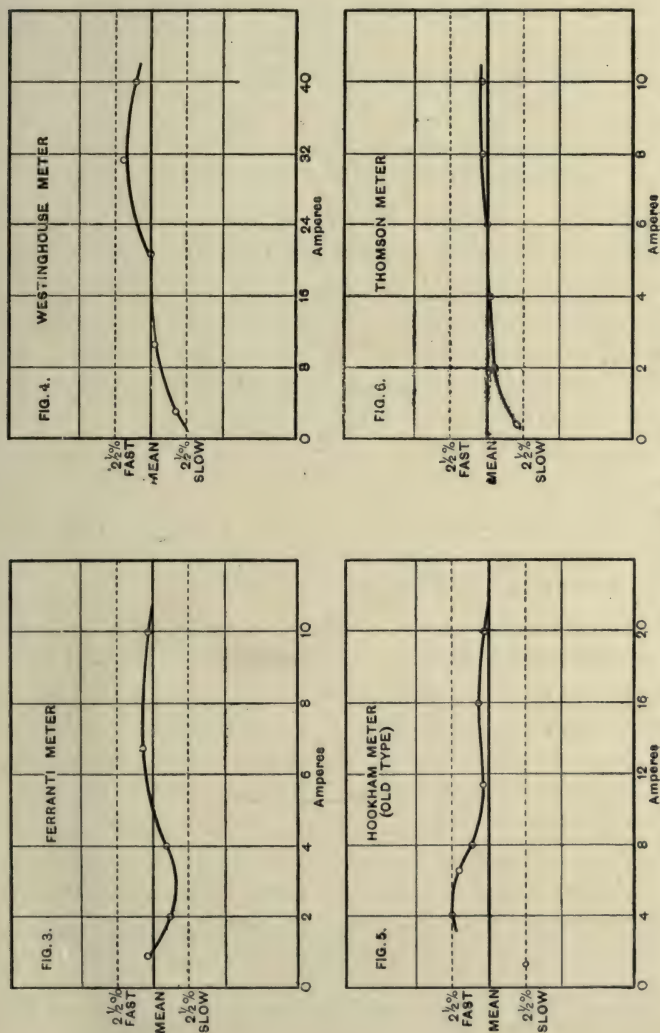


Fig. 36.

former have an advantage in one way from the point of view of the consumer, and the latter from that of the supply company. The shunt coil losses in the case of various meters have been stated to be as follows\* :—

Meter.	Shunt Loss in Watts.
Hookham alternating-current	... 1.5—2.5
Thomson watt-hour	... 2.3—3.3
Westinghouse watt-hour	... 1.3
Edmondson zero torque	... 1.0
Schæffer	... 1.0

The losses in the series coil are important at full load, but those in the shunt coil continue all the time. Hence it is important to the consumer to know whether the meter registers these internal losses against him or against the undertakers or supply company.

One most important matter is whether the shunt coil is so connected that the meter can “run on the shunt.” If the current which passes through the shunt passes first through the main coil, then it is clear that, if the pivot friction is diminished by vibration, the meter may start and register units when no lamp or consumer current is passing. This possibility should be examined.

7. *Variation of Reading with Power-Factor.*—In the case of meters used on alternating-current circuits an important point is, whether the reading is equally correct on a circuit of unit power-factor and on one with a fractional power-factor. If not, then it cannot meter correctly energy delivered to alternating-current motors.

8. *Testing a Number of Meters.*—The tests of all new meters, or meters suspected of inaccuracy, should be conducted systematically at a supply station. If the meters are ampere-hour meters they may be strung up in series, and the same measured currents passed through them for observed times, the meter readings being taken at the beginning and end of

\* “Electricity Meters” by T. P. Wilmshurst, *The Electrician*, Vol. XXXIX., p. 409.

the time. If energy meters are being tested, a similar connection may be adopted for the series coils, but both ends of every shunt coil must be connected by a separate pair of potential wires back to the main terminals of the first meter so that each shunt gets the same potential difference on it and the shunt currents are not reckoned into the main current. The record of these tests should then be entered up in a meter test book in such a form as is shown on pp. 98, 99.

9. *Testing Polyphase Energy Meters.*—Meters such as the Westinghouse polyphase energy meter are intended to record true energy delivered to a polyphase circuit, no matter what may be the power-factor. Hence they must be carefully checked, not only on a non-inductive circuit but on an inductive circuit of power-factor less than unity. As it is a difficult matter to construct an inductive circuit of known small power-factor and known large power-absorption, the best method is to check the polyphase energy meters on a circuit built up with transformers lightly loaded and with lamps in parallel, or, if possible, with condensers in parallel with lamps. The power delivered to this mixed circuit must be carefully measured by one or other of the methods suitable for this purpose described in Chap. V., Vol. I., of this HANDBOOK.

If the meter is a single-phase meter a carefully standardised dynamometer wattmeter may be employed, or else a quadrant electrometer as there described. In this way the true power delivered to the circuit in a given time may be measured, and, if the power delivery is kept constant, then the product of power and time gives the true energy delivered, and this can be compared with the meter reading. In the case of polyphase energy meters, the checking requires the assistance of two carefully standardised suitable wattmeters, employed as described in Chap. V., Vol. I., to measure the polyphase power.

The measurement of alternating energy on inductive polyphase circuits can be effected by one energy meter such as

former have an advantage in one way from the point of view of the consumer, and the latter from that of the supply company. The shunt coil losses in the case of various meters have been stated to be as follows\* :—

Meter.	Shunt Loss in Watts.
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Westinghouse watt-hour	... 1·3
Edmondson zero torque	... 1·0
Schæffer	... 1·0

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\* “Electricity Meters” by T. P. Wilmshurst, *The Electrician*, Vol. XXXIX., p. 409.

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The measurement of alternating energy on inductive polyphase circuits can be effected by one energy meter such as

that of Shallenberger, provided the branches of the circuit are equally loaded. In this case the meter has two series coils under the revolving aluminium disc (*see* § 6, p. 86). and one shunt coil, and these are connected up to the circuit branches as shown in Fig. 30. If, however, the branches of a polyphase circuit are unequally loaded, then the measurement of the energy absorption can only be effected by the use of two single-phase watt-hour meters of the Westinghouse type, which have their shunt and series joined up to the circuits in the same manner as if they were used on single-phase circuits (*see* Fig. 29).

Accordingly, if we have to measure energy delivered to an unequally loaded polyphase circuit, two meters of the Shallenberger form must be arranged with their series coils in the two separate circuits and their shunt coils connected through proper inductance coils across the circuits in quadrature as shown in Fig. 29. Two standardised wattmeters may then be arranged with their series and shunt coils in a similar manner, and be employed to check the readings of the energy meter. In this case the sum of the energy meters will be the whole energy delivered to the polyphase power-absorbing circuit.

The results of a meter test are entered up in a test sheet as follows:—

#### METER TEST. No.....

Name of maker of meter .....  
 Number of meter .....  
 Date of test .....  
 Name of tester .....  
 Place where test was made .....  
 Type of meter (*e.g.*, ampere-hour or watt-hour) .....  
 Class of meter (*e.g.*, electrolytic, intermittent self-integrating, or continuously self-integrating) .....  
 Remarks as to special form of meter (*e.g.*, two or three-wire, single-phase or polyphase, motor or graphic, prepayment or self-recording, direct-reading or with constant multiplier) .....  
 Range of meter in amperes .....  
 Voltage of meter, if an energy meter.....  
 Starting current in amperes .....

Starting current as percentage of full-load current.....	
Resistance of main coil.....	
Resistance of shunt coil ..	
Volt drop over meter .....	
Watts lost in series coil ..	
Watts lost in shunt coil .....	
How shunt coil is connected ( <i>i.e.</i> , whether on lamp or supply side of series coil).....	
Whether meter will run on shunts with tapping or raising volts .....	
Insulation of meter .....	
Tests for accuracy.....	
Meter reading when 0·1 of full current is passed for 10 hours .....	
Meter reading when 0·5 of full current is passed for 2 hours .....	
Meter reading when full current is passed for 1 hour .....	
Meter constant at tenth load .....	
Meter constant at half load.....	
Meter constant at full load .....	
General remarks as to performance or failure .....	

§ 8. **Battery Testing.**—In the testing of primary and secondary cells one portion of the investigation is generally concerned with the determination of the *quantity* and *energy efficiency* of the cell or battery. In the case of secondary cells this involves making a measurement of the electric quantity put into, and given out from, the cell under specified conditions, and also of the electric energy taken in and given out. In the case of primary batteries we are concerned with the quantity of energy given out in relation to the consumption of chemical materials in the cell.

The *efficiency* of any electrical device is the ratio between the energy or quantity given out by it and the energy or quantity taken in. Hence efficiency measurements of any kind always involve quantity or energy measurements which have to be made by some of the methods already described in this chapter. If the efficiency measurement to be made is that of a primary cell or battery, we have to ascertain the consumption of chemical material in the cell and determine how much of this chemical action is effective in producing the electric quantity or energy delivered to the external circuit.

Suppose the cell to be tested is one of the numerous forms in which the electro-positive element is zinc, the electro-negative graphitic carbon, and the electrolyte an acid or alkaline oxidising solution such as dilute sulphuric acid, having in it a so-called depolariser consisting of potassic bichromate, nitric acid, hypochlorate of soda, potassic permanganate or some one of the many secret solutions which have been devised. It is desired to test the cell. We proceed as follows :—

The zinc plate or rod is first carefully cleaned and amalgamated with mercury. All stray globules of mercury are wiped off and the plate is dried. It is then carefully weighed. The electrolyte is measured out and if possible its composition and cost is ascertained. The cell is then set up and the external circuit is closed through a known resistance coil, which must be made of platinoid or manganin and be of sufficient cross section or surface not to heat sensibly with the current employed. This resistance must be adjusted so that the cell sends the desired current through it. From the ends of this resistance potential wires are taken to a potentiometer, so that the P.D. at the terminals can be measured against that of a Clark or Weston cell.

Let  $V$  be the potential difference at any instant on the cell terminals and  $R$  the external resistance, then  $V/R$  is the current and  $V^2/R$  is the power given to the external circuit. If, then, a curve is plotted the ordinates of which are the values of  $V/R$  or  $V^2/R$  and the corresponding abscissæ are the times from the moment of starting the cell, the areas included by this curve and the base time-line and any extreme ordinates will represent to scale the quantity or energy given out by the cell.

This test should continue until the terminal voltage of the cell or the current has fallen to such fraction of its initial value that the cell may be considered to be exhausted. This will of course depend upon the work the cell is set to do. In the case of a primary cell for working miners' lamps,

the cell must be considered to be exhausted when the terminal voltage has fallen to 90 per cent. of its original value, such voltage being taken when the cell circuit is closed by a resistance which allows the passage of a current equal to that of the incandescent lamp which will be used with the cell. In the case of a cell used for telegraphic or bell purposes the exhaustion of current or terminal voltage might proceed much farther before the cell could be considered to be "run down."

At the conclusion of the test the zinc plate may be again washed, dried and weighed, and the loss in the weight of zinc determined.

Let us suppose that  $Q$  represents the time-integral of the current  $C = V/R$ , that is

$$Q = \int_{T_1}^{T_2} \frac{V}{R} dt,$$

where  $T_1$  and  $T_2$  are the times when the experiment began and ended, so that  $T_2 - T_1$  is the duration of the test. Then, if  $V$  is expressed in volts and  $R$  in ohms and  $T_2 - T_1$  in seconds,  $Q$  will be expressed in coulombs. Corresponding to this delivery of electric quantity to the external circuit, let  $W$  be the loss in weight, in grammes, of the zinc used up. The electro-chemical equivalent of zinc in coulombs per gramme is 2,967.10 (*see* Vol. I., p. 420). Hence, if the zinc used had all been usefully expended we should have had a delivery from the cell of 2,967.1  $W$  coulombs. We have, however, a delivery of  $Q$  coulombs. Hence the quantity efficiency of the cell is  $\frac{Q}{2,967.1 W}$ . Since the electro-chemical

equivalent of zinc in grammes per coulomb is 0.00033696, it follows that the difference between the weight  $W$  and the product 0.00033696  $Q$  is the weight of zinc uselessly consumed in the cell in local action. Hence the ratio  $\frac{0.00033696 Q}{W}$  may be called the zinc efficiency of the cell, as

it is the ratio of the useful consumption of zinc to the total consumption.

If this measurement is made with the same cell sending out the same electric quantity but in the form of various currents lasting different times, it will be found that the greater the duration of the experiment, the less is the zinc efficiency of the cell. This must obviously be, because the local action or wasteful consumption of zinc is continually taking place, and therefore bears a greater proportion to the useful consumption of zinc the greater the time over which the experiment is prolonged. In the absence, therefore, as to details, of the time of the experiment, a mere statement of the battery efficiency means nothing.

The results should always be stated by giving the weight of zinc used to produce a coulomb or ampere-hour of electric quantity, coupled with a mention of the time occupied by the delivery of that electric quantity. This gives the means of calculating the cost of producing one ampere-hour as any number of ampere-hours at a certain current rate, provided we know also the cost of this solution per gallon and the volume of it which is equivalent to a certain weight of zinc.

The chief facts which must be ascertained in order to appraise any primary battery, are (i.) The variation of its terminal voltage with time when sending certain measured currents. This can be set forth in the form of a series of curves, the abscissæ being *time in hours* and the ordinates of each curve the terminal P.D. of this cell when sending a certain current in amperes which is marked on the curve. We require to know—(ii.) The weight of zinc and electrolyte consumed in producing an ampere-hour or one watt-hour for various currents. This may be set out in the form of a curve, the abscissæ of which are currents in amperes and the ordinates the cost of producing one ampere-hour or one watt-hour as the case may be.

Measurements of the internal resistance of primary cells are not of much practical use or often required. A more

important matter is an inquiry into the rate at which the cell recovers its E.M.F. after having been called upon to give a current of known value. Minor points of importance are the weight of the whole cell for a given useful energy output in watt-hours—that is, an output within certain limits of decrease of terminal P.D.

For further information as to the cost of transformation of energy by primary batteries and on the subject of internal resistance measurement the reader is referred to the following sources of information :—

W. R. COOPER. “Primary Batteries : Their Theory, Construction and Use.” “The Electrician” Printing and Publishing Co., London.

E. WYTHE SMITH. “On the Measurement of the Internal Resistance of Cells.” See *Proc. Phys. Soc.*, London, 1892, Vol. XI., p. 342; also *The Electrician*, Vol. XXIX., p. 258.

The author describes a method of measuring the internal resistance of a cell which is a modification of the well-known Mance method. The internal resistance can be measured corresponding to any current sent out from the cell, and also enables us to find the resistance of the cell without altering the current through it.

HOLDEN and PITKIN. “An Instrument for Testing Primary Batteries.” See *The Electrician*, Vol. XXXI., p. 200.

This is a direct-reading voltmeter which enables a cell terminal-potential difference in volts to be immediately ascertained, corresponding to an external resistance of 8 ohms and of 2 ohms. From a simple geometrical diagram, the internal resistance is then obtained, or supposed to be obtained, as follows :—On squared paper is set off to the left hand abscissæ respectively of 8 and 2 units, representing the external circuit resistance. Along a perpendicular axis are then set off distances proportional to the corresponding observed cell terminal-potential differences. Inclined lines are then drawn as shown in the diagram, (see Fig. 37), and produced until they meet. The distance to the right of the axis of the point of intersection gives the internal resistance. Since, however, the latter is a function of the current the cell is sending, the value obtained for the internal resistance cannot have very much real value.

For a criticism of the above method see a letter by Mr. E. C. Rimington in *The Electrician*, Vol. XXXI., p. 295.

H. S. CARHART. “The Variation of Internal Resistance of a Voltaic Cell with Current.” *The Electrician*, Vol. XXXV., p. 18, also the *Physical Review*, Vol. II., No. 5.

PAUL SCHOOP. "On the Internal Resistance of Secondary Galvanic Elements." *The Electrician*, Vol. XXXV., p. 187.

The author gives measurements of the internal resistance of a secondary cell and discusses the various causes affecting it.

In testing secondary cells or batteries, we have to measure the electric quantity or energy put into the cell and compare it with that which can be recovered, the charging and discharging having certain specified values. If a single cell has to be tested, the most accurate method of measuring the in-going or out-coming current and terminal voltage is by the

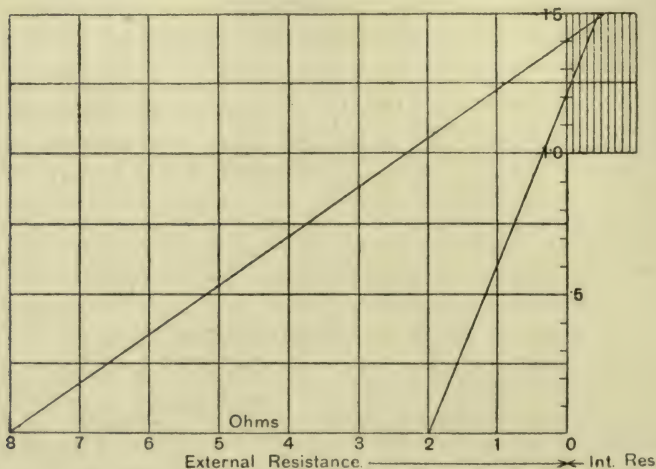


FIG. 37.

potentiometer. If a large battery has to be tested, then some form of graphic recording ammeter and voltmeter, such as those of Holden-Pitkin or Elliott Bros., is very useful. (See Vol. I., pp. 390 and 450.)

If these instruments are first carefully checked, the curve drawn on the paper gives us by its integration the quantity in ampere-hours taken in or given out and at the same time the current value itself is recorded. Then the simultaneous record of the terminal voltage is taken, and enables us at once to find the energy taken in and delivered.

In the case of a secondary cell, every test should be preceded by a careful measurement of the weight of the cell or battery and of the total weight of all the plates, since the results must be stated in watt-hours or ampere-hours capacity per pound of plates or per pound of cell.

In connection with a secondary cell we have two efficiencies to consider—viz., the *ampere-hour* or *quantity efficiency* and the *watt-hour* or *energy efficiency*. Since the cell requires 2·5 impressed volts to charge it and gives out current at an average voltage of about 1·9 or 2 volts, it is clear that the energy efficiency can never exceed 80 per cent. The quantity efficiency may, however, be made to be much higher provided the cell is charged and discharged slowly.

We have furthermore to define what is to be understood by the terms “a charged cell” and “a discharged cell.” A cell is said to be charged when the electrolyte boils freely and has a milky appearance, due to the free evolution of oxygen and hydrogen gases against the plates. A cell is considered to be discharged when its terminal voltage has fallen down to or below 1·8 volts, as thereafter the voltage is not useful for incandescent electric lighting. The capacity in ampere-hours must always be stated for a particular current or duration of discharge. The usual plan is to state the capacity of the battery or cell in ampere-hours for a 9-hour discharge for a 6-hour, 3-hour and 1-hour rate. The greater the rate at which the cell is discharged the less will be its apparent capacity in ampere-hours. The charging rate must always be stated also in defining the ampere-hour efficiency.

The process, therefore, of testing the ampere-hour efficiency is as follows :—

The cell or battery is first slowly discharged, and the charging is then commenced. The charging current is kept constant and at a known value and the ampere-hours noted which have to be put in at this rate before the cell or cells boil. This quantity is the charge at a certain ampere rate or time rate. The cell or cells are then discharged in a

certain time (9, 6, 3 or 1 hour as the case may be) and a record kept of the ampere-hours delivered before the cell voltage falls to 1.8 or to an average of 1.8 per cell. This last quantity is the discharge.

The ratio of discharge to charge is the ampere-hour efficiency, and it may be stated in a percentage for, say, a 9-hour charge and 6-hour discharge, or  $x$  hours charge and  $y$  hours discharge, coupled with a statement of the charge put in. Thus, we might say a certain cell had given to it 200 ampere-hours at a uniform 10-hour rate and its ampere-hour efficiency is 90 per cent. at a 9-hour discharge, 80 per cent. at a 6-hour discharge, 70 per cent. at a 3-hour rate discharge, and 50 per cent. at a 1 hour discharge.

The energy measurements can be taken at the same time as the quantity measurements if the voltage is observed. The most important energy measurement is the determination of the energy stored up in the cell or battery per pound weight when fully charged and the fraction of this which can be released when the battery is discharged at a certain rate.

At the same time that the charge and discharge readings are taken, the specific gravity of the electrolyte should be observed and compared with the electric quantity left in the cell at that time. The condition of the plates after heavy discharges should be noted, and any deposit of active material which falls out of the plates to the bottom of the cell should be collected and weighed. Additional facts concerning the cell that are required in its appraisalment are—

- (i.) Its ability to withstand vibration without damage.
- (ii.) Its ability to yield heavy discharges without plate deterioration or capacity deterioration.
- (iii.) Power to remain unused, or but little used, without great loss in capacity.
- (iv.) Power to recuperate its capacity under treatment after being but little used for some time.

(v.) Effect of time in the positive and negative plates respectively.

(vi.) General durability under specified conditions of use.

The majority of the above questions can only be answered by tests lasting a very considerable time, and any attempt to rest an opinion of a secondary cell *entirely* upon brief laboratory tests generally results in disappointment. The experience of use for a considerable time is the *only* test on which the electrical engineer can properly judge of the value of a secondary cell. Hence, whilst innumerable forms of secondary cells have appeared, the vigorous survivors amongst them are few.

Results of secondary battery tests may be set down in the following form:—

#### TEST OF A SECONDARY CELL (OR BATTERY).

Place of test .....	
Tester's name.....	
Date or dates of test.....	
Type of cell (e.g., whether pasted plates or Planté plates) .....	
Maker of cell or battery .....	
Number of cells tested .....	
Size of cell .....	
Material of containing vessel (glass, lead, wood or ebonite).....	
Number of plates per cell.....	positives ..... negatives
Size of plates .....	
Weight of the whole cell .....	lbs. ....ozs.....
Weight of all the plates .....	lbs. ....ozs.....
Specific gravity of the acid .....	
Normal charging current.....	
Actual charging current .....	
Time of charge .....	
Ampere-hours put into the cell .....	
Cell voltage at beginning of charge .....	
Cell voltage at end of charge .....	
Specific gravity at the beginning of charge .....	
Specific gravity at the end of charge .....	
Discharge current .....	
Time of discharge .....	
Total ampere-hours taken out of the cell down to 1·8 volts at terminals...	
Ampere-hour efficiency at a discharge of.....	amperes.....
Energy in watt-hours put into cell at a charging current of.....	
amperes .....	

Energy in watt-hours taken out of cell at a discharge of.....	amperes	
Energy efficiency at a discharge of.....	amperes.....	
Ampere-hour capacity per pound of cell .....		
Ampere-hour capacity per pound of plates .....		
Watt-hour storage per pound of cell.....		
Watt-hour storage per pound of plates .....		
Ampere-hour capacity at a 1-hour discharge .....		
"    "    "    3    "		
"    "    "    6    "		
"    "    "    9    "		
Watt-hour storage at a 1-hour discharge .....		
"    "    3    "		
"    "    6    "		
"    "    9    "		
General remarks on the make of cell and on the effect of use on its plates, &c., &c., and matters referred to in paragraphs numbered (i.) to (vi.) above .....		

For further information on secondary cells the reader is referred to the following Papers and books:—

G. H. ROBERTSON. "The Chemistry of the Secondary Cell."  
*The Electrician*, Vol. XXVII., p. 165.

He gives a valuable resumé of early work and references to the most important Papers published up to 1891.

GLADSTONE and TRIBE. "On the Cause of the Changes of Electromotive Force in Secondary Batteries," *The Electrician*, Vol. XXIX., pp. 67, 95. See also *Journal Inst. Elec. Eng.*, Vol. XXI., p. 412.

This is a valuable Paper, read before the Institution of Electrical Engineers in 1892. It deals with the chemical changes in the cell during action and repose.

W. E. AYRTON, C. G. LAMB and E. W. SMITH. "Notes on the Chemistry of Secondary Cells." *Journal Inst. Elec. Eng.*, Vol. XIX., p. 660, 1890.

E. J. WADE. "The Chemical Theory of Accumulators." *The Electrician*, Vol. XXXIII., pp. 603, 625, 657, 688, 722.

This is an excellent summary of the subject.

W. R. COOPER. "The Electrochemical Reactions of Secondary Cells." *The Electrician*, Vol. XXXV., p. 290.

A discussion of the double sulphating theory of the secondary cell—viz., that the material formed and broken up on both plates is plumbic sulphate.

"Additional Notes on the Theory of the Lead Secondary Cell." *Phil. Mag.*, Vol. XXXI., 1891, p. 42.

*The Electrician*, Vol. XXXIV., p. 339. "A List of Secondary Battery Patents Expiring in 1895."

H. A. EARLE. "Storage Batteries." *The Electrician*, Vol. XXXV., p. 685.

A very useful summary of the subject, with good diagrams.

M. BARNETT. "The Evolution of the Storage Battery." *The Electrician*, Vol. XXXVII., p. 90; also *Journal of the Franklin Institute*, Vol. CXLII., No. 844.

The author gives a description of various forms of cell and plate. The discussion on the Paper is useful, and deals with the difficulties of the various theories of chemical actions taking place in the lead secondary cell.

E. J. WADE. "Storage Batteries." *The Electrician*, Vol. XXXIX., pp. 716, 823; also Vol. XL., pp. 124, 192, 286, 350, 444, 591 and 757.

E. J. WADE. "Secondary Batteries; Their Theory, Construction and Use." "The Electrician" Printing and Publishing Co., London.

CAHEN and DONALDSON. "A Comparison between the Effect of Charging a Secondary Cell at Constant Pressure and at Constant Current." *The Electrician*, Vol. XLI., pp. 675 and 710.

The authors show that charging at a constant voltage gives the cell a larger capacity than charging at constant current.

*The Electrician*, Vol. XXXIX., p. 368.

A diagram and description of a small accumulator cell used for voltmeter testing at the Berlin Reichsanstalt.

E. S. JACOB. "A Method of Measuring the Insulation of Storage Cells." *The Electrician*, Vol. XXXV., p. 855.

C. L. WEBER. "A Convenient Miniature Accumulator or Secondary Cell for Testing Purposes." *The Electrician*, Vol. XXVIII., p. 218.

B. HOPKINSON. "The Losses of Energy in Accumulators." *The Electrician*, Vol. XLVIII., p. 211.



## CHAPTER II.

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### THE MEASUREMENT OF CAPACITY AND INDUCTANCE.

§ 1. **Definition of Capacity.**—The electrical capacity of a conductor is measured by the quantity of electricity or electrical charge which must be given to it to raise it to unit potential when all other surrounding conductors are at zero potential. Hence, if  $Q$  is the charge,  $V$  the potential of the conductor under the above conditions, and  $C$  the capacity, then we have  $Q=CV$  or  $C=Q/V$ .

If the dielectric surrounding the conductor in question is a vacuum or a gas of some kind, then the value of  $C$  will be independent of  $V$  and independent also of the time of application of the charging E.M.F. to the conductor. If, however, the dielectric is a solid body, and especially if it is glass, resin or paraffin, then the capacity is not an absolutely definite quantity, but is dependent on the manner in which the charge has been given to the conductor, upon the potential and upon the time which has elapsed since the charging, and, generally, upon the whole previous history of the dielectric.

The energy stored up in the conductor is measured by  $\frac{1}{2}CV^2=\frac{1}{2}QV$ , and is affected by the same causes as the capacity. If the conductor is raised to unit potential very suddenly, and immediately discharged, the quantity of electricity recovered is a measure of the *instantaneous capacity*. If, however, the E.M.F., or charging potential, is continued for some time, an additional charge is gained, which is called the *soakage* or *absorption*.

If, then, the conductor is connected to earth, an instantaneous charge comes out, and finally if the conductor is insulated and left for some time, an additional discharge can be subsequently obtained, which is called the *residual discharge*. In the same manner, by successive discharge and insulation, a whole series of diminishing residual discharges can be obtained. The appearance of this residual discharge is facilitated by tapping or vibrating the dielectric. In the case, therefore, of solid dielectrics the term *capacity* has no definite meaning unless the circumstances of the charge and discharge are completely defined.

A *condenser* consists of a metal plate of some kind, which is surrounded by some dielectric, and this again by a conductor, which envelops the first more or less, and is kept at zero potential.

If the dielectric is air, the condenser is called an *air condenser*. If it is glass it is generally called a Leyden jar, or Leyden pan, and if of any other material, such as mica or paraffin paper, it is called a mica or paraffin condenser. A condenser may be employed merely as a standard of capacity, or as a means of storing electrostatic energy.

If the capacity can be altered by moving the plates, or in any other way, the condenser is called a *sliding condenser*. If the plates take such a form that the capacity can be calculated from the dimensions, the condenser is called an *absolute condenser*.

If the dielectric has some true electric conductivity superimposed on its dielectric quality, the condenser is called a *leaky condenser*.

The quality in virtue of which a condenser stores up electrostatic energy is called its *permittance*, and that in virtue of which it allows a true current of conduction to pass through it is called its *conductance*.

**§ 2. Standards of Capacity.**—The most simple absolute standard of capacity is a metal sphere insulated in air, all

surrounding conductors being at a great distance and at zero potential. Under these conditions the capacity of the sphere in electrostatic units is numerically equal to its radius in centimetres.

Since one electromagnetic unit of capacity is equal to  $v^2$  electrostatic units, where  $v = 3 \times 10^{10}$ , and since the British Association practical unit of capacity, viz., the farad, is equal to  $1/10^9$  of an electromagnetic unit, it follows that the usual working unit of capacity, the microfarad ( $= 1/10^6$  of a farad), is equal to  $9 \times 10^5$  electrostatic units. Hence, a sphere whose radius is 900,000cms., or 9,000 metres, or 9kms., or about 5.6 miles, has a capacity in free space of one microfarad. The capacity of the whole earth, considered as a sphere in space, is about 800 microfarads, and is equal to about the capacity of an Atlantic cable.

Hence, to reduce a capacity measured in electrostatic units or centimetres to the value expressed in microfarads, we have to divide the former number by  $9 \times 10^5$ .

As a standard of practical capacity, the objections to a sphere are that it is difficult to construct, and, moreover, a very large sphere is required to provide even a moderate capacity.

Another form of simple standard of capacity is a circular thin metal disc. It can be shown that the electrical capacity of a circular thin metal plate in electrostatic units, when insulated in space and at a great distance from all other conductors, is equal to  $2r/\pi$ , where  $r$  is the radius of the disc in centimetres; in other words, it is  $1/1.571$  of that of a sphere of the same radius. The above fraction is very nearly equal to two-thirds, and accordingly we can obtain a capacity not very different from that of a sphere, but in a much more convenient form, by employing a flat circular disc of metal.

If a flat circular metal disc is made, having a diameter of  $d$  centimetres, then, when hung up in infinite space, its capacity in microfarads is

$$\frac{d}{\pi \times 9 \times 10^5} = \frac{d}{2,827,431} \text{ mfd.}$$

If the diameter of the circular disc is  $d'$  inches, then the electrical capacity will be  $\frac{d'}{1,113,160}$  mfd.

For all practical purposes, therefore, we may say that a circular disc having a diameter of 283cm., or 2·83 metres, or rather more than 9ft., has a capacity in free space of  $\frac{1}{100000}$ th of a microfarad. To possess the same capacity a sphere must have a diameter of 180cm., or 1·8 metres.

The capacity of a sphere or a circular disc in infinite space can be deduced from the general mathematical expression for the capacity of an ellipsoid, of which the sphere and circular disc are special cases. The mathematical expression for the electrical capacity  $C$  of an ellipsoid having semi-axes  $a$ ,  $b$  and  $c$ , when charged and insulated in infinite space, has been given by Prof. Chrystal (see article "Electricity," *Encyclopædia Brit*, 9th edition). It is as follows :—

$$\frac{1}{C} = \frac{1}{2} \int_0^\infty \frac{du}{\sqrt{(a^2+u)(b^2+u)(c^2+u)}} \quad \dots \dots \dots (i)$$

If we put  $a=b=c=r$  in the above expression, we have

$$\frac{1}{C} = \frac{1}{2} \int_0^\infty \frac{du}{(r^2+u)^{\frac{3}{2}}}$$

Now

$$\int \frac{du}{(r^2+u)^{\frac{3}{2}}} = \frac{2}{(r^2+u)^{\frac{1}{2}}},$$

and this integral between the limits 0 and  $\infty$  has the value  $\frac{2}{r}$ . Hence, for the sphere, we have the well-known result

$$\frac{1}{C} = \frac{1}{2} \cdot \frac{2}{r} \text{ or } C = r. \quad \dots \dots \dots (ii)$$

Again, if we put in the general expression for the capacity of an ellipsoid,  $a=b$  and  $c=0$ , we have

$$\frac{1}{C} = \frac{1}{2} \int_0^\infty \frac{du}{(a^2+u)\sqrt{u}}$$

The integral

$$\int \frac{du}{(a^2+u)\sqrt{u}} = \frac{2}{a} \tan^{-1} \frac{\sqrt{u}}{a},$$

and this last, between the limits 0 and  $\infty$ , is equal to  $\frac{\pi}{a}$ . Hence, for the circular disc, the capacity is

$$\frac{1}{C} = \frac{1}{2} \frac{\pi}{a} \text{ or } C = \frac{2a}{\pi}. \quad \dots \dots \dots (iii)$$

It is, however, particularly to be noted that these expressions for the capacity of a sphere and a disc are only true if surrounding conductors are at great distances. A disc of metal 5ft. or 6ft. in diameter, hung up evenly in the centre

of a large room, would have a capacity from 10 to 20 per cent. greater than that given by the formula  $C = 2a/\pi$ . The same would hold good for a sphere. Even if out of doors, and yet held within a short distance of the ground, the actual capacity of a sphere or disc would certainly exceed by a notable percentage the capacity given by the formulæ  $C = r$  or  $C = 2a/\pi$ . Accordingly, these forms of standard capacity, though theoretically simple, are yet not practically very useful.

We may, however, obtain larger capacities in small bulk and more exact practical standards, the capacity of which is independent of locality, by using two conductors.

As a first example of such a non-variable standard of electrical capacity let us take the case of two concentric metal spheres. Let the radius of the inner sphere be  $R_1$  and that of the inside of the outer sphere be  $R_2$ . Then the capacity  $c$  in electrostatic units of the two spheres, the outer being maintained at zero potential and the inner at potential  $V$ , is given by

$$c = \frac{1}{\frac{1}{R_1} - \frac{1}{R_2}} \text{ electrostatic units,}$$

where  $R_1$  and  $R_2$  are measured in centimetres. Hence, the capacity,  $C$ , in microfarads of the two concentric spheres of radius  $R_1$  and  $R_2$  centimetres is given by the expression

$$C = \frac{R_1 R_2}{9 \times 10^5 (R_2 - R_1)} \text{ mfd.}$$

In the next place, let us take the case of two parallel conducting planes separated by air. It is not possible to calculate precisely the capacity of the condenser so formed unless we know the exact distribution of electric force at the edges of the planes, and this is difficult to determine. We can, however, annul the difficulty by the device of a *guard ring*.

Let us suppose a glass plate to be covered with tinfoil, and a very narrow circular or rectangular cut to be made so as to create a disc of metal surrounded by a ring or enclosing

metal plate in the same plane, but separated from the first by a very narrow air-gap. Let another sheet of metal, much larger than the delimited area of the first plate be placed parallel to the first-named plate and at a distance  $d$  centimetres from it. Let the delimited area of the first plate be  $S$  square centimetres. Then, if the plate and its surrounding guard-plate are kept at a potential  $V$  and the opposed plate at zero potential, we have a condenser formed of the area  $S$  of the one plate opposed to the larger second plate, such that all the lines of electric force which proceed from the plate of area  $S$  to the other are parallel and the electric force everywhere the same.

If the quantity of electricity in the delimited condenser plate is equal to  $Q$ , then its density is  $Q/S$ , and the electric force between the plates is equal to  $4\pi Q/S$  and also to  $V/d$ . Hence, we have

$$\frac{4\pi Q}{S} = \frac{V}{d} \text{ or } c = \frac{Q}{V} = \frac{S}{4\pi d}$$

where  $c$  is the capacity of the condenser so formed in electrostatic units.

If  $C$  is the capacity in microfarads, then

$$C = \frac{S}{9 \times 10^5 \times 4\pi d} = \frac{S}{11,309,724d} \text{ mfd.}$$

Suppose, for instance, the area  $S$  is 1 sq. ft and  $d$  is 1mm. Then the capacity of the air condenser so formed is  $\frac{9}{11300}$  or  $\frac{1}{1250}$  microfarad nearly.

A condenser of the above kind can be practically made by silvering two large plates of flatted plate glass. With a sharp knife a rectangular or circular cut or scratch is made on one plate so as to delimit a square or circular area. This cut is best made of the shape shown in Fig. 1, being left unclosed so as to leave a tail piece to conduct a charge to the inner region. The two plates are then placed with silvered faces towards each other and are separated by fragments of microscopic glass placed at the four corners. The thickness

of these fragments must be the same, and must be determined by the sphereometer.

In charging and discharging the condenser to obtain its capacity by any method, the guard plate must not be discharged by the same route as the central portion but must have its charge removed simultaneously by some other circuit.

Another useful form of condenser consists of two co-axial metal cylinders. If we neglect the distribution of electric force at the ends of the co-axial cylinders, and consider merely a length,  $l$  centimetres, of the cylinders taken in the middle,

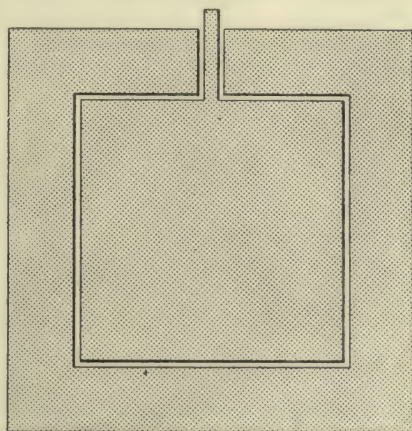


FIG. 1.

then, if  $R_2$  is the radius of the inside of the outer cylinder and  $R_1$  is the radius of the outside of the inner cylinder, and if the dielectric between them is air, the capacity  $c$  of a length  $l$  centimetres of the cylinders in electrostatic units is

$$c = \frac{l}{2 \log \frac{R_2}{R_1}}.$$

If the capacity in microfarads is  $C$ , and if the logarithms used are to the base 10, then, if  $M$  is the modulus 2.303, we have

for the capacity of a length of  $l$  centimetres the value

$$C = \frac{l}{18 \times 10^5 M \log_{10} \frac{R_2}{R_1}} = \frac{l}{4,145,400 \log_{10} \frac{R_2}{R_1}} \text{ mfd.s.}$$

If the dielectric between the cylinders has a dielectric constant, or specific inductive capacity,  $K$ , then the capacity of the length  $l$  centimetres of the cylinders is

$$C = \frac{Kl}{4,145,400 \log_{10} \frac{R_2}{R_1}} \text{ mfd.s.}$$

(Tables of the specific inductive capacities or dielectric constants of various solids and liquids are given at the end of this chapter.)

In practice, however, we have to eliminate the effect of the uncertain distribution of electric charge at the ends of the cylinders. This can be done in the following manner:—Let two cylindrical condensers be made, each consisting of two co-axial tubes of the same sizes, the condensers, however, being of different lengths,  $L_1$  and  $L_2$ . If, then, we measure in any manner the apparent capacities of these two cylindrical condensers and take the *difference* of these values, it will give us the capacity of a cylindrical condenser of the same radii but of length equal to  $L_2 - L_1$  and having no uncertain distribution at the ends.

We can practically construct a single condenser of the above type, having a calculable and therefore known capacity, by the employment of the principle of the guard ring, only in this case it becomes a *guard tube*.

A stout brass tube is made perfectly cylindrical on the outside, and is parted in two places, so as to make two short lengths and one long one.

The long cylinder (*see* Fig. 2) has its ends bevelled down and the short cylinders have one end similarly treated. These cylinders then have ebonite plugs fitted into them by means of which they are held slightly separated on a long

iron screw rod. A pair of ebonite end pieces are fixed outside the end cylinders by means of which an outer tube, made perfectly cylindrical on the inside, is then fixed over the plugs. The whole arrangement is held together by the screw rod with nuts on the ends, which passes down the centre. An insulated wire is soldered to the inside of the middle inner tube, and another to both the guard ring extensions, and these are brought out through holes in the ebonite plugs. We have, by this arrangement, a concentric cylinder condenser. If the inner and outer cylinders are kept at a difference of potential  $V$ , and the middle part of the inner cylinder is charged and discharged through a galvanometer, whilst the guard-tube parts are discharged through a separate bye-

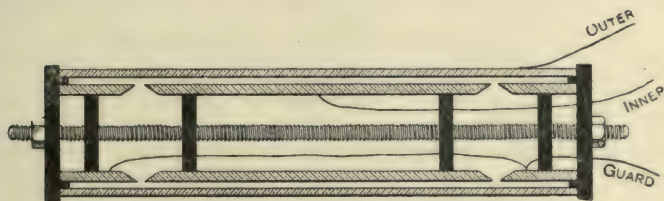


FIG. 2.

pass and not sent through the galvanometer, the quantity of electricity,  $Q$ , given to and taken from the middle inner cylinder can be measured, and the capacity of this part is  $Q/V$ . If, then, the radii of the outer cylinder (inside) and inner cylinder (outside) are known, and the length of the inner cylinder, we have the means of calculating the capacity of this middle part.

A condenser of the above kind was made for the Pender Electrical Laboratory, University College, London, and had the following dimensions:—The inner middle brass tube had a mean outer diameter as determined from 24 measurements of 1.977in., or 5.021cm. The length of this tube was 21.99cm. The length of the guard-tube extensions was about 1in. at each end, and the thickness of the insulation

between the effective or central part of the inner tube and the guard tubes was 0.085cm. Hence the effective length of the inner tube was taken as  $21.99 + 0.085$ cm., or 22.075cm.

The mean inside diameter of the outer tube was obtained by filling the tube with distilled water and measuring the volume of this water in a burette. The value so obtained was 710.8 cubic cm. Hence, the mean inside diameter of the outer tube was found to be 5.285cm.

Accordingly, in the above formula for the capacity of the concentric cylinder condenser, we have, in this case,

$$l = 22.075 \text{ cm.}$$

$$R_2 = 5.285 \text{ cm.}$$

$$R_1 = 5.021 \text{ cm.}$$

and, therefore,

$$C = \frac{1}{9 \times 10^5} \times \frac{22.075}{2 \log_e \frac{5.285}{5.021}} = 0.0002398 \text{ mfd.},$$

or, say, 0.00024mfd.

The above brass tube condenser forms a convenient *small* standard air condenser, its capacity being nearly  $\frac{1}{4000}$ th of a microfarad. It has been found to be a convenient comparison standard for the determination of the capacity of such condensers as small Leyder jars which are not required to be measured with any very great accuracy.

There are two interesting problems in electrical capacity calculations which meet us in telegraphy and in alternating-current work, viz., the determination of the capacity of a very long cylindrical wire insulated in infinite space and also the capacity of two parallel cylindrical wires considered as the coatings of an air condenser. The following is a preliminary lemma:—

(i.) *Let a metallic filament be uniformly charged with electricity so that it has  $q$  electrostatic units of charge per unit of length. It is required to calculate the potential at any point, P, outside the filament. From the point P drop a perpendicular on the wire and let it intersect it at O. Reckon distance along the wire from O. Let  $x$  be the distance of any element of the wire and  $dx$  the length of the element. Then  $qdx$  is its charge and the electric force at P, due to the element reckoned in the direction OP, is equal to  $\frac{rqdx}{(r^2 + x^2)^{\frac{3}{2}}}$ , where  $r$  is the length OP.*

Hence the total electric force at P in the direction OP due to the whole wire is

$$2 \int_0^{\infty} \frac{rqdx}{(r^2 + x^2)^{\frac{3}{2}}}.$$

The integral

$$\int \frac{r dx}{(r^2 + x^2)^{\frac{3}{2}}} = \frac{x}{r \sqrt{r^2 + x^2}},$$

and this, between the limits  $x=0$  and  $x=\infty$ , is  $1/r$ . Hence the whole electric force is  $2q/r$ .

Accordingly, if the potential at P is called V, we have

$$-\frac{dV}{dr} = \frac{2q}{r},$$

or

$$V = -2q \log r + C,$$

where C is the constant of integration.

This gives us the value of the potential at any distance  $r$  from an infinitely long charged filament.

(ii.) Let there be two circular-sectioned parallel straight wires at a distance D apart from centre to centre. Let the radius of each wire be  $r$ . Let them be considered as the surfaces of an air condenser. It is required to calculate the capacity per unit of length on the assumption that the ratio  $r/D$  is small.

Let us call one wire A and the other B, and let them be charged with charges of opposite sign. Then the potential at the centre of A =  $V_A$  is given by

$$V_A = (-2q \log r + C) - (-2q \log D + C),$$

and the potential at the centre of B is similarly given by

$$V_B = -(-2q \log r + C) + (-2q \log D + C).$$

Hence the difference of potential of the wires is

$$V_A - V_B = 4q(\log D - \log r) = 4q \log \frac{D}{r}.$$

Hence the capacity per unit of length is given by

$$c = \frac{1}{4 \log_e \frac{D}{r}} \text{ electrostatic units.}$$

This capacity, expressed in microfarads and ordinary logarithms, gives

$$C = \frac{1}{4 \times 2.303 \times 9 \times 10^9 \times \log_{10} \frac{D}{r}} \text{ mfd.},$$

or

$$C = \frac{0.0000001208}{\log_{10} \frac{D}{r}} \text{ mfd.},$$

where C in the last two equations is the value of the capacity per centimetre of run of the wires expressed in microfarads.

(iii.) To calculate the capacity per unit of length of a long (telegraph) wire placed parallel to the earth's surface and at a height  $h$  above it.

The earth's surface is a zero potential surface, and, therefore, the difference of potential between the wire and the earth is half that between two wires, one charged wire and the other at a distance D apart. Hence, since then  $2h=D$ , it follows that the capacity of a telegraph wire per unit of length having a radius  $r$  and suspended at a height  $h$  above the earth is

$$c = \frac{2}{4 \log_e \frac{D}{r}} \text{ electrostatic units.}$$

If we call the diameter of the wire  $d$ , then the capacity per unit of length is

$$c = \frac{1}{2 \log \epsilon \frac{4h}{d}}.$$

If we employ ordinary logarithms, and express the capacity per centimetre in microfarads, it becomes

$$\begin{aligned} C &= \frac{1}{2 \times 2.303 \times 9 \times 10^5 \log_{10} \frac{4h}{d}} \text{ mfd.s.} \\ &= \frac{0.0000002415}{\log_{10} \frac{4h}{d}} \text{ mfd.s.} \end{aligned}$$

(iv.) To calculate the capacity of a long circular-sectioned wire suspended in infinite space.

This is a reduced case of the ellipsoid. The capacity  $C$  of an ellipsoid in infinite space is given (see above) by the expression

$$C = \frac{1}{2} \int_0^\infty \frac{du}{\sqrt{(a^2+u)(b^2+u)(c^2+u)}},$$

where  $a$ ,  $b$  and  $c$  are the semi-axes. If we make  $b=c$ , then the expression gives us the capacity of an ellipsoid of revolution round its axis  $2a$ . The integral in this case becomes (see article "Electricity," *Encyclopædia Britannica*, 9th edition)

$$C = \frac{2 \sqrt{a^2 - b^2}}{\log \epsilon \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}}.$$

Let  $e$  be the eccentricity of the principal section, then

$$e = \frac{\sqrt{a^2 - b^2}}{a}.$$

Hence, if the ellipsoid is very long, in which case it becomes a rod, we have  $e$  nearly equal to unity, and hence

$$1+e=2 \text{ and } (1-e)(1+e)=2(1-e) \text{ very nearly.}$$

Therefore, in the case when  $b/a$  is a small fraction,  $a(1-e^2)=2a(1-e)$

$$\therefore \frac{a}{2} \frac{b^2}{a^2} = a - ae,$$

or

$$\frac{b^2}{2a} = a - \sqrt{a^2 - b^2}.$$

But

$$\log \frac{2a}{b^2} = \log \frac{4a^2}{b^2} = 2 \log \frac{2a}{b}.$$

Hence,  $\log \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}$  becomes equal to  $2 \log \frac{2a}{b}$ , when  $b/a$  is very small.

If, then, we have a circular-sectioned wire of length  $l$  and diameter  $d$ , where  $d/l$  is very small we may treat it as an ellipsoid of revolution whose

semi-axes are  $l/2$  and  $d/2$ , and the expression for its capacity,  $c$ , is

$$c = \frac{l}{2 \log_e \frac{2l}{d}} \text{ electrostatic units.}$$

If we employ ordinary logarithms, and express the capacity in microfarads, all dimensions being in centimetres, then the capacity  $C$  is given by

$$C = \frac{l}{2 \times 2.303 \times 9 \times 10^5 \log_{10} \frac{2l}{d}} \text{ mfd.s.,}$$

or

$$C = \frac{0.0000002415l}{\log_{10} \frac{2l}{d}}.$$

A form of standard air condenser has been designed by Dr. A. Muirhead. It consists of a number of concentric brass tubes set upon a stepped base to preserve them in position (*see* Fig. 3). Alternate tubes are connected together so as to form a multiple-plate air condenser. This condenser is not of a type of which we can predetermine the capacity by calculation. It is simply designed as an air condenser standard whose capacity has to be determined by appropriate methods, and has the advantage that it has no sensible leakage or residual charge. Hence its capacity is independent of time of charging or voltage. This is the only type of condenser which is really satisfactory as a laboratory standard. No condenser made with a solid or liquid dielectric is free from leakage or conductance and residual charge or soakage.

Nevertheless, an electrical laboratory should be provided with condensers of various capacities, the most convenient on the whole being those made with selected mica as dielectric. It is convenient to have a half microfarad standard (*see* Fig. 4) and also a graded condenser giving values from 10 to  $\frac{1}{16}$  microfarad.

For some purposes condensers made with paraffined paper can be employed. The best paper to use for this purpose is a tough paper called *butterskin*, but a well selected linen paper is also good. At least two thicknesses of paper should

be employed as a dielectric, to avoid leakage by minute holes. The paper should be dried at a temperature of  $120^{\circ}\text{C}.$ , to

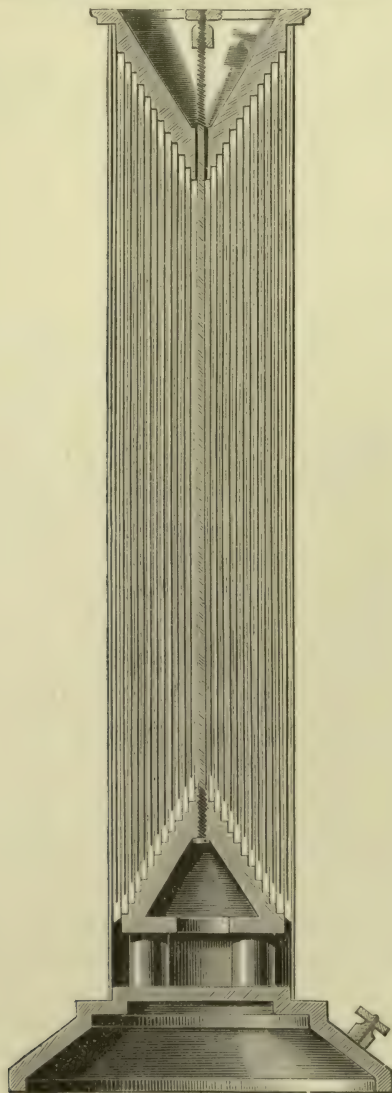


FIG. 3.—Section of a Muirhead Standard Air Condenser constructed by the Cambridge Scientific Instrument Company.

expel moisture, and be then well boiled in good melted paraffin wax from which all water has been removed.

The paper having been selected, dried and waxed, is cut into sheets, and pieces of tinfoil of the same length, but 2 in. less in width, are interposed. The paper and tinfoil sheets are laid so that each tinfoil sheet overhangs the paper by 1 in. on one side alternately, and, between every tinfoil, two sheets of paper are placed. When a sufficient number have been piled up, the whole mass is compressed in a hydraulic press and bound together. The projecting tinfoils on the two sides are then pinched together, and the whole mounted

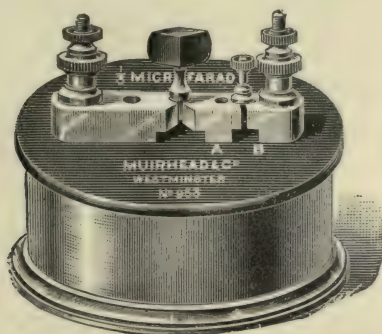


FIG. 4.—Standard Mica Condenser.

in a wooden box with ebonite lid. The box is run in with melted paraffin wax. Wires, previously soldered to the sets of tinfoil sheets, are brought out to terminals on the lid of the box.

Condensers made as above of paraffined paper are not suitable for working at pressures above 100 volts, although mica condensers can be charged up to a pressure of 1,000 volts.

For working at high tensions, and for use with induction coils in experiments on electrical oscillations, or for wireless telegraphy, the only dielectrics of any use are glass or micanite.

A condenser suitable for working with a pressure of 20,000 volts may be made as follows:—Glass plates are procured of

a quality known as 21oz. lime glass, and must be selected as far as possible free from bubbles and flaws. The plates may be conveniently square shape, and 15in. or 40cm. in the side.

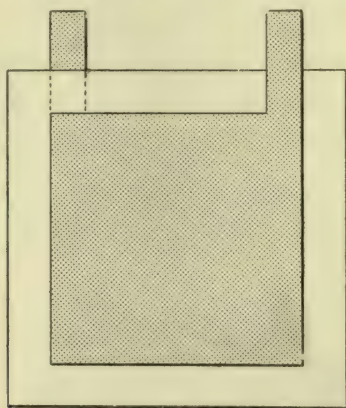


FIG. 5.

These plates should then have their sharp edges ground off and be made perfectly clean. Good thick tinfoil sheets are then cut 12in. square, and provided with a lug strip

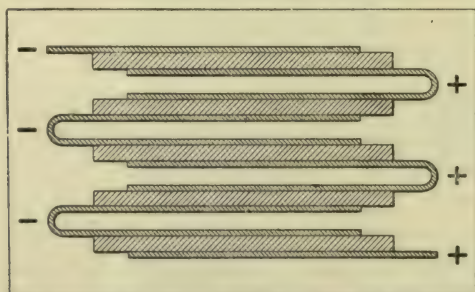


FIG. 6.

1in. wide and 6in. long. These tinfoils are to be affixed to the glass by shellac varnish or seccotine, with the lugs not opposite, but as shown in Fig. 5. Each glass sheet may

conveniently have strings or tapes fixed to the sides by which to lift it. A number of these plates are then placed in a stoneware box and filled in with linseed or vaseline oil.

In arranging the plates, the lugs must be so situated that adjacent sheets of tinfoil not separated by glass are at the same potential, and the relative disposition of glass and tinfoil sheets is as shown in Fig. 6. The whole of the lugs on each side respectively are pinched together and connected to a terminal.

If the glass sheets are, on an average,  $\frac{1}{8}$  in. thick, a condenser made as above described will stand charging with an E.M.F. of 20,000 volts.

### § 3. Determination of Capacity in Absolute Measure.—

If we are given a standard air condenser the capacity of which cannot be calculated from its dimensions, it can yet be determined in absolute measure by certain processes which involve only measurements referable to known and existing standards. We can thus determine the capacity in microfarads or fractions of a microfarad on an air condenser of the Muirhead pattern, and then employ this in comparison with other condensers.

We have thus two classes of capacity measurement: First, those which are generally called *absolute methods*, which require only the determination of a length, or time, or in addition of a current or potential, and, secondly, those which are called *comparison methods*, which merely tell us the ratio of one capacity to another supposed to be known.

The absolute methods are only available for application to standard air condensers in which the question of time of charging does not enter. The principal absolute methods are:—

(i.) By the ballistic galvanometer.

(ii.) By alternate charge and discharge at a known rate through a calibrated galvanometer.

(iii.) By alternate charge and discharge at a known rate and the use of a differential galvanometer.

(iv.) By using a condenser and vibrating switch, as one arm of a Wheatstone's bridge.

**§ 4. The Measurement of Capacity by the Ballistic Galvanometer.**—The ballistic galvanometer selected for this purpose may be either a movable coil or movable needle galvanometer, and should be set up as described in § 3, Chap. I. of this volume.

It must then have its *ordinary* logarithmic decrement  $\lambda$  carefully determined in the manner explained in § 3, Chap. I., of this volume (*see* p. 15), and also the complete periodic time,  $T$ , of its movable portion measured.

The angular deflection,  $\phi$ , of the movable part, produced by a voltage  $V$  acting through a resistance  $R$ , including that of the galvanometer, must be observed, and from these quantities the relation between the quantity of electricity  $Q$  and the resulting "throw"  $\theta$ , calculated by the formula

$$Q = \frac{T}{2\pi} \frac{V}{R \sin \phi} \theta \left( 1 + M \frac{\lambda}{2} \right),$$

where  $M$  is the modulus 2.303.

If the deflections are taken by the mirror and scale it must not be forgotten that the angular deflection of the ray of light is twice that of the mirror. Hence if the scale is, at a distance  $d$  from the mirror, and  $x$  is the displacement of the image on the scale, both  $x$  and  $d$  being in the same units, we have  $x/d = \tan 2\phi$ , where  $\phi$  is the angular deflection of the mirror or movable part of the galvanometer.

If, then, a standard air condenser having a capacity  $C$  is charged to a potential  $U$  and discharged through the galvanometer, and if this discharge produces an angular deflection of the needle or coil equal to  $\theta$  in absolute or circular measure, we have

$$CU = Q = \frac{T}{2\pi} \frac{V}{R \sin \phi} \theta \left( 1 + M \frac{\lambda}{2} \right),$$

or 
$$C = \frac{T}{2\pi} \frac{V}{U} \frac{\theta}{R \sin \phi} \left(1 + M \frac{\lambda}{2}\right).$$

If the battery used to charge the condenser is the same as that used to create the steady galvanometer deflection  $\phi$ , then  $U = V$ , and

$$C = \frac{T}{2\pi} \frac{\theta}{R \sin \phi} \left(1 + M \frac{\lambda}{2}\right).$$

If the angle  $\phi$  is small, we may, without sensible error, take  $\sin \phi = \phi$ , and the formula reduces to

$$C = \frac{T\theta}{2\pi R \phi} \left(1 + M \frac{\lambda}{2}\right).$$

This expression gives us the capacity,  $C$ , of the condenser in farads, if  $R$  is measured in ohms. If, however,  $R$  is measured in *megohms*, then  $C$  is expressed in *microfarads*. In the above formula  $T$  is the complete periodic time in seconds of the vibrating part and  $\lambda$  its logarithmic decrement in ordinary logarithms to the base 10.

The above described method is not applicable to any condenser which has a sensible soakage or residual discharge, because the discharge is not all completed before the galvanometer coil or needle has moved from its zero position, and the assumed conditions under which the formula was obtained no longer hold good. Hence, it is only applicable to air condensers of not very small capacity.

One difficulty which presents itself in making capacity measurements of air condensers is that we cannot employ very high charging voltages without running the risk of sparking across between the plates. Hence, generally speaking, the ballistic galvanometer method is not suitable for air condensers of very small capacity.

The relative capacity of two air condensers can be obtained by the ballistic galvanometer, by taking the "throws" given by each condenser when charged to the same voltage. The ratio of these throws is the ratio of the capacities.

The ballistic galvanometer must, however, be used only with caution and discrimination when making measurements

with condensers with solid dielectrics. The "throw" will then vary with the time of charging and with the interval of time which elapses between the end of the charge and the beginning of the discharge, and any comparison between condensers of different "soakages" are apt to yield figures of very doubtful value.

**§ 5. The Measurement of Capacity by Rapid Charge and Discharge through a Galvanometer.**—If a condenser of which the capacity is  $C$  microfarads is charged with a voltage  $V$  volts, and then discharged through a galvanometer,

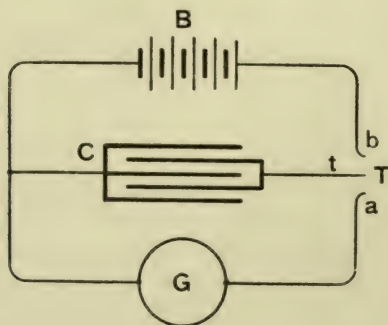


FIG. 7.

and if this process is repeated rapidly  $n$  times per second, the galvanometer is traversed by a series of discharges each equal to  $CV$  microcoulombs, which, following each other at the rate of  $n$  per second, produce an effect on the galvanometer equal to that of a current of  $CVn$  microamperes or  $CVn/10^6$  amperes. This process can be effected by several kinds of switch or rotating contact-maker, the effect of which is to alternately connect the one condenser terminal  $t$  to one galvanometer,  $a$ , and one battery terminal,  $b$ , the remaining condenser, galvanometer and battery terminals being connected constantly together (see Fig. 7).

The device which has been most often suggested and used for the purpose of making this rapid alternate contact is a tuning-fork driven by an electromagnet. Suppose such a tuning-fork provided, which makes, say, 100 vibrations per second, then one way in which we may arrange the apparatus is as follows:—The tuning-fork should be provided with a light flexible steel stylus, *t*, making contact between two fixed insulated pieces of metal, *a* and *b*, which may be screwed to a slip of ebonite (see Fig. 8). Then, as the tuning-fork vibrates, its stylus will make contact alternately between the top contact connected to the galvanometer *G* and the bottom connected to the battery *B*. The tuning-fork and its driving battery must

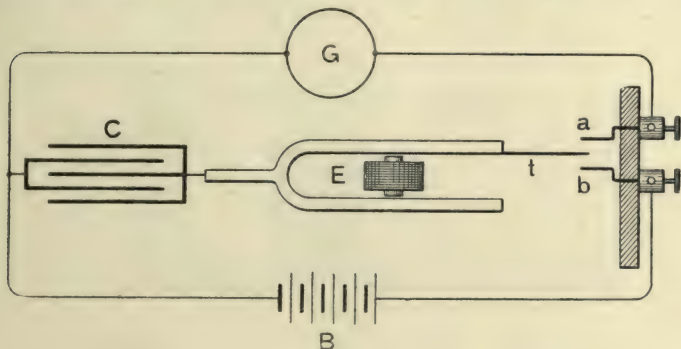


FIG. 8.

all be placed on a sheet of ebonite to insulate it, and the condenser the capacity of which is to be measured is connected to the tuning-fork. When these arrangements are made, on setting the fork in action the galvanometer should show a steady deflection.

There are, however, several practical objections to the above arrangement. The tuning-fork and its battery all form part of the capacity to be measured, and offer considerable opportunities for leakage. If the condenser to be measured has a small capacity, then considerable corrections would be necessary. Moreover, the stylus may make very uncertain

contact. It is apt to bounce on the contacts, owing to the violence of its blow. The condenser may, therefore, not be fully charged or discharged at each oscillation, and there is no means of ascertaining the exact duration of the contact.

Various other contact devices have been tried, such as providing each prong of the tuning-fork with a steel needle dipping into a mercury cup so that the fork is alternately connected to one of two mercury cups; but the splashing of the mercury soon makes the contacts uncertain.

Another plan which has been tried, with more success, is to use the fork to drive, in synchronism with itself, an electromagnetic vibrating break, like the vibrating contact interrupter of an induction coil, but having two contacts.

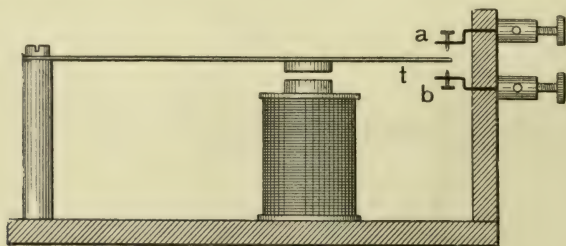


FIG. 9.

In this case a steel plate or rod is held in a fixed position over the poles of an electromagnet, and it vibrates between two screw contacts, *a* and *b* (see Fig. 9). The electromagnet is traversed by the same intermittent current which drives the tuning-fork, and therefore the steel plate vibrates in step with the fork. The steel plate therefore moves at the same rate, and makes contact alternately between its two screw stops. The mass or stiffness of this steel plate must be adjusted so that its natural free period of vibration is the same as that of the tuning-fork.

This device of a vibrating contact-maker driven by an electrically controlled tuning-fork has been used by Prof. J. J. Thomson (see *Phil. Trans.*, Roy. Soc. Lond., 1883) and

by Dr. R. T. Glazebrook (*Phil. Mag.*, August, 1884), and by the Author. It is, however, open to the objection that there is no means of controlling or knowing the exact time during which contact is made. Hence a better device is a revolving contact, of which various forms have been devised by the Author. One form was employed by Dr. Glazebrook in an investigation on the capacity of a standard air condenser, and consisted of a contact-breaker driven by a water motor (see Fig. 10).

In this case the speed of the commutator was determined by means of a stroboscopic disc attached to its pulley, this

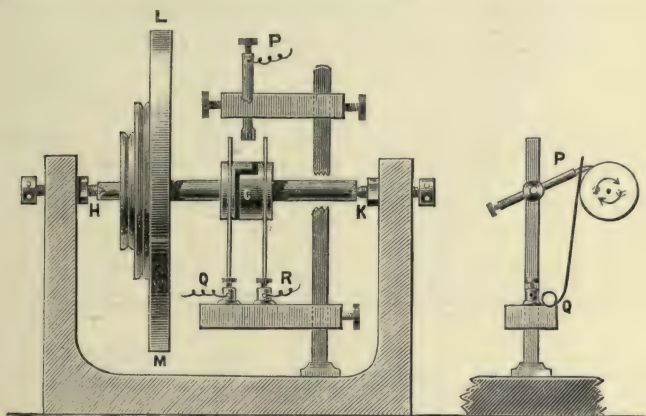


FIG. 10. —Rotating Commutator Arrangement employed in connection with the determination of the capacities of Air Condensers.

disc being observed through slits in the overlapping metal plates attached to the prongs of an electrically-driven tuning-fork (see a Paper read by Dr. Glazebrook before the British Association at Leeds, 1890, or *The Electrician*, Vol. XXV., p. 616).

After a long experience of different devices of the above kind, the Author, in conjunction with Mr. W. C. Clinton, has devised a form of motor-driven rotating commutator which is in every respect very satisfactory for use in condenser capacity

determinations. This arrangement, as shown in Fig. 11, consists of an electric motor of  $\frac{1}{4}$  H.P., with an associated starting and controlling resistance, which can be run off any constant voltage circuit of 100 or 200 volts. The speed of the motor may conveniently be 1,500 revs. per min.\* To the shaft of the motor is connected, by a flexible coupling, a commutator, which consists of two gunmetal discs, A, B (see Fig. 11), each having four projecting lugs, like a crown wheel. Between these is placed another wheel, I, with eight teeth. These three wheels are insulated from the shaft and from each other, and are turned up perfectly true on the surface. Against this drum press three brass wire brushes,

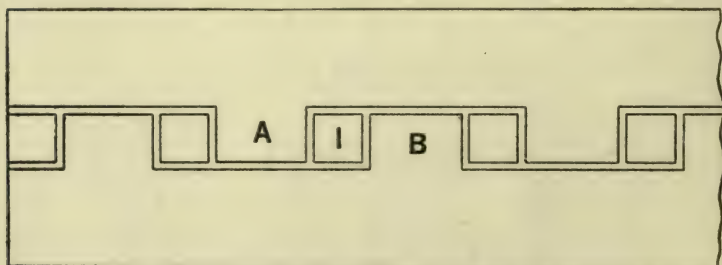


FIG. 12.

which are carried on insulating pillars, R. Two of these make contact on the outer flanges of the crown wheels, and the middle one on the central portion. The developed surface of part of the triple wheel is shown in Fig. 12.

It will be seen, therefore, that as the drum revolves the centre brush alternately makes contact four times in each revolution, or 100 times a second, first with the brush on the right and then with the brush on the left. The time of contact is also accurately known. There is no bouncing or uncertain contact, but a smooth, steady contact all the time.

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\* It is better, if possible, to employ a motor of larger size. The steadiness with which the commutator runs is increased by having plenty of power to drive it.

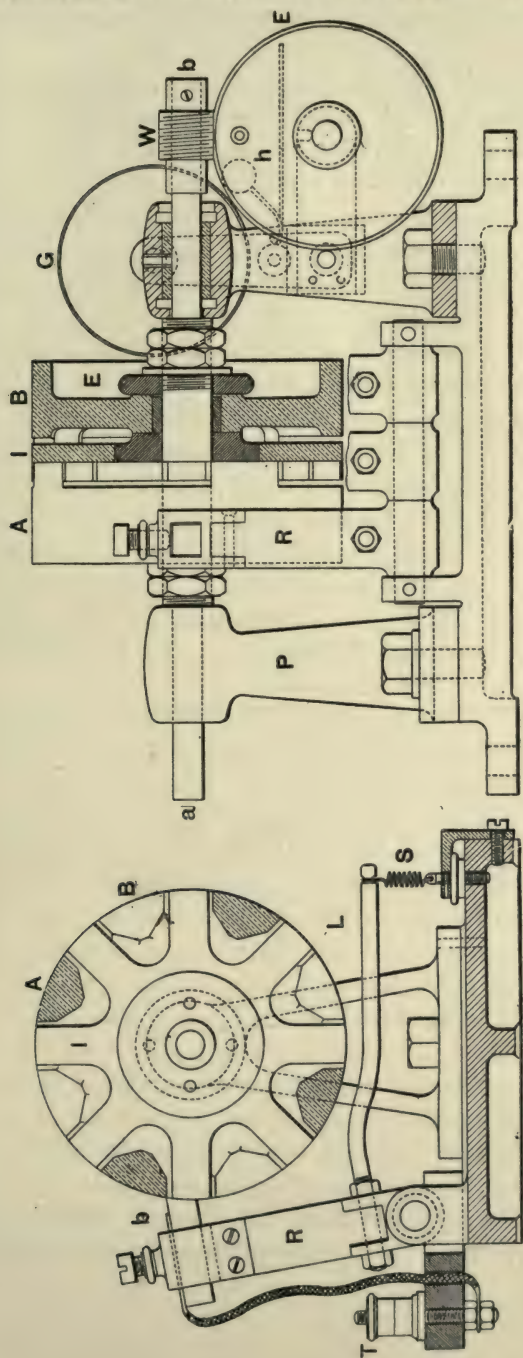


FIG. 11.—Rotating Commutator for Capacity Measurements. (Fleming and Clinton.)

The speed of the commutator is determined by attaching to the shaft a screw, W, and inter-gear wheel, E, so that the wheel makes one revolution for every 100 revolutions of the shaft. At each revolution a pin on the wheel is made to lift a lever and strike a blow on a gong, G. By means of a stop watch the time of 10 revolutions of the wheel, and therefore of 1,000 revolutions of the commutator, can be ascertained with an accuracy of less than 1 per cent. Hence there is no uncertainty either as to the number of changes of contact per second made by the commutator or as to the duration of each contact. This piece of apparatus, when well made, has proved itself to be far more satisfactory than any arrangement involving an electrically-driven tuning-fork. By its aid capacities as small as  $\frac{1}{100000}$ th of a microfarad can be measured with ease. For additional details concerning this commutator the reader is referred to a Paper by Dr. J. A. Fleming and Mr. W. C. Clinton, "On the Measurement of Small Capacities and Inductances" (see *Phil. Mag.*, May, 1903, Series VI., Vol. V., p. 493).

It remains, then, to describe the process of measurement. The condenser of which the capacity is to be measured has one terminal connected to the middle brush. A battery of small secondary cells of suitable voltage is connected to one of the outer brushes, and a movable coil galvanometer of the required sensibility has one terminal connected to the other brush. The remaining terminals of condenser, galvanometer and battery are connected together. The battery, condenser and galvanometer must be well insulated by being placed on sheets of ebonite. Care must be taken that no current from the battery passes direct into the galvanometer, but the latter must be affected only by the discharge of the condenser. If the commutator is working properly when set in action, it will cause a set of discharges to take place through the galvanometer, which will have all the effect of a steady current and create a deflection which remains constant. The galvanometer selected must be of

such sensibility, and the charging voltage be so adjusted, that a convenient, steady galvanometer deflection is obtained. This deflection is observed and also the number of discharges made per second.

We have, then, to ascertain the value of the unvarying current which will give the same galvanometer deflection. This may be done either by potentiometer or by a shunt-and-series resistance in connection with the galvanometer. If the required resistances are at hand, the standardisation of the galvanometer may be effected as follows:—Place a large resistance,  $R$ , in series with the galvanometer and a small shunt,  $S$ , across the terminals, and let  $G$  be the resistance of the galvanometer itself. Then apply to the terminals of the united resistance a voltage,  $V$ , which is preferably but not necessarily the same as that employed in charging the condenser. Adjust  $R$  or  $S$  until the deflection given by the galvanometer is the same as that obtained from the rapid discharge of the condenser through it. Then the current through the galvanometer is equal to  $\frac{V}{R + \frac{GS}{G+S}}$ . If the condenser was

charged with the same voltage its intermittent current is equal to  $\frac{CVn}{10^6}$ . Hence we have,

$$\frac{Cn}{10^6} = \frac{1}{R + \frac{GS}{G+S}},$$

or 
$$C = \frac{10^6(G+S)}{n\{R(G+S) + GS\}} \text{ mfd.s.}$$

If the galvanometer is very sensitive, the value of  $R$  necessary to create the required deflection may have to be very large. It is not advisable to employ a shunt resistance  $S$  which is exceedingly small compared with the galvanometer resistance  $G$ , because then the ratio of  $G$  to  $S$  is not easily ascertained with the required accuracy. Hence, in some cases,

if a potentiometer is to hand, it is better to standardise the galvanometer and ascertain the ampere-value of the deflection given by the condenser in the manner described in § 12, Chap. III., Vol. I., of this HANDBOOK.

If, then,  $A$  is this ampere-value, we have for the capacity of the condenser the expression

$$C = \frac{10^6 A}{nV} \text{ mfd.},$$

where  $V$  is the charging potential and  $n$  the number of discharges per second.

In employing this method to measure the capacity of condensers with solid or liquid dielectrics it is necessary to ascertain whether there is any leakage or true conductivity in the dielectric which would vitiate the accuracy of the method. Apart altogether from the effect of frequency on the capacity, it is clear that if there is any leakage the condenser will be partly discharged during the time it is insulated and on the way to be discharged after being charged. This can be ascertained by changing the connection of the galvanometer so as to make it take the series of charging currents and not the discharge currents. This is achieved by placing the galvanometer in series with the battery and using merely a short-circuiting wire in the original place of the galvanometer. If, when this change is made, the same condenser capacity is obtained as before, then sensible leakage is absent, but if, in the latter case, the apparent capacity is greater than in the former, then the results cannot be trusted to give the true capacity of the condenser.

The above described method may be employed to determine the capacity of Leyden jars, on the understanding that the number so obtained is the capacity under the special limitation of charging time determined by the speed of the commutators.

The capacity of a so-called *gallon* Leyden jar is roughly about  $\frac{1}{300}$ th of a microfarad.

A modification of the above method has been suggested by the Author which employs a special form of differential galvanometer. If we suppose a differential galvanometer constructed with absolutely no leakage between the wires, and so wound that when the same current flows in opposite directions round the coils no deflection of the needle is produced, we can proceed to make a capacity measurement as follows:—Let one of the coils of the galvanometer be employed, in connection with the rotating commutator already described, to take the series of discharge currents from a condenser of capacity  $C$ , charged by a battery of E.M.F.  $V$ , the frequency being  $n$ . Then the current through that coil is given by the expression  $CVn/10^6$ . If, then, we connect the other coil of the galvanometer through a resistance  $R$  ohms with the same battery, shunt this coil with a shunt of resistance  $S$ , and arrange the value of  $R$  so that the galvanometer needle remains at rest, we have as before the equality

$$\frac{CVn}{10^6} = \frac{V}{R + \frac{GS}{G+S}},$$

or 
$$C = \frac{10^6(G+S)}{n\{R(G+S) + GS\}}.$$

For the purposes of the above test the ordinary differential galvanometer is not suitable. In the usual form of this instrument two silk-covered wires are wound side by side on the same bobbin, and so adjusted that when the same current flows in opposite directions through the two wires there is no magnetic field in the centre of the coil. It is impossible, however, to maintain a very high insulation between two paraffined silk-covered wires of very great length in contact through their whole length. Hence, an ordinary differential galvanometer cannot be employed in any test in which practically perfect insulation between the coils is required. A differential galvanometer can, however, be constructed on a plan devised by the Author and Mr. W. C. Clinton, in which

there is nearly perfect insulation between the two coils. The galvanometer coil is made up, like the secondary coil of an induction coil, of a number of flat coils of insulated wire wound on very thin ebonite spools or between shellaced paper discs. These coils are assembled on an ebonite tube, and the first, third, fifth, seventh, &c., coils are connected in series, and the second, fourth, sixth, &c., so as to form two coils which are sandwiched in between each other. The winding in each flat coil must be joined up in the proper direction with that of its fellows. By this plan nearly all leakage is annulled, and a differential galvanometer can be constructed in which each coil is quite independent electrically of the other.

In using high voltage on such a galvanometer, electrostatic effects sometimes make their appearance, which can only be cured by enclosing the needle system in a metal case and keeping the whole needle system at the earth potential.

For use in the above-described manner a differential movable coil galvanometer has also been devised by us consisting of two movable coil Holden-Pitkin galvanometers mounted one over the other. In proximity to the poles of each fixed magnet is placed a curved piece of soft iron, which can be moved by a screw in or out so as to shunt part of the magnetic field. The two coils are joined in series in opposition and the magnetic fields regulated until the same current passed through both coils creates no deflection. The coils are then separated and their ends brought to two pair of terminals, and can be used as a differential galvanometer to adjudicate on the equality or otherwise of the two independent electric currents. (*See* Dr. J. A. Fleming and Mr. W. C. Clinton, "On the Measurement of Small Capacities and Inductances," *Phil. Mag.*, May, 1903, or *Proc. Phys. Soc. London*, February, 1903.

**§ 6. The Measurement of Capacity by Balancing a Condenser and Commutator on a Wheatstone's Bridge.**—Maxwell suggested another method in which the capacity of

a condenser can be measured provided we have a tuning-fork or rotating commutator to charge and discharge it rapidly. Such a rapidly charged and discharged condenser resembles in effect a resistance, because it allows a certain quantity of electricity to pass per second—that is, a certain current to flow through it under a fixed electromotive force. Hence we may treat the condenser and commutator taken together as a resistance, and measure it on a Wheatstone's bridge. This method is briefly described in Maxwell's treatise on "Electricity and Magnetism," Vol. II., second edition, p. 385, but more completely by the Author in a Paper on "The Distribu-

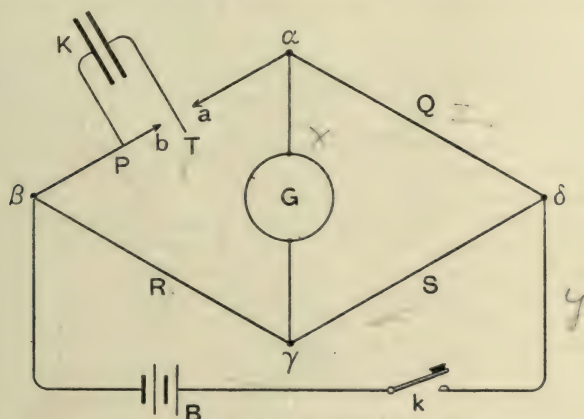


FIG. 13.

tion of Electric Currents in Networks of Conductors," published in the *Proceedings* of the Physical Society of London, Vol. VII., 1885, and in the *Phil. Mag.* for September, 1885.

As a first simple explanation, we may notice that, if a condenser of capacity  $C$  microfarads is charged  $n$  times per second at a voltage  $V$ , then the quantity of electricity it transmits per second is  $CVn/10^6$ , and since this is equal to a current of  $A$  amperes, we have

$$CVn = 10^6 A,$$

$$\text{or} \quad \frac{V}{A} = \frac{10^6}{Cn}$$

But, since  $V/A$  is the equivalent ohmic resistance, it follows that the condenser and commutator taken together are equivalent to a resistance  $10^6/Cn$  ohms. If, then, we connect a condenser and commutator to a Wheatstone's bridge, as shown in Fig. 13, we can balance the bridge by adjusting the arms.

The theory of this method, taken from notes of a lecture by Maxwell, has been given by the Author in a Paper "On Problems in the Distribution of Electric Currents in Networks of Conductors" (see *Phil. Mag.*, Sept., 1885, Vol. XX., p. 221; also *Proc. Phys. Soc. London*, 1885). The method of proof is based upon the employment of the network equations of Maxwell already explained in Vol. I., Chap. II., of this HANDBOOK.

Let  $\alpha, \beta, \gamma, \delta$  be the four points of a Wheatstone's bridge (see Fig. 13), and let the branch between  $\alpha$  and  $\beta$  be interrupted at  $ab$ , and a Leyden jar or condenser inserted provided with some rapid commutator, such as a tuning-fork, so that whilst the outside of the jar is kept permanently attached to  $\beta$ , the inside is alternately joined to  $\alpha$  and  $b$ .

If a tuning-fork is used, and its prongs have small metal styles which just come down to the surface of the mercury in two little cups, when the fork vibrates, as the prongs come together, the upper point dips in; and as they separate, the lower one dips in; hence the shank of the fork is alternately connected with one and the other cup, the interval between the time of connection being exactly half the time of a complete oscillation of the fork.

Let the cyclical symbols of the meshes of the network be  $x+z, z$  and  $y$ ; then  $x$  is the current through the galvanometer, and  $y$  is the current through the battery. When the arrangement is made as in the diagram, and the fork set vibrating, the vibrating fork and the condenser act together like a resistance, and a certain current or quantity of electricity per second passes through it.

As the condenser receives its charge by electricity flowing into it, it builds up an opposing E.M.F. in the  $z$  circuit which at any instant is equal to the

value of  $\frac{\int z dt}{K}$ , where  $K$  is the capacity of the jar, the integral being integrated from the instant when the charging commenced up to the instant considered. If the fork makes  $n$  vibrations a second when the steady state is set up, the current  $z$  which flows into the jar has a mean value  $\bar{z}$ ; and therefore  $\frac{\bar{z}}{nK}$  is the opposing E.M.F. in that branch.

Accordingly, the condenser and associated commutator behave like a voltmeter inserted in the branch  $\alpha\beta$ , or like a resistance with a counter E.M.F. in it. A combined jar and fork differs, however, from an ordinary metallic resistance in this, that its apparent resistance is not constant, but depends

on two things, the speed of commutation or charge and recharge and the capacity of the condenser; whilst the counter E.M.F. depends on the current  $z$ , and, being represented by  $\frac{z}{nK}$ , is dependent not only on  $n$  and  $K$ , but also on the values of all the other resistances in the branches.

In the first place, we require an expression for the E.M.F. charging the condenser. Let the difference of potential between  $a$  and  $b$  be called  $e$ . Then, considering the network formed by the five conductors  $R$ ,  $S$ ,  $Q$ ,  $G$  and  $B$  with the E.M.F. in the branch  $B$ , write down the network equations for this  $z$  mesh network. They are

$$\begin{aligned}(B + R + S)y - S(x + z) &= E, \\ -Sy + (Q + S + G)(x + z) &= 0.\end{aligned}$$

Hence  $y = \frac{E(Q + S + G)}{\delta}$ ; and  $x + z = \frac{ES}{\delta}$ , where  $\delta$  = the determinant

$$\begin{vmatrix} B + R + S, & -S \\ -S, & Q + S + G \end{vmatrix},$$

which is equal to  $S(Q + G) + (R + B)(Q + S + G)$ .

The difference of potential  $e$  between  $a$  and  $b$  when the condenser is just beginning to be charged  $G(x + z) + Ry = e$ ;

$$\therefore e = \frac{ESG}{\delta} + \frac{ER(Q + S + G)}{\delta}$$

$$= E \frac{SG + R(Q + S + G)}{S(Q + G) + (R + B)(Q + S + G)} = E \frac{\begin{vmatrix} -G, & -R \\ Q + S + G, & -S \end{vmatrix}}{S(Q + G) + (R + B)(Q + S + G)},$$

or

$$e = E \frac{\begin{vmatrix} -G, & -R \\ Q + S + G, & -S \end{vmatrix}}{\delta}.$$

If the E.M.F.  $e$  be employed  $n$  times in a second to charge a jar of capacity  $K$ , the average current flowing into the jar is  $nKe = z$ .

To find  $z$  we have to consider the distribution of currents when the fork or commutator is in operation, and the condenser allowing a flow of electricity to take place through it.

Let  $P$  be the resistance which could equivalently replace the jar and fork—that is, would allow an equal quantity of electricity to pass per second; then, since  $\frac{z}{nK}$  is the opposing E.M.F. in this branch, we have the following equation for the three cycles  $x$ ,  $x + z$ , and  $y$  :—

$$\begin{aligned}-Sx + (R + S + B)y - (R + S)z &= E, \\ -Gx - Ry + (P + R)z &= -\frac{z}{nK}, \\ (Q + S + G)x - Sy + (Q + S)z &= 0.\end{aligned}$$

Let  $\Delta$  stand for the determinant

$$\begin{vmatrix} -S, & R + S + B, & -(R + S) \\ -G, & -R, & P + \frac{1}{nK} + R \\ Q + S + G, & -S, & Q + S \end{vmatrix}.$$

Then the solution of the above equations for  $z$  and  $x$  are

$$z = \frac{E \begin{vmatrix} -G, & -R \\ Q+S+G, & -S \end{vmatrix}}{\Delta},$$

and

$$x = \frac{E \begin{vmatrix} -R, P + \frac{1}{nK} + R \\ -S, Q+S \end{vmatrix}}{\Delta},$$

in which  $z$  is the average current flowing through the condenser and  $x$  is the current through the galvanometer. Let the resistances  $R, S$  and  $Q$  be so varied that the current through the galvanometer is zero, then  $x=0$ ; and therefore

$$\begin{vmatrix} -R, P + \frac{1}{nK} + R \\ -S, Q+S \end{vmatrix} = 0,$$

or

$$R(Q+S) = S \left( P + \frac{1}{nK} + R \right),$$

or

$$\frac{RQ}{S} = P + \frac{1}{nK}.$$

We next insert this value for  $P + \frac{1}{nK}$  in the determinant  $\Delta$  above, and calculate its value, and we arrive at the expression

$$\Delta = \frac{\{B(Q+S) + Q(R+S)\} \{G(R+S) + R(Q+S)\}}{S}.$$

We have now, by substitution of this value of  $\Delta$  in the value obtained above for  $z$ , an expression for the value of the average current through the condenser when the bridge is balanced, and it is

$$z = \frac{ES \begin{vmatrix} -G, & -R \\ Q+S+G, & -S \end{vmatrix}}{\{B(Q+S) + Q(R+S)\} \{G(R+S) + R(Q+S)\}}.$$

Equating this to the other value for  $z$ , namely,

$$z = nK\epsilon = nKE \frac{\begin{vmatrix} -G, & -R \\ Q+S+G, & -S \end{vmatrix}}{S(Q+G) + (R+B)\{Q+S+G\}},$$

we have

$$nK = \frac{S \{S(Q+G) + (R+B)(Q+S+G)\}}{\{B(Q+S) + Q(R+S)\} \{G(R+S) + R(Q+S)\}},$$

which gives us a value for  $nK$  in terms of  $B, Q, R, S, G$ .

It is interesting to note that we may otherwise write the above expression for  $nK$ ,

$$\frac{1}{nK} = \frac{\Delta}{\delta},$$

where  $\Delta$  is the determinant

$$\begin{vmatrix} R+B+S, & -S, & -R+S \\ -S, & Q+S+G, & R\left(\frac{Q}{S}+1\right) \\ -R, & -G, & Q+S \end{vmatrix},$$

and  $\delta$  is its first minor

$$\begin{vmatrix} R+B+S, & -S \\ -S, & Q+S+G \end{vmatrix},$$

and  $\frac{1}{nK}$  is of the dimensions of a resistance.

The value for  $nK$  writes out by a simple transformation into another form,

$$nK = \frac{S \left\{ 1 - \frac{S^2}{(Q+S+G)(R+B+S)} \right\}}{RQ \left\{ 1 + \frac{SB}{Q(R+B+S)} \right\} \left\{ 1 + \frac{SG}{R(Q+S+G)} \right\}};$$

which is the expression given by Prof. J. J. Thomson, and quoted by Dr. R. T. Glazebrook in his memoir on a "Method of Measuring the Capacity of a Condenser."\*

**§ 7. Comparison Methods of Capacity Determination: De Sauty's Method.**—In addition to the above-described absolute methods of capacity determination, there are a number of useful methods for the comparison of one capacity with another.

The difficulty which presents itself in the case of condensers with *different* solid dielectrics is that of the unequal residual charge or soakage effects, and these must be taken into account if any determination of real value is to be made.

One of the simplest comparison methods is that due to De Sauty. In this method the two condensers of which the relative capacities  $C_1$  and  $C_2$  are to be determined take the place of two arms of a Wheatstone's bridge (see Fig. 14). Two non-inductive resistances,  $R_1$  and  $R_2$ , form the two other arms, and these resistances are altered in value until the galvanometer in the bridge circuit gives no "kick" on making or breaking the battery circuit. When this is the case, we have  $C_1 R_1 = C_2 R_2$ .

A complete discussion of the theory of this method has been given by Dr. R. T. Glazebrook (see *Phil. Mag.*, May, 1881, p. 370).

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\* This method of Maxwell's of obtaining the capacity of a condenser has been practically employed, with most excellent results, by Dr. R. T. Glazebrook, F.R.S., and the full details of the tests to which he subjected the method are given in his Paper in the *Proceedings* of the Physical Society, Vol. VI., Part III., p. 204 (June 28, 1884).

Let  $v_0$  be the potential of the common terminal of the condensers, and  $v$  that of both terminals of the galvanometer when balance is established. Let  $v_1$  be the potential at the common terminal of the resistances. Then the quantities of electricity flowing through the resistances  $R_1$  and  $R_2$  during a small time  $\delta t$  will be  $(v_1 - v)\delta t/R_1$  and  $(v_1 - v)\delta t/R_2$ , and the corresponding increases in the potentials at the galvanometer terminals in this time will be  $(v_1 - v)\delta t/C_1 R_1$  and  $(v_1 - v)\delta t/C_2 R_2$ . Hence, if the galvanometer continues to give no deflection, we must have the relation  $C_1 R_1 = C_2 R_2$ .

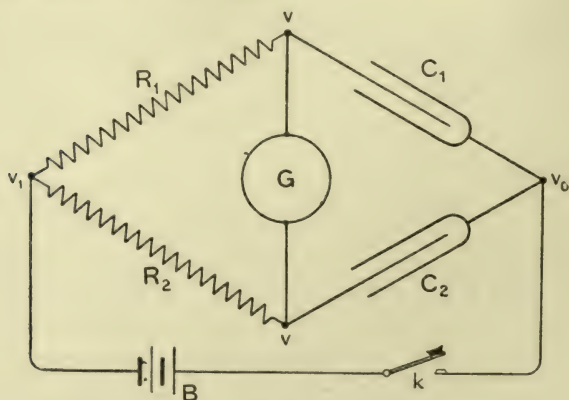


FIG. 14.

The complete discussion above referred to shows that the current through the galvanometer during the time of charging is

$$i = \frac{(v_1 - v_0)(R_1 C_1 - R_2 C_2)}{G + R_1 + R_2}.$$

Hence the error in the result when using a given galvanometer is least when  $R_1$  and  $R_2$  are as large as possible, and the value of  $G$  (the galvanometer resistance) should be equal to  $R_1 + R_2$  to obtain the greatest sensitiveness.

The effects of dielectric absorption (if present) interfere very much with the test, because it is then impossible to obtain a sharply-marked zero galvanometer reading. The

needle gives a "kick" in one direction and a slow glide or movement in the other direction when the balance is nearly established.

To evaluate the absorption "kick," disconnect one condenser and replace the other by a known small capacity in the form of an air-condenser, and observe the throw. This gives us a means of standardising the galvanometer. If there is any conductivity or leakage in the condensers, then Dr. Glazebrook's investigation shows that the capacity ratio of the condensers is

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} \left( 1 - \frac{2GK}{E} \tan \delta \right),$$

where  $G$  is the galvanometer resistance,  $K$  the galvanometer constant,  $E$  the E.M.F. of the battery, and  $\delta$  the permanent deflection when the initial "kick" is zero.

**§ 8. Comparison of Capacities by the Methods of Mixtures. Thomson's and Gott's Methods.**—A method of comparing capacities much used in cable factories is some modification of the "method of mixtures" first suggested by Lord Kelvin (*see* Vol. I. *Jour. Soc. Tel. Eng.*, London, pp. 397-398). In this method the two condensers to be compared are joined in series and connected to a battery which is shunted by a wire of high resistance. Some point in this wire is then connected to the junction point of the condensers. The condensers are then joined in parallel and connected to a galvanometer to observe if there is any outstanding charge. The three operations are spoken of as the *charge*, *balance*, and *mix* or *discharge*.

The arrangement of the two condensers  $C_1$  and  $C_2$  and the high resistance ( $R_1$  and  $R_2$ ) and battery and galvanometer is as shown in Fig. 15, and is virtually a Wheatstone's bridge arrangement, or the De Sauty arrangement just described (§ 7) with the galvanometer and battery interchanged. The battery  $B$  sends a current through the bridge arms  $R_1$  and

$R$  and the contact point  $a$  is shifted or  $R_1$  and  $R_2$  varied until, when the key  $K_1$  (battery) is closed first and then

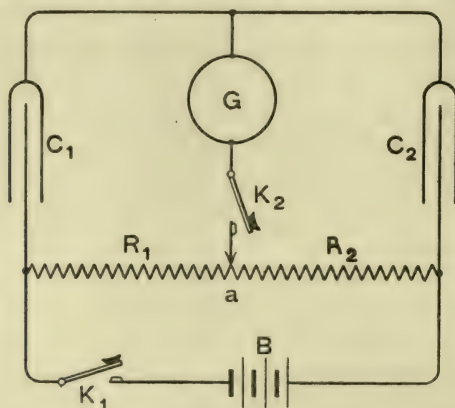


FIG. 15.

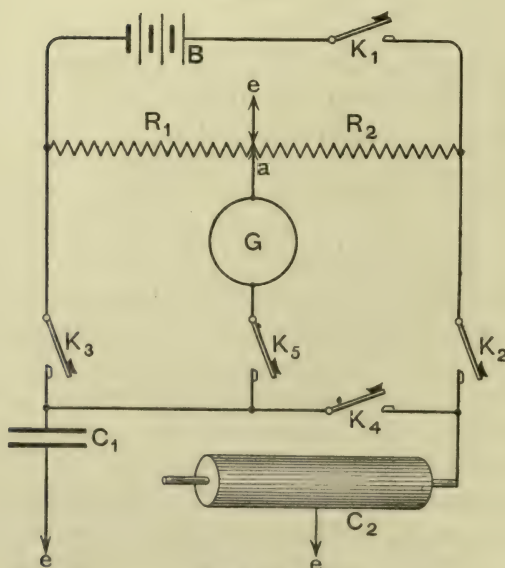


FIG 16

$K_2$  (galvanometer), there is no galvanometer deflection, or "kick." Then, under these conditions, we have  $C R_1 = C_2 R_2$ .

When the capacity of a concentric cable is being measured, or that of a single insulated cable laid in water, the connections are as shown in Fig. 16, where  $C_2$  is the cable and  $C_1$  a standard condenser. In this case there are, in fact, five keys, and the operations are as follows :—

(i.) Close key  $K_1$  and produce a fall of potential by means of the battery B down the resistance  $R_1 + R_2$ .

(ii.) Close keys  $K_2$  and  $K_3$  together for a fixed time. This charges the condensers.

(iii.) Open keys  $K_2$  and  $K_3$  and close key  $K_4$  for a fixed time to mix the charges.

(iv.) Keeping the key  $K_4$  closed, close key  $K_5$  and test the common potential.

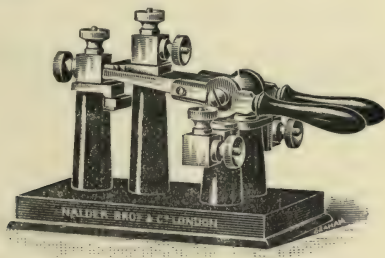


FIG. 17.

Keys have been devised which perform all these operations in the proper order, such as those of Mr. Lambert or Mr. Saunders.

For a description of the Saunders key see *The Electrician* Vol. XXV., p. 170.

The difficulties introduced by dielectric absorption into this test are fully discussed by Dr. Muirhead and by Dr. Glazebrook (see *The Electrician*, 1890, Vol. XXV., pp. 487 and 637).

If the condensers compared have different dielectric absorptions, then any delay in obtaining a balance or reading the resistance ratio will cause the potential of the junction to

vary, because the capacities will have virtually altered in relative magnitude. This has been investigated as follows by Dr. Muirhead :—

Let  $V$  be the whole potential fall down the resistance  $R_1 + R_2$ , and let  $v$  be the fall down one section ; then we have  $\frac{V-v}{v} = \frac{R_2}{R_1}$ . But if  $Q$  is the quantity put in, then  $Q = vC_1 = (V - v)C_2$ . Therefore, if there is no absorption we have  $\frac{C_1}{C_2} = \frac{R_1}{R_2}$ .

Next, suppose that dielectric absorption virtually increases the capacities to  $C_1 + c_1$  and  $C_2 + c_2$ . Then the charges will

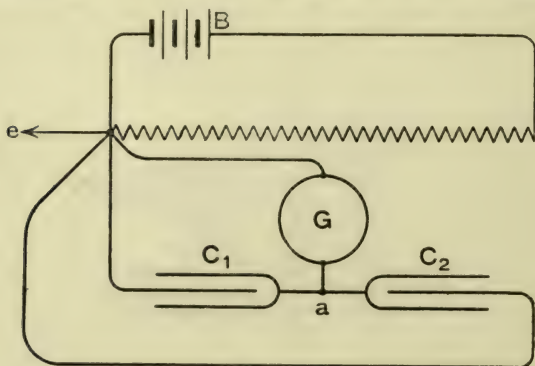


FIG. 18.

be  $(C_1 + c_1)v_1$  and  $(C_2 + c_2)(V - v_1)$ , where  $v_1$  is the altered potential of the common point.

To obtain the values  $C_1$  and  $C_2$ , Dr. Muirhead disconnects one condenser from the resistance and joins it to earth, and then immediately after tests the junction potential at  $a$  (see Fig. 18) with the galvanometer and finds it to be, say,  $v_2$ ; then we have

$$C_1 v_1 - C_2 (V - v_1) = (C_1 + C_2) v_2,$$

or

$$(C_1 + C_2) v_1 - C_2 V = (C_1 + C_2) v_2.$$

Therefore

$$(C_1 + C_2)(v_1 - v_2) = C_2 V;$$

therefore

$$\frac{C_1}{C_2} = \frac{V - (v_1 - v_2)}{(v_1 - v_2)}.$$

The diagram of the actual connections employed in this test is as shown in Fig. 19. The operations of taking a reading are as follows :—

(i.) Press down the *charge key*  $k_1$ , and the condensers are connected in series with the resistances.

(ii.) Press the *balance key*  $k_3$ , and test the equality or otherwise of potential of the condenser junction and resistance junction.

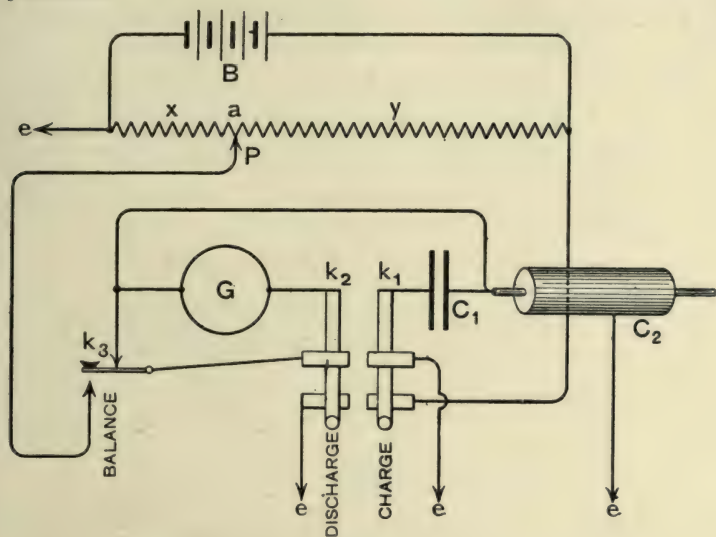


FIG. 19.

(iii.) Press the *discharge key*  $k_2$ , and the condensers are put in parallel with the galvanometer, and the correction for  $v_2$  in the equation above is obtained.

Suppose in the last case we obtain a galvanometer deflection  $\alpha$ . Then we evaluate this as follows:—Move up the contact point P by a small amount, equivalent to a resistance  $r$ , and let  $R$  be the whole resistance  $x+y$  and  $V$  the whole potential fall down it; then  $\frac{r}{R}V$  is the corresponding increment in potential.

If we then press the discharge key again, we get a throw on the galvanometer equal, say, to  $\beta$ , and we have the same capacity—viz., the two condensers in parallel charged to a known potential. Hence, if a throw  $\beta$  corresponds to a charging potential  $\frac{\gamma}{R}V$ , we know by a simple proportion the value of the potential  $v_2$  which produced a throw  $\alpha$ .

For detailed diagrams illustrating the various stages of the above test the reader should consult an article by Dr. Muirhead in *The Electrician* for September 5, 1890, Vol. XXV., p. 487.

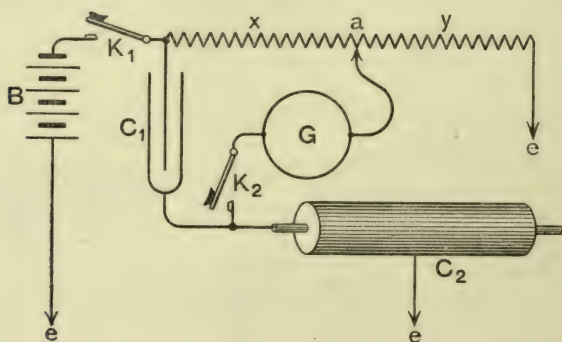


FIG. 20.

A modification of Thomson's (Kelvin's) method was given by Mr. J. Gott in 1881 (see *Proc. Soc. Tel. Engineers*, Vol. X., p. 278). For a discussion of the Thomson and Gott methods the reader may be referred to remarks by Mr. J. Elton Young in a Paper on "Capacity Measurements of Long Submarine Cables" (see *The Electrician*, Vol. XLIII., 1899, pp. 45 and 80).

Gott's arrangement of circuits is as shown in Fig. 20. B is an earthed battery and  $x, y$  the sections of a high resistance,  $C_1$  and  $C_2$  is the condenser and cable to be compared, G is a galvanometer. The key  $K_1$  is first closed to charge for, say, five seconds, and the point of contact  $a$  on the slide resistance is shifted until on closing the key  $K_2$  the galvanometer gives

no "kick." Under these circumstances we have  $C_1/C_2 = x/y$ . If the galvanometer deflects, the condensers must be thoroughly discharged and the test recommenced.

**§ 9. Rules for Adding Capacities.**—In dealing with arrangements of condensers, it is useful to bear in mind the rule that *capacities in parallel* are added together by the same rule as *resistances in series*. On the other hand, *capacities in series* are added together by the same rule as *resistances in parallel*. Thus, if  $C_1, C_2, C_3, \&c.$ , be the capacities of various condensers, then, if we place these condensers in parallel, the joint or resultant capacity is  $C_1 + C_2 + C_3 + \&c.$  If, on the other hand, we place them in series, then the resultant capacity is

$$\frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \&c.}$$

Condensers so joined in series are said to be *in cascade*. Hence, if we have  $n$  condensers, each of capacity  $C$ , the resultant capacity when all are joined in parallel is  $nC$ , and when all are joined in series is  $C/n$ .

If we have a condenser of small capacity  $c$ , and we desire to measure its capacity with a ballistic galvanometer; if its capacity is not sufficiently large to cause it to give a sensible "throw" on the galvanometer, we may proceed as follows:—Let the small condenser be charged and then connected in parallel with a condenser of much larger capacity,  $C$ , and let this process be repeated  $n$  times. At each connection the small condenser shares its charge with the large condenser, and the charge is divided between them in the ratio of their capacities. Hence, if  $V$  is the charging voltage, the initial charge of the small condenser is  $cV$ , and, on paralleling with the empty large condenser, this latter gains a charge equal to

$$cV \frac{C}{C+c}.$$

If this process is repeated  $n$  times, the charge in the large condenser becomes

$$Q = \frac{ncVC}{C+c}$$

Suppose, then, that we discharge this large condenser so charged through the ballistic galvanometer; we obtain a "throw"  $D$  proportional to  $Q$ , or, say,  $Q = GD$ , where  $G$  is the galvanometer ballistic constant.

Suppose, then, that we charge the large condenser directly with the voltage  $V$ ; we have a quantity  $Q'$  in it such that

$$Q' = VC.$$

Let the large condenser so charged be discharged through the galvanometer; we obtain a "throw"  $D'$  such that

$$Q' = GD'.$$

Hence, since 
$$Q = \frac{nVCc}{C+c} = GD,$$

and 
$$Q' = VC = GD',$$

we have 
$$\frac{D}{D'} = \frac{nc}{C+c},$$

or 
$$\frac{nD'}{D} - 1 = \frac{C}{c},$$

or 
$$c = C \frac{D}{nD' - D}.$$

We can determine, therefore,  $c$  in terms of  $C$  and  $n$ ,  $D$  and  $D'$ .

**§ 10. The Determination of Capacity by Alternating Currents.** — If a simple periodic E.M.F.,  $v = V \sin pt$  of frequency  $n$  (where  $p = 2\pi n$ ) is applied to a condenser having a capacity  $C$ , and if  $q$  is the charge in it at any instant and  $v$  the potential difference of the plates, then we have

$$q = Cv = CV \sin pt.$$

If  $i$  is the current at any instant flowing into the condenser, then  $i = \frac{dq}{dt}$ . Hence  $i = CVp \cos pt$ .

Therefore the maximum value,  $I$ , of the current during the period must be such that

$$I = CVp = 2\pi nCV.$$

If  $C$  is measured in microfarads,  $V$  in volts, and  $I$  in amperes, then we have

$$I = 2\pi nCV/10^6.$$

Also, it is clear that if  $I_{(R.M.S.)}$  and  $V_{(R.M.S.)}$  stand for the root-mean-square or instrumental values of the current and voltage, we have

$$I_{(R.M.S.)} = 2\pi nCV_{(R.M.S.)},$$

where  $C$  is the capacity in farads.

Suppose that the E.M.F. is not a simple periodic function of the time, nevertheless we still have always

$$i = C \frac{dv}{dt}.$$

Let  $I_{(T.M.)}$  stand for the true mean value of the current into the condenser, and  $I_{(R.M.S.)}$  for the root-mean-square or instrumental value. Then we have

$$I_{(T.M.)} = 4VCn,$$

where  $n$  is the frequency. Now let

$$\frac{I_{(R.M.S.)}}{I_{(T.M.)}} = f = \text{the form factor of the current curve.}$$

$$\text{Then } I_{(R.M.S.)} = 4fnCV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is easy to show that, if  $f$  is the form factor of any single-valued curve, such as that in Fig. 21,  $\Delta$  is the area enclosed between the curve and the axis of  $x$ , and  $Y$  is the height of the centre of gravity of the curve above the axis of  $x$ ,

$$f = \sqrt{\frac{2YX}{\Delta}},$$

where  $X$  is the length of the axis of  $x$  intercepted by the curve.

Thus, for instance, if the voltage curve is an isosceles triangle of height  $H$  and base  $X$ , we have

$$Y = \frac{1}{3}H, \quad \Delta = \frac{1}{2}XH.$$

$$\therefore f = \sqrt{\frac{2}{3}} = 1.16.$$

Let  $g$  stand for the *amplitude factor* of the voltage curve—that is, for the ratio of the root-mean-square (R.M.S.) value of the ordinate to the maximum ordinate or maximum value during the phase. Then the equation (1) above may be written

$$I_{(R.M.S.)} = \frac{4f}{g} Cn V_{(R.M.S.)},$$

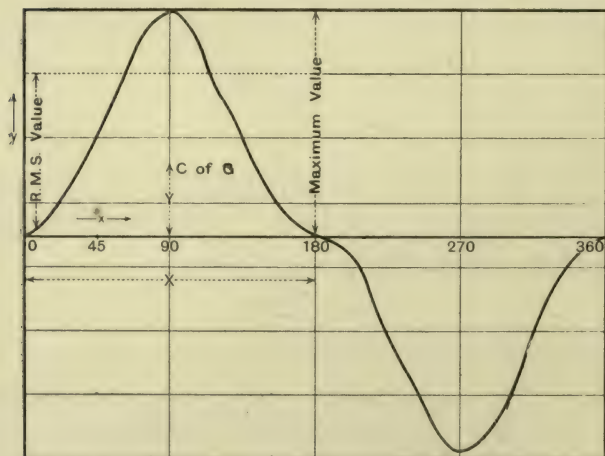


FIG. 21.

where  $f$  is the form factor of the current curve, and  $g$  is the amplitude factor of the voltage curve, the instantaneous values of the current and voltage being connected by the equation  $i = C \frac{dv}{dt}$ .

For values of the form factor and amplitude factor for various curves, see "The Alternate-Current Transformer" (Fleming), Vol. I., 3rd edition, pp. 584–586.

For example, if the voltage curve is a series of isocles triangles, then the current curve is a series of rectangles, as in Fig. 22. In this case the value of  $g$  for the voltage curve is  $g = \frac{1}{\sqrt{3}}$ , and the value of  $f$  for the current curve is  $f = 1$ ;

$$\therefore \frac{4f}{g} = 4\sqrt{3},$$

and

$$I_{(R.M.S.)} = 4\sqrt{3}CnV_{(R.M.S.)}.$$

The factor  $4f/g$  varies, of course, for different curves, but it has a minimum value equal to  $2\pi$  when the voltage curve is a simple sine curve. Hence, then, as above shown,

$$I_{(R.M.S.)} = 2\pi nCV_{(R.M.S.)}.$$

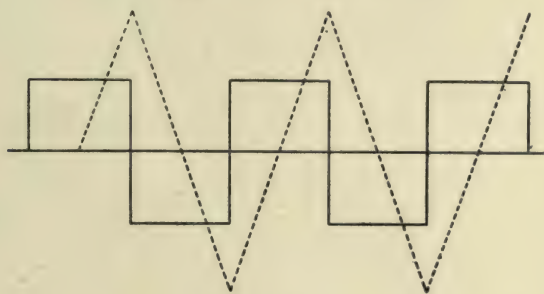


FIG. 22.

If the voltage curve is a peaky curve, then the value of  $4f/g$  becomes much greater than  $2\pi$ .

It will be seen, therefore, that the R.M.S. value of the current flowing into and out of a condenser under the action of a periodic E.M.F. of which the R.M.S. value is  $V_{(R.M.S.)}$  can only be calculated when the form of the E.M.F. curve is known.

It has been shown by Prof. H. F. Weber that in the case of capacity measurements with alternating currents the condenser current (R.M.S. value) is connected with the charging E.M.F. (R.M.S. value) by an equation of the form

$$I_{(R.M.S.)} = XCnV_{(R.M.S.)}$$

where  $X$  is a factor identical with the factor  $4f/g$ . Prof. Weber has shown that, if  $E_1$ ,  $E_3$ ,  $E_5$ , &c., are the maximum values of the various harmonic constituents in the E.M.F. curve, then

$$X^2 = \frac{E_1^2 + 9E_3^2 + 25E_5^2 + \&c.}{E_1^2 + E_3^2 + E_5^2 + \&c.}.$$

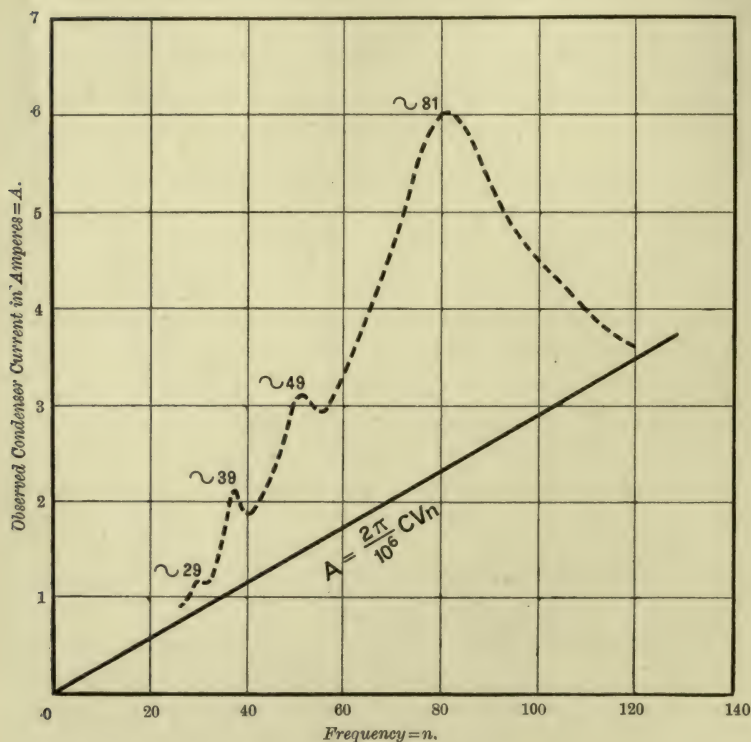


FIG. 23.

See *Science Abstracts*, Vol. I., p. 407, or *Ann. Phys. Chem.* 63, 1, p. 366. See also F. G. Baum "On the Effect of Wave Form on Capacity Current," *Electrical World*, Vol. XXXVI., p. 610, or *Science Abstracts*, Vol. IV., p. 205, Abstract No. 455,

February, 1901. See also remarks by Mr. A. Russell in a Discussion at the Institution of Elec. Eng., London, *Journal*, Vol. XXX., p. 439, April, 1901.

It is important, however, to notice that, if the condition called electrical resonance exists, the real capacity current may be very much greater than that pre-determined by the above formula.

An important investigation was carried out to illustrate this point by Mr. T. Mather, which was described in the *Electrical Review* for May 31, 1901. This investigation, entitled "Capacity Currents at Various Frequencies," followed upon a discussion on a Paper by Mr. W. M. Mordey on "Capacity in Alternate-Current Working (see *Jour. Inst. Elec. Eng.*, London, Vol. XXX., p. 364, January, 1901). Mr. Mather supplied current to a condenser of 50 microfarads capacity from a Pyke and Harris 6kw. alternator run at various speeds, but excited so as to always give 90 volts (R.M.S. value) between the machine terminals. The current into the condenser at various frequencies was then observed, and the results plotted down in a curve. He found that this curve departed greatly from the straight line (see Fig. 23) represented by the equation  $A = \frac{2\pi}{10^6} CVn$ , and had several maximum points on it corresponding to the frequencies 29, 37, 49 and 81.

Since,  $81 \times 3 = 243,$

$49 \times 5 = 245,$

$37 \times 7 = 259,$

and  $29 \times 9 = 261,$

this suggested at once that the peaks on the curve are due to resonating harmonics, and this is confirmed by a calculation of the inductance of the condenser circuit from the known equation  $LCp_2 = 1$ , prescribing the conditions of

resonance. Calculating the value of  $L$  for various frequencies, it is found to be

Frequency $n$ .	Inductance $L$ (henrys).	Exciting currents (amperes).
81 .....	0.0087	..... 0.29
49 .....	0.0086	..... 1.06
37 .....	0.0076	..... 1.25
29 .....	0.0075	..... 2.2

The actual measured value of  $L$  came out 0.0077 for 2.25 amperes.

These experiments clearly showed that, when the impressed E.M.F. is non-sinoidal, there may be resonating harmonics in its curve which cause the condenser current to be very different, and even many times greater than that calculated from the simple formula  $A = \frac{Cp}{10^6} V$ .

For example, in the above experiments at a frequency of 81 the observed condenser current was 6.15 amperes, whilst the calculated value, according to the above formula, is only 2.3 amperes. If the condenser dielectric has any true conductivity or leakage, then the condenser current will still more depart from the value given by the above simple formula. For, if  $K$  is the conductance of the dielectric, then the equation for the current into the condenser is

$$i = Kv + C \frac{dv}{dt},$$

and, if

$$v = V \sin pt,$$

then we have  $i = V \sqrt{K^2 + C^2 p^2} \sin (pt - \theta)$ .

$$\therefore I = \sqrt{K^2 + C^2 p^2} V.$$

The quantity  $\sqrt{K^2 + C^2 p^2}$  has been called by Mr. O. Heaviside the *admittance* of the condenser, and it is analogous to the *impedance* of an inductive circuit.

For additional discussion on the flow of alternating currents into condensers the reader may consult "The Alternate-

Current Transformer" (Fleming), Vol. I., p. 182, *et seq.*, and Vol. II., p. 372, *et seq.* Also a Paper on "Current Flow in Circuits having Capacity and Inductance," by J. A. Fleming, *Jour. Inst. Elec. Eng.*, 1891, Vol. XX., p. 362. Also "The Measurement of Capacity Condensers with Direct and Alternating Currents," by J. Sahulka, *The Electrician*, Vol. XXXII., p. 453.

From the above discussion it will be seen that the practical precaution to be taken in measuring capacity by means of alternating currents, is to be sure to employ an E.M.F. varying in a simple sinoidal manner, and to see that the inductance, frequency and capacity in the circuit are not so related that resonance is set up; in other words, the conditions expressed by the formula  $LCp^2=1$  must *not* hold good if the formula  $A=\frac{Cp}{10^6}V$  is to hold good.

As a mechanical illustration, we may notice that, whilst the deflection of a beam produced by placing on it a given weight is definitely related to that weight, yet, if a resonance effect is produced by applying and removing the weight in time with the natural free period of vibration of the loaded beam, the actual deflection may greatly exceed the calculated steady deflection when the weight is not put on and off.

**§ 11. The Measurement of Cable Capacity.**—A capacity measurement which most frequently concerns the electrical engineer is that of an insulated cable of some kind. This may, for instance, consist of two or more copper rods or stranded wires embedded side by side in a cylindrical mass of dielectric, surrounded generally with a metallic covering called the sheath. The conductors may be circular-sectioned or have any other appropriate sectional form. If they are separate and placed symmetrically within the sheath, then the cable is known as a *two-core*, *three-core* or *four-core* cable; &c. If, however, the conductors are in the form of cylinders surrounding one another, then the cable is called a *concentric*

cable. In the case of all these forms of cable there are various capacities which can be measured or calculated, and it is usual to express these in microfarads or fractions of a microfarad per mile run of the cable. It is only in one or two cases that the capacity per mile can be calculated from the dimensions and given section of the cable, combined with a knowledge of the dielectric constant of the insulating material.

The most simple case is that of the twin concentric cable. In this case, if  $D$  is the inside diameter of the outer tubular conductor and  $d$  is the outside diameter of the inner tubular conductor, both expressed in centimetres, then the capacity per mile in microfarads of this concentric cable is given by the formula

$$C = \frac{0.0388K}{\log_{10} D/d},$$

where  $K$  is the dielectric constant of the insulator. Hence, if the ratio  $D/d$  is about 2, and if  $K$  has a value between 2 and 3, the value of  $C$  is nearly 0.3. It will be found that this is the case for most paper-insulated cables, and, in fact, for the generality of insulators used the value of  $C$  lies between 0.3 and 0.4.

The only other case which is amenable to simple mathematical methods is that of two circular-sectioned rods lying parallel to each other in a dielectric of infinite extent; but for most forms of core and concentric cable it is impossible to predetermine entirely by calculation the capacities which are required to be known. We can, however, lay down certain general principles as follow :—

If a number of conductors are in proximity to each other, then, as shown by Clerk Maxwell,\* we may express the value of the electric charge  $q_1$  on any one of them by a function of the form

$$q_1 = K_{1.1}v_1 + K_{1.2}v_2 + K_{1.3}v_3, \text{ \&c.,}$$

where  $v_1$  is the potential of the conductor considered, and  $v_2, v_3, \text{ \&c.,}$  are the potentials of the neighbouring conductors.

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\* "Electricity and Magnetism," Vol. I., § 87.

The quantities  $K_{1,1}$ ,  $K_{1,2}$ ,  $K_{1,3}$ , &c., are constants called coefficients of induction of the nature of capacities.

If  $v_2$ ,  $v_3$ , &c., are zero and if  $v_1=1$ , then  $q_1=K_{1,1}$ . Under these circumstances the coefficient  $K_{1,1}$  is numerically equal to the charge on the conductor and it is called its capacity. Hence we define the capacity of a conductor as its charge when it is itself at unit potential and all other neighbouring conductors are at zero potential. The other coefficients,  $K_{1,2}$ ,  $K_{1,3}$ , &c., are all negative quantities, and the sum of them can never be greater than  $K_{1,1}$ , or the capacity coefficient, which is always a positive quantity.

If we apply the above principles to the case of a multiple core cable having a metallic sheath we see that the capacity of any conductor or member is the charge it takes when all the other conductors and the sheath are connected to the earth, and the conductor considered is itself raised to unit potential. The various coefficients of induction between the different conducting members of the cable, and between these and the sheath, can be determined from certain capacity measurements made as described below.

Let us consider the case of a three-core cable with metallic sheath, as used for three-phase current transmission. Let each conductor be a circular-sectioned rod, and be symmetrically placed with respect to the two other conductors and the sheath. Then it is clear that each conductor has capacity with respect to the sheath and with respect to each of the other conductors, and that we might exactly imitate the capacity effects of the cable if we considered the conductors themselves to have no capacity but six condensers to be connected in between conductors and sheath.

The three-phase cable of section, as shown in Fig. 24, is equivalent, as far as capacity is concerned, to a Wheatstone bridge arrangement of condensers such as those shown in Fig. 25. From the symmetry of the cable it is evident that the condenser capacity between each conductor and the sheath is the same, and may be denoted by  $K$ . Also the

capacity between each pair of copper conductors is the same and may be denoted by  $k$ . Accordingly, the actual capacity between any two groups of conductors formed of the three conductors and the sheath can be calculated by the simple

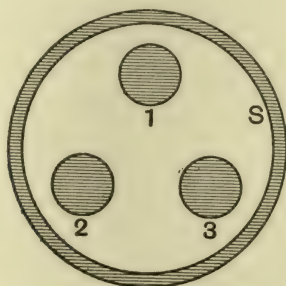


FIG. 24.

rules for adding capacities in series or in parallel. Let it be remembered that the joint capacity of two or more condensers joined in parallel is the sum of their separate capacities, and that the resultant capacity of any two condensers joined in series is equal to the quotient of the product by the sum of

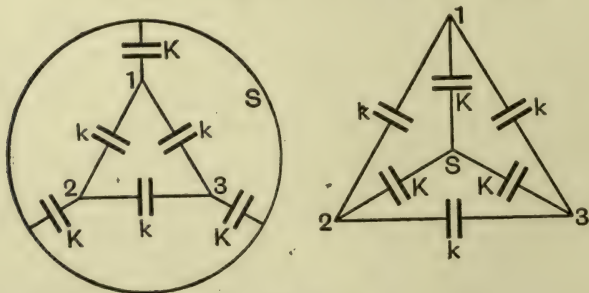


FIG. 25.

the separate capacities. If, then, we have a complex arrangement of condensers as in Fig. 25, it is quite easy to calculate by the above rules the resultant capacity between any two points.

In the case of the six condensers arranged as shown, there are eight resultant capacities which may be measured. If we denote by 1, 2 and 3 the three copper conductors, and by S the sheath, then we may group these six capacities in the following ways and measure the resultant capacity. We may, for instance, connect the conductors 1 and 2 together and measure the capacity between them and the sheath S, the conductor 3 being insulated. This arrangement is indicated by the expression "Capacity taken between 1+2 and S, 3 insulated," and if the capacity between each conductor and the other is  $k$  and each conductor and the sheath is  $K$ , then the resultant compound capacity of the above arrangement is easily seen to be equal to

$$\frac{2k \cdot K}{K + 2k} + 2K.$$

The following table gives the resultant capacity for the eight arrangements, these being indicated by the letters  $C_1$ ,  $C_2$ , &c. :—

No.	Capacity taken between Conductors.	Conductor Insulated.	Resultant Capacity.	Symbol.
1	2+3+S and	None	$K + 2k$	$C_1$
2	1+S and 2+	None	$2(K + k)$	$C_2$
3	1+S and 2	3	$\frac{(K + k)(K + 3k)}{K + 2k}$	$C_3$
4	1 and 2+3	S	$\frac{2}{3}(3k + K)$	$C_4$
5	1 and 2	S, 3	$\frac{1}{2}(3k + K)$	$C_5$
6	S and 1+2+3	None	$3K$	$C_6$
7	S and 1+2	3	$\frac{2k \cdot K}{K + 2k} + 2K$	$C_7$
8	S and 1	2+3	$\frac{2kK}{K + k} + K$	$C_8$

In the first case, the capacity between one conductor and the other two in parallel with the sheath is called the capacity of that conductor. Let us denote this by the symbol  $K_{1,1}$ . Then we have  $K + 2k = K_{1,1}$ . Again, the capacity between the conductors which we have called  $k$  is really the coefficient of

induction between these conductors with negative sign. Hence, if we call this coefficient  $K_{1,2}$ , we have  $k = -K_{1,2}$ .

In the expressions for the resultant capacity given in the fourth column of the above table let us substitute for  $K+2k$  the value  $K_{1,1}$ , which is equivalent to writing for  $K$  the value  $K_{1,1}+2K_{1,2}$ , and for  $k$  the value  $-K_{1,2}$ . We then obtain for the eight resultant capacities the following more simple expressions in terms of these two coefficients of induction :

$$\begin{aligned} C_1 &= K_{1,1}. \\ C_2 &= 2(K_{1,1} + K_{1,2}). \\ C_3 &= \frac{1}{K_{1,1}}(K_{1,1} + K_{1,2})(K_{1,1} - K_{1,2}). \\ C_4 &= \frac{2}{3}(K_{1,1} - K_{1,2}). \\ C_5 &= \frac{1}{2}(K_{1,1} - K_{1,2}). \\ C_6 &= 3(K_{1,1} + 2K_{1,2}). \\ C_7 &= \frac{2(K_{1,1} + 2K_{1,2})(K_{1,1} - K_{1,2})}{K_{1,1}}. \\ C_8 &= \frac{(K_{1,1} + 2K_{1,2})(K_{1,1} - K_{1,2})}{K_{1,1} + K_{1,2}}. \end{aligned}$$

From the above equations it is seen that  $K_{1,1} = C_1$  and  $K_{1,2} = C_1 - 2C_5$ . Also from the symmetry of the three-phase cable it is clear that all the nine coefficients of induction reduce to two, for we have

$$K_{1,1} = K_{2,2} = K_{3,3} \text{ and } K_{1,2} = K_{2,1} = K_{2,3} = K_{3,2} = K_{1,3} = K_{3,1}.$$

Accordingly, every possible measurable capacity which can be made in connection with a symmetrical three-core sheathed cable becomes known if we know the capacities denoted above by  $C_1$  and  $C_5$ , or the induction coefficients denoted by  $K_{1,1}$  and  $K_{1,2}$ . We have, therefore, these rules :—

(i.) Measure the capacity between any one conductor and the other two joined with the sheath. This gives  $C_1 = K_{1,1}$ .

(ii.) Measure the capacity between two of the conductors joined together and the third joined to the sheath. This gives  $C_5$  and  $C_1 - 2C_5 = K_{1,2}$ .

From the calculated values of  $K_{1,1}$  and  $K_{1,2}$  we can pre-determine the capacity of any other grouping of the conductors and sheath.

The makers of a three-core cable should always be asked to furnish the values per mile of cable of these two principal coefficients of induction.

The justification of this assumption that the actual capacity of the cable can be replaced by an arrangement of condensers is as follows:—

From the fundamental expression of Maxwell we have for the charge  $Q_1$  taken by any conductor the equation

$$Q_1 = K_{1,1}V_1 + K_{1,2}V_2 + K_{1,3}V_3, \dots \dots \dots (i.)$$

and two other similar equations for  $Q_2$  and  $Q_3$  the charge taken by each of the other conductors.

If, then, as above, we consider these capacity coefficients replaced by the capacity of actual condensers joined in between the conductors and the sheath, and assume a symmetrical arrangement, then, with the notation already used, we have

$$Q_1 = k(V_1 - V_2) + k(V_1 - V_3) + KV_1,$$

and similar equations for  $Q_2$  and  $Q_3$ .

Hence, if the sheath is connected to earth and the potentials balanced, we have

$$V_1 + V_2 + V_3 = 0.$$

Therefore,

$$Q_1 = (K + 2k)V_1 - kV_2 - kV_3, \dots \dots \dots (ii.)$$

Accordingly, by equating the coefficients of (i.) and (ii.), we have

$$\begin{aligned} K + 2k &= K_{1,1} \\ -k &= K_{1,2} = K_{1,3}. \end{aligned}$$

It is, therefore, clear that the assumption of actual capacities between the conductors and the sheath is equivalent in effect to the mathematical method which involves the employment of coefficients of induction.

We may treat in precisely the same manner the simpler case of a two-core cable enclosed in a metal sheath. As before, let  $k$  represent the equivalent capacity or condenser between each of the two conductors and  $K$  that between each and the sheath. Then the two-core cable is equivalent, so far as capacity is concerned, to an arrangement of condensers as in Fig. 26.

If  $K_{1,1}$ ,  $K_{1,2}$ , &c., represent the coefficients of induction, then we have for the charge  $Q_1$  on arc conductor the expression

$$Q_1 = K_{1,1}V_1 + K_{1,2}V_2,$$

where  $V_1$  and  $V_2$  are the potentials of the two conductors.

Hence the capacity between one conductor and the sheath in parallel with the other conductor is equal to  $K_{1,1}$  and obviously

$$K_{1,1} = K + k.$$

Also 
$$K_{1,2} = -k.$$

The capacity between the two conductors when the sheath is earthed is equal to  $\frac{1}{2}(K + 2k) = \frac{1}{2}(K_{1,1} - K_{1,2})$ , an expression identical with that for any pair of conductors of the three-core cable.

These capacity measurements are important, because they enable us to calculate the capacity current taken by the cable under an impressed simple periodic E.M.F.

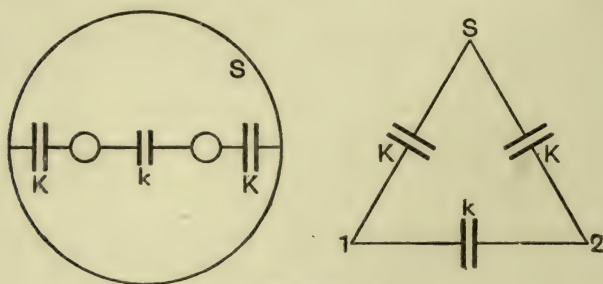


FIG. 26.

If  $q_1$  is the charge of any conductor, then  $dq_1/dt$  is the current flowing into it. Hence the differentiation with regard to time of the expression for the charge gives us the capacity current when we assume that there is no leakage.

If  $i_1$  is the instantaneous value of the capacity current flowing into conductor 1, in the case of a two-core cable, then, if  $v_1$  is its potential, we have

$$i_1 = \frac{1}{2}(K_{1,1} - K_{1,2}) \frac{dv_1}{dt}.$$

Hence, if  $v_1$  varies in a simple periodic manner, we have for the root-mean-square value (R.M.S.) of the capacity current the expression

$$A = \frac{1}{2}(K_{1,1} - K_{1,2})pE,$$

where  $p=2\pi$  times the frequency and  $E$  is the R.M.S. value of the impressed E.M.F.

An expression of the same form enables us to calculate the capacity current for a three-core symmetrical cable.

The practical use of these coefficients of induction in the case of cable work may be illustrated by the following example taken from a Paper by Mr. A. Russell, M.A., "On the Capacities of Polyphase Cables."\* The specification of the cable was as follows:—

*Type*.—Three-core lead-covered cable.

*Section of Conductors*.—0.15 sq. in. each.

*Insulation*.—Prepared paper.

*Working pressure*.—6,500 volts between each pair of conductors, and 3,750 between any conductor and the sheath.

The capacity per mile was measured as follows:—

(i.) Between conductors 1 and 2 in parallel and conductor 3 and sheath, viz., 1+2 and 3+S. This corresponds with the capacity  $C_2$  in the table above, and was found to be 0.436 microfarad.

(ii.) Between conductors 1, 2, 3 together, and the sheath. This corresponds with the capacity  $C_6$ , and was found to be 0.498 microfarad.

If  $K_{1.1}$  and  $K_{1.2}$  are the coefficients of induction, then, from the table, we see that

$$C_2 = 2(K_{1.1} + K_{1.2}) = 0.436,$$

$$\text{and} \quad C_6 = 3(K_{1.1} + 2K_{1.2}) = 0.488.$$

$$\text{Hence} \quad K_{1.1} = 0.273,$$

$$\text{and} \quad K_{1.2} = -0.0553.$$

From these we can calculate all the other capacities. Thus the capacity  $C_1$ , or that between 1 and 2+3+S is, by calculation, equal to 0.273 microfarad, and was found by measurement to be 0.268.

\* See *Journal Inst. Elec. Eng.*, Vol. XXX., p. 1,022, June, 1901.

The capacity  $C_5$ , or that between conductors 1 and 2, is, by calculation, 0.164 microfarad, and was found by measurement to be 0.165.

From this last capacity we can predetermine the capacity current per mile (I) at 6,500 volts pressure between each pair of conductors at a frequency  $n=50\sim$ , and it is given by  $I = CpV \times 10^{-6}$ ,

$$\begin{aligned}\therefore I &= 2\pi n C_5 \times 6,500 \times 10^{-6} \\ &= 314.15 \times 6,500 \times 0.165 \times 10^{-6} \\ &= 0.436 \text{ ampere.}\end{aligned}$$

In connection with this subject the reader may be referred to the following original Papers:—

C. E. GUYE. “Les Courants de Capacité dans les Lignes Polyphasées Symétriques.” *L’Éclairage Électrique*, January 20, 1900, or *Science Abstracts*, Vol. III., p. 437.

Prof. Guye points out that concentric cables are very unsuitable for polyphase work, because the capacities between the conductors are relatively large and not distributed symmetrically between the conductors and the sheath.

A. RUSSELL. “The Capacities of Polyphase Cables.” *Journal Inst. Elec. Eng.*, Vol. XXX., p. 1,022.

A. DELLA RICCIA. “Capacity of Polyphase Cables.” *Soc. Belge Elec. Bulletin*, 19, p. 318, May, 1902; or *Science Abstracts*, Vol. V., p. 809.

## § 12. The Measurement of Inductance.—Definitions.—

The inductance of a circuit is a quality of it in virtue of which energy is stored up in connection with the circuit when a current is flowing in it. For circuits surrounded by non-magnetic matter and consisting of non-magnetic materials the inductance is a constant quantity depending only on the geometrical form of the circuit. It is denoted by  $L$  and defined by the equation

$$L \frac{di}{dt} + Ri = e,$$

where  $i$  and  $e$  are the instantaneous values of the current and impressed E.M.F. and  $R$  is the ohmic resistance of the circuit.

If  $e$  is a simple harmonic function of the time, so that  $e = E \sin pt$ , then we have

$$i = \frac{E}{\sqrt{R^2 + p^2 L^2}} \sin (pt - \theta),$$

and, therefore,

$$\frac{E}{I} = \sqrt{R^2 + p^2 L^2}.$$

This ratio of  $E/I$  is called the impedance of the circuit.

Inductance is measured in terms of a unit called the *henry*, and one henry is defined to be the inductance of a circuit of one turn, such that when a circuit of one ampere flows in it a magnetic flux of one weber is self-linked with the circuit.

The mathematical *dimension* of an inductance in electromagnetic measure is a *length*. Hence the absolute electromagnetic unit of induction is *one centimetre*. The henry is equal to  $10^9$  centimetres, or to ten million metres. Hence it is sometimes called a *quadrant*, and has also been named a *secohm*.

When the electric circuit consists of or is surrounded by magnetic material, then the inductance is not constant, but is a function of the current in the circuit. Hence circuits are divided into those of constant and non-constant inductance.

It is only in a few cases that we can easily predetermine the inductance of a circuit. One such case is that of a straight wire of non-magnetic material suspended in space with the return circuit at a great distance from it. This is the case of an overhead telegraph or power transmission line with an earth return. A calculation of the value of the inductance of such a circuit has been given by Hertz and also by Sir Oliver Lodge and Dr. Howard. From a Paper by the latter authors (see *Phil. Mag.*, Series 5, Vol. XXVIII., July, 1889, p. 63) we take the investigation below:—

*Inductance of a Straight Circular Sectioned Wire.*—Consider two straight filamentary currents parallel to each other and at a distance  $c$ . Let an element of length of one conductor be

denoted by  $da$  and that of the other by  $db$ . Let  $a$  and  $b$  be measured from fixed points (*see* Fig. 27). Then the distance  $r$  of the elements is  $\sqrt{(a-b)^2 + c^2} = r$ . The mutual potential energy of these elements is, by Neumann's formula (*see* Maxwell's "Treatise on Electricity and Magnetism," Vol. II., 2nd edition, p. 160),

$$\frac{da \cdot db \cos \theta}{r},$$

where  $\theta$  is the angle between the elements, which in this case is zero. Therefore, in the above case we have

$$M = \int_0^l \int_0^l \frac{da \cdot db}{\sqrt{(a-b)^2 + c^2}}$$

as an expression for the whole mutual potential energy of the two filamentary currents, the length of each being  $l$ .

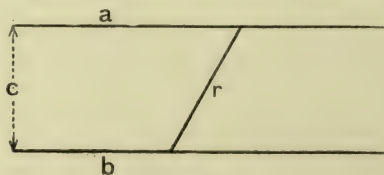


FIG. 27.

Integrating the above expression with respect to  $a$ , we obtain

$$M = \int_0^l \log \frac{\sqrt{\{(l-b)^2 + c^2\}} + (l-b)}{\sqrt{(b^2 + c^2)} - b} \cdot db.$$

Integrating again with respect to  $b$ , we have

$$M = 2l \log \frac{\sqrt{(l^2 + c^2)} + l}{c} - 2 \{ \sqrt{(l^2 + c^2)} - c \}.$$

Now, suppose that  $c$  is small compared with  $l$ , then

$$M = 2l \left\{ \log \left( \frac{2l}{c} + \frac{c}{2l} \right) - \left( 1 - \frac{c}{l} + \frac{c^2}{2l^2} \right) \right\}.$$

If we can neglect  $c/l$  and higher powers, we have, finally,

$$M = 2l \left\{ \log \left( \frac{2l}{c} \right) - 1 \right\}.$$

This is the mutual potential energy of two straight parallel currents of length  $l$  at a distance  $c$  apart.

To obtain the whole inductance of a straight circuit we have, in accordance with principles explained by Maxwell (*see* "Electricity and Magnetism," Vol. II., p. 298, § 691, 2nd edition), to put for  $c$  the *geometric mean distance* (G.M.D.) of all filaments into which the total current can be divided, or of all points on the cross-section of the conductor. If we assume the conductor to have a circular section and the current to keep to the surface, as in the case of rapid oscillations or Leyden jar discharges, then the G.M.D. of all the filamentary currents arranged in a circle is  $\frac{d}{2}$ , where  $d$  is the diameter of this circle. Put, then,  $c = \frac{d}{2}$  in the above formula, and we have, for the inductance  $L$  of a straight rod of length  $l$  and circular section, when the current keeps to the surface, the expression

$$L = 2l \left\{ \log \frac{4l}{d} - 1 \right\}.$$

If, however, the current is steady or slowly periodic it is uniformly distributed over the cross-section, and for this case we must take  $c$  to be the G.M.D. of all points on the surface of a circle. Maxwell shows that this is equal to  $\frac{d}{2}e^{-\frac{1}{4}}$ , where  $e$  is the base of the Napierian logarithms. Hence, in this case, we have

$$L = 2l \left\{ \log \frac{4l}{de^{-\frac{1}{4}}} - 1 \right\},$$

or

$$L = 2l \left\{ \log \frac{4l}{d} - \frac{3}{4} \right\}.$$

This last equation, therefore, gives us the inductance of the straight circular-sectioned rod of length  $l$  and diameter  $d$  for uniform distribution of the current over the cross-section. For long thin wires the value of  $L$  in the two cases is not very different.

*Inductance of a Circular Conductor of One Turn.*—Two other cases are important—viz., when the circular-sectioned wire

is bent into the form of a circle and a square. Let  $R$  be the radius of the circle, and let  $R$  be large compared with  $d$ , the diameter of the wire. Then  $l=2\pi R$ .

Consider the circular conductor to be split up into parallel, closely adjacent circular filaments. Then Maxwell shows ("Electricity and Magnetism," Vol. II., p. 314, 2nd edition) that the mutual potential energy of two such circles is

$$M=4\pi a\left\{\log\frac{8a}{c}-2\right\},$$

where  $a$  and  $a+c$  are the radii of the circles.

Hence, to calculate the inductance of the circular-sectioned conductor in the form of a circle of radius  $R$ , we have to insert for  $r$  in the formula

$$L=4\pi R\left\{\log\frac{8R}{r}-2\right\},$$

the G.M.D. of all points on the section or of the filaments of the current. In other words, we must put  $r=\frac{d}{2}$  when the current is distributed only on the surface and  $r=\frac{d}{2}e^{-1}$  when it is distributed uniformly over the cross-section.

In the first case (peripheral distribution) we have

$$\begin{aligned} L &= 4\pi R\left\{\log\frac{16R}{d}-2\right\} \\ &= 2l\left\{\log\frac{4l}{d}-2-\log\frac{\pi}{2}\right\} \\ &= 2l\left\{\log\frac{4l}{d}-2.45\right\}; \end{aligned}$$

in the second case (uniform distribution) we have

$$\begin{aligned} L &= 4\pi R\left\{\log\frac{8R}{\frac{d}{2}e^{-1}}-2\right\} \\ &= 2l\left\{\log\left(\frac{4l}{d}\right)-2-\log\left(\frac{\pi}{2}e^{-1}\right)\right\} \\ &= 2l\left\{\log\left(\frac{4l}{d}\right)-2.20\right\}. \end{aligned}$$

*Inductance of a Square Circuit of One Turn.*—If the circuit is bent into the form of a square, Prof. H. F. Weber has shown (*Electrical Review*, Vol. XIX., 1886, p. 33) that the inductance is given by the formula

$$L = 2l \left\{ \log \frac{4l}{d} - 2.6 \right\}.$$

Hence, for all cases, straight, circular and square conductor of circular cross-section we have for the inductance a formula of the same form

$$L = 2l \left\{ \log \frac{4l}{d} - \alpha \right\},$$

where  $\alpha=1$  for a straight wire and  $\alpha=2.5$  nearly for a circle or a square circuit.

In this formula, if  $l$  and  $d$  are measured in centimetres, and the logarithms are Napierian, then  $L$  is given also in centimetres, and is converted to henrys by dividing by  $10^9$ .

Since  $\log_e(2.118)=0.75$  and  $\log_e(2.718)=1.0$ , we can write the formula for the inductance of the straight wire, when the distribution of currents is uniform over the cross-section, in the form

$$L = 2l \left\{ \log_e \frac{4l}{d} - \log_e(2.12) \right\},$$

which is very nearly the same as the expression

$$L = 2l \left\{ \log_e \frac{2l}{d} \right\}.$$

Again, since  $\log_e(12.19)=2.5$ , we can write the expression for the inductance of the wire, bent into a circle or a square, as

$$L = 2l \left\{ \log_e \frac{4l}{d} - \log_e(12.19) \right\},$$

which is very nearly the same as

$$L = 2l \left\{ \log_e \frac{l}{3d} \right\}.$$

*The Inductance of an Endless Solenoid.*—Another simple case in which we can calculate the inductance of a circuit is

that of an endless solenoid, the diameter of which is small compared with the length of the mean perimeter. Consider a circular-sectioned ring of non-magnetic material, on which silk-covered copper wire is wound in one layer, so as to form an endless solenoid. Let the mean perimeter of the ring be  $l$  centimetres, and the number of times of wire be  $N$ , and let the area included by one turn of the wire be  $S$  square centimetres. In taking this area we must suppose it to be that included by the central axis of the wire: hence the thickness of the wire must be taken into account. Then let  $L$  be the inductance of the circuit formed of this endless solenoid. By the fundamental equation we have—

$$L \frac{di}{dt} + Ri = e,$$

where  $i$  and  $e$  are the instantaneous values of the current and terminal potential difference of the circuit.

Let  $b$  be the magnetic flux density in the interior of the winding. Then we must have

$$SN \frac{db}{dt} + Ri = e.$$

Hence, 
$$SN \frac{db}{dt} = L \frac{di}{dt}.$$

But, since the permeability of the space inside the winding is unity, we have  $b = h$ , where  $h$  is the magnetising force, and  $h = 4\pi Ni/l$  where  $i$  and  $h$  are in absolute C.G.S. electromagnetic measure. Hence

$$L = 4\pi N^2 \frac{S}{l} \text{ centimetres,}$$

or 
$$L \text{ (in henrys)} = \frac{4\pi N^2 S}{10^9 l}.$$

*The Inductance of a Straight Finite Solenoid.*—Prof. J. Perry has given (*Phil. Mag.*, Sept., 1890, or *Proc. Phys. Soc.*, London, Vol. XI., p. 15) an empirical formula for calculating the inductance  $L$  of a straight cylindrical coil of wire of resistance  $R$ .

If  $b$  is the length of the coil parallel to the axis,  $c$  is the depth of winding radially, and  $a$  is the mean radius of the coil, then he finds that  $L/R$  can be found by the formula

$$\frac{L}{R} = \frac{V}{728a + 1330c + 1500b},$$

where  $V$  is the volume of the copper in the coil in cubic centimetres,  $L$  is the inductance in henrys and  $R$  the resistance in ohms of the coil. The dimensions  $a$ ,  $b$  and  $c$  must be measured in centimetres.

**§ 13. Standards of Inductance.**—As an absolute standard of inductance of which the value can be predetermined, the endless solenoid wound on a non-magnetic core is the form most easily constructed. To make this standard, a large ring should first be turned out of wood or ebonite. The ring should have a truly circular section, and the mean diameter of the ring should be at least 12 times the diameter of the cross-section. A number of measurements should be taken to determine exactly these mean diameters.

Silk-covered copper wire is then wound closely on the ring in one layer, and the adjacent ends brought out to terminals. The first and last turn of wire should not be allowed to touch each other. The diameter over the silk is then to be taken, and from the diameter of the bare ring and the over-all diameter the mean diameter of one circular turn can be calculated, and hence the area included by one turn of the axis of the wire.

We have then the mean length of the solenoid  $=l$  and the mean area of one turn  $=S$ , and the number of turns. Hence the inductance  $L$  can be calculated in microhenrys by the formula

$$L = \frac{4\pi}{10^3} N^2 \frac{S}{l}.$$

It will in general be found difficult to secure in this manner anything but a small inductance. In order to make

the inductance large we must make  $N$  large, and this involves using a very fine wire. In the same way  $S$  should be large and  $l$  small. This may be secured by using a *cylinder*, and not a ring, on which to wind the wire. If we employ a thin cylinder of ebonite, of which the length is two or three times the mean diameter, we may obtain a relatively large value of the ratio  $S/l$ , which is what we require. It is, however, a much more troublesome matter to wind such a cylinder with covered wire, so as to form an endless solenoid of rectangular section than in the case of a circular-sectioned ring.

It has been suggested by Frölich (see *The Electrician*, Vol. XL., p. 543, or *Science Abstracts*, Vol. I., p. 412) to employ as a core a square-sectioned ring of marble wound with one layer of insulated wire.

Another form of standard inductance is that of an insulated wire wound up in a rectangular-sectioned groove in the outer edge of a ring of non-magnetic material. Let  $a$  denote the mean radius of the windings,  $n$  the number of windings,  $b$  the axial breadth, and  $c$  the radial depth of the groove. A long formula is given in Vol. II. (2nd edition) of Maxwell's treatise on "Electricity and Magnetism," in Appendix III. to Chapter XIV., for the inductance of such a ring. It may be given in the general form

$$L = 4\pi n^2 a \left( \log \frac{8a}{R} - 2 \right),$$

where  $R$  is the geometric mean distance (G.M.D.) of the section of the coil from itself (see Maxwell's "Electricity and Magnetism," Vol. II., 2nd edition, p. 298, for definition of G.M.D.).

For a more complete expression the reader can consult Mascart and Joubert's "Electricity and Magnetism" (English translation by Atkinson, Vol. II., p. 152), where it is shown that if insulated wire is wound up in a rectangular-sectioned coil having a mean radius  $a$  and a radial depth of groove  $2c$

and an axial width  $2b$ , the inductance for a coil of  $n$  turns is

$$L = 4\pi an^2 \left[ \left( 1 + \frac{36^2 + c^2}{24a^2} \right) \log_{\epsilon} \frac{4a}{\sqrt{b^2 + c^2}} - \lambda_2 + \frac{6^2}{4a^2\mu} \right],$$

where  $\lambda$  and  $\mu$  are functions of  $c/b$  which are tabulated by Mascart and Joubert. If  $b=c$ , then  $\lambda=0.84834$  and  $\mu=0.2169$  and thus

$$L = 4\pi a n^2 \left[ \left( 1 + \frac{36^2 + c^2}{24a^2} \right) \log_{\epsilon} \frac{4a}{\sqrt{b^2 + c^2}} - 0.84834 + \frac{6^2}{4a^2 \cdot 0.2169} \right]$$

... of two circular ... one fixed and the other movable ... a vertical axis, to which is attached an index-needle to show the angle made by the planes of the coils with each other. These wire circuits are joined up in series, and can be so placed that a current flowing through both is going in the same or in opposite directions according as the coils are placed.

In Fig. 39 (*see* p. 212) is shown a variable inductance standard of the above description.

Another variable inductance of the same kind consists of two solenoids, one of which can be slipped more or less into the other. They are wound in opposite directions, so that when separated entirely the inductance of the two joined in

the inductance large we must make  $N$  large, and this involves using a very fine wire. In the same way  $S$  should be large and  $l$  small. This may be secured by using a *cylinder*, and not a ring, on which to wind the wire. If we employ a thin cylinder of ebonite, of which the length is two or three times the mean diameter, we may obtain a relatively large value of the ratio  $S/l$ , which is what we require. It is, however, a much more troublesome matter to wind such a cylinder with covered wire so as to form an endless solenoid

For

$$L = 4\pi an^2 \left[ \left( 1 + \frac{36^2 + c^2}{24a^2} \right) \log_{\epsilon} \frac{4a}{\sqrt{b^2 + c^2}} - \lambda_2 + \frac{6^2}{4a^2\mu} \right],$$

Read

$$L = 4\pi an^2 \left[ \left( 1 + \frac{3b^2 + c^2}{24a^2} \right) \log_{\epsilon} \frac{4a}{\sqrt{b^2 + c^2}} - \lambda + \frac{b^2}{4a^2\mu} \right].$$

And for

$$L = 4\pi an^2 \left[ \left( 1 + \frac{b^2}{6a^2} \right) \log_{\epsilon} \frac{4a}{b\sqrt{2}} - 0.84834 + \frac{6^2}{3.2648a^2} \right].$$

Read

$$L = 4\pi an^2 \left[ \left( 1 + \frac{b^2}{6a^2} \right) \log_{\epsilon} \frac{4a}{b\sqrt{2}} - 0.84834 + \frac{b^2}{3.2648a^2} \right].$$

To face page 179.]

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$$L = 4\pi n^2 a \left( \log \frac{R}{r} - 2 \right),$$

where  $R$  is the geometric mean distance (G.M.D.) of the section of the coil from itself (*see* Maxwell's "Electricity and Magnetism," Vol. II., 2nd edition, p. 298, for definition of G.M.D.).

For a more complete expression the reader can consult Mascart and Joubert's "Electricity and Magnetism" (English translation by Atkinson, Vol. II., p. 152), where it is shown that if insulated wire is wound up in a rectangular-sectioned coil having a mean radius  $a$  and a radial depth of groove  $2c$

and an axial width  $2b$ , the inductance for a coil of  $n$  turns is

$$L = 4\pi an^2 \left[ \left( 1 + \frac{36^2 + c^2}{24a^2} \right) \log_{\epsilon} \frac{4a}{\sqrt{b^2 + c^2}} - \lambda_2 + \frac{6^2}{4a^2\mu} \right],$$

where  $\lambda$  and  $\mu$  are functions of  $c/b$  which are tabulated by Mascart and Joubert. If  $b=c$ , then  $\lambda=0.84834$  and  $\mu=0.8162$ , and thus

$$L = 4\pi an^2 \left[ \left( 1 + \frac{b^2}{6a^2} \right) \log_{\epsilon} \frac{4a}{b\sqrt{2}} - 0.84834 + \frac{6^2}{3.2648a^2} \right].$$

Suppose, for instance, that the coil has 100 turns, and that  $a=50\text{cm.}$  and  $b=5\text{cm.}$ ; then, from the above formula, we find  $L=0.0157$  henry. A standard of inductance of this kind can be prepared by winding up silk-covered wire in a square groove cut in the edge of a disc of wood built up of sections like a railway carriage wheel. It is preferable not to employ a metal former, on account of the eddy currents which would be set up in its mass.

In addition to absolute standards of inductance, it is convenient to be provided with forms of variable inductance of which the value is known or can be found, but not necessarily predetermined by calculation. These are called *variable inductances*. One of the most convenient of these is a form due to Profs. Ayrton and Perry. It consists of two circular concentric coils of wire, one fixed and the other movable round a vertical axis, to which is attached an index-needle to show the angle made by the planes of the coils with each other. These wire circuits are joined up in series, and can be so placed that a current flowing through both is going in the same or in opposite directions according as the coils are placed.

In Fig. 39 (*see* p. 212) is shown a variable inductance standard of the above description.

Another variable inductance of the same kind consists of two solenoids, one of which can be slipped more or less into the other. They are wound in opposite directions, so that when separated entirely the inductance of the two joined in

series is equal to the sum of them separately; but when made coaxial by being pushed one over the other, the resultant inductance is reduced to a minimum value. The resultant inductance may be regarded as made up of the sum of the inductances of each coil separately *minus* twice their mutual inductance if the coils are so wound that the current in one opposes the current in the other in creating a magnetic field along their common axis.

It is a comparatively difficult matter, however, to secure anything more than a small inductance, or one which can be suitably reckoned in millihenrys, when using coils of wire without any iron core.

For standards of inductance it is, of course, quite inadmissible to employ any coils with iron cores. The only form in which a standard of high inductance can be made is to wind up very fine silk-covered insulated wire in well-insulated bobbins, like the secondary coil of an induction coil with the iron core and primary coil removed.

For many purposes in connection with experiments in electrical oscillations, coils of wire are required of small but variable inductance. These can be made by winding brass wire upon a glass or fibre tube. The tube used is much larger than the spiral. This latter is fixed at one end, but can be pulled out so as to form a spiral of open or close order of turns—in fact, of variable pitch. As the coils are stretched out so the inductance is reduced, but the resistance remains, of course, the same. Coils of this description have been called by the Author *accordion inductance coils*, from their resemblance to a well-known musical instrument.

§ 14. **Measurement of Inductance.**—*The Time-constant of a Circuit.*—The measurement of inductance is generally made to depend upon one of three methods: (i.) A measurement of the electromagnetic energy associated with the inductive circuit when a known current is flowing in it. (ii.) A comparison between the time-constants of the given inductive

circuit and a circuit containing a known capacity. (iii.) A comparison between the impedance of a circuit and its ohmic resistance when tested with a simple periodic current. In all cases we are concerned with the *time-constant* of the inductive circuit.

Consider the fundamental equation for the current  $i$  existing at any instant in an inductive circuit of inductance  $L$  and resistance  $R$  after the application of a constant E.M.F.  $E$ . We have, then, the differential equation for the current

$$L \frac{di}{dt} + Ri = E.$$

The solution of this for  $i$  is

$$i = I \left( 1 - e^{-\frac{R}{L}t} \right),$$

where  $I = E/R$ , and is the full value to which the current  $i$  will rise after an infinite time. Hence, the defect of the actual current from this full value is  $I - i = Ie^{-\frac{R}{L}t}$ .

If we integrate this last expression from 0 to  $\infty$ , we obtain the value of the whole quantity of electricity which is, so to speak, stored up in connection with the circuit and is discharged through it when the steady E.M.F. is suddenly removed. This quantity is—

$$Q = \int_0^{\infty} Ie^{-\frac{R}{L}t} dt = \frac{L}{R} I.$$

The ratio  $L/R$ , or *inductance/resistance*, is called the *time-constant* of the circuit ( $=T$ ), and is evidently the time during which the actual full current would have to flow to convey the quantity of electricity which is actually discharged through the circuit on removing the E.M.F.

Graphically, it may be represented thus :—Let the horizontal line in Fig. 28 be a time line, and let the curve represent by its ordinates the gradual rise of current in the circuit when a constant E.M.F. is applied. This curve will

be asymptotic to a line drawn at an altitude representing to scale the full current  $I$ . Then the shaded portion or area represents the total quantity of electricity called  $Q$  above. The time-constant is a time,  $OT$ , such that the rectangular area,  $OI \cdot OT$ , is equal to the whole shaded area.

### § 15. Lord Rayleigh's Method of Measuring Inductance.

—Let  $R$  be the resistance of a coil of which the inductance  $L$  is to be measured. Balance this coil on a Wheatstone bridge with three other coils or resistances  $P$ ,  $Q$  and  $S$ , and obtain the steady balance. Then we have  $P/Q = R/S$  as the condition fulfilled. The galvanometer will then give no deflection

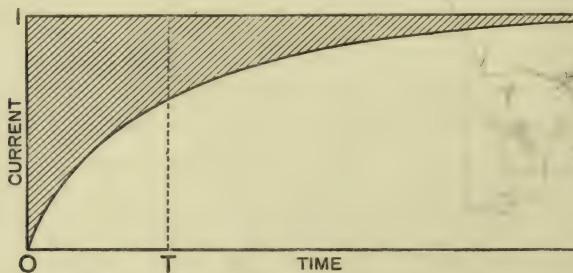


FIG. 28.

if the battery key is put down first and the galvanometer key afterwards. If, however, the galvanometer key is pressed first and then the battery key we obtain a "kick" on the galvanometer due to the passage through it of an induced current. Let  $e$  be the E.M.F. induced in the inductive resistance ( $L$ ,  $R$ ) at any instant, then  $e = L \frac{di}{dt}$ , where  $i$  is the instantaneous current in that branch. Hence  $\int e dt = LI$ , the integral being taken between the limits of time corresponding to  $i = 0$  and  $i = I$ ; that is, the time integral of the induced E.M.F. is equal to the product of the inductance  $L$  and the value  $I$  attained by the current in that branch of the bridge when the current became steady.

It has already been shown that if a quantity of electricity  $Q$  is sent suddenly through a ballistic galvanometer it creates a "throw"  $\theta$  of the coil or needle such that

$$Q = \left( \frac{T}{2\pi} \frac{V}{R \tan \phi} \right) \theta$$

(see equation 11, § 3, chap. I., p. 23 of this volume), where  $T$  is the complete periodic time of the needle and  $V$  is the potential difference which must be applied to the ends of the galvanometer circuit having a resistance  $R$  to create a steady deflection  $\phi$ . But the quantity  $Q$  may be looked upon as the quotient of the time integral of a sudden E.M.F. applied to the galvanometer circuit by its resistance, and if we denote by  $e$  the instantaneous value of this E.M.F. we have

$$Q = \frac{1}{R} \int e dt = \frac{T}{2\pi} \frac{V}{R \tan \phi} \theta;$$

or, writing  $2 \sin \frac{\theta}{2}$  for  $\theta$ , we have,

$$\frac{\int e dt}{V} = \frac{T}{2\pi} \frac{1}{\tan \phi} 2 \sin \frac{\theta}{2}.$$

Hence the expression on the right-hand side of the above equation is the value of the ratio of the time integral of a rapidly varying E.M.F. to the value of a steady E.M.F. in terms of the "throw"  $\theta$  and steady deflection  $\phi$  respectively produced by them on the same galvanometer.

Returning, then, to the bridge arrangement, we notice that, if the bridge is balanced and the galvanometer key kept down, then, on pressing the battery key, we obtain a "throw,"  $\theta$ , of the galvanometer corresponding to the product  $LI$ , where  $I$  is the steady current in the branch  $R$ , this product being equal to the time integral of the E.M.F. in that branch.

Suppose, then, that the resistance of the arm  $R$  is increased by a small amount,  $\delta R$ , which does not sensibly alter the value of the current  $I$  in it. This will create a steady deflection of the galvanometer equivalent to the introduction

of a steady E.M.F.,  $I\delta R$ , in that branch. Hence, we have

$$\frac{LI}{I\delta R} = \frac{T}{2\pi} \frac{2 \sin \frac{\theta}{2}}{\tan \phi},$$

or 
$$L = \frac{T}{2\pi} \frac{\delta R}{\tan \phi} 2 \sin \frac{\theta}{2}$$

This equation gives us the value of  $L$  in terms of  $\delta R$  and  $T$  and the deflections.

If, instead of waiting for the steady deflection  $\phi$ , we may notice the "throw"  $\beta$  which occurs when the arm  $R$  is increased to  $R + \delta R$  and the battery key is put down *first* and then the galvanometer key, we can write the following equation

$$L = \frac{T\delta R}{2\pi} \frac{2 \sin \frac{\theta}{2}}{\tan \frac{\beta}{2}},$$

which is the form in which the formula was given by Lord Rayleigh. (See *Proc. Roy. Soc. Lond.*, Vol. XXXII. (1881), p. 116, in a Paper by Lord Rayleigh and Prof. A. Schuster, "On the Determination of the Ohm in Absolute Measure.")

The practical proceeding is then as follows:—To determine by the above method the inductance of a coil, balance the coil on a Wheatstone bridge and adjust the arms until the steady balance is obtained and there is no deflection of the galvanometer when the battery key is pressed *before* the galvanometer key.

Then, in the next place, keeping the galvanometer circuit closed, press the battery key and notice the "throw"  $\theta$  of the galvanometer needle or coil.

Thirdly, increase the resistance of the arm containing the inductive resistance by adding to it a small non-inductive resistance  $\delta R$ , and observe the steady deflection  $\phi$  or the throw  $\beta$  which takes place when the battery circuit is first made, and then afterwards the galvanometer key pressed.

Lastly, observe the complete periodic time  $T$  of the galvanometer, and compute the inductance from the formulæ given above.

§ 16. **Theorem Concerning the Time-constants of Bridge Arms.**—Before proceeding to describe other methods of determining inductance, the reader will find it an advantage to have studied a theorem connected with the use of Wheatstone's bridge when the arms are inductive.

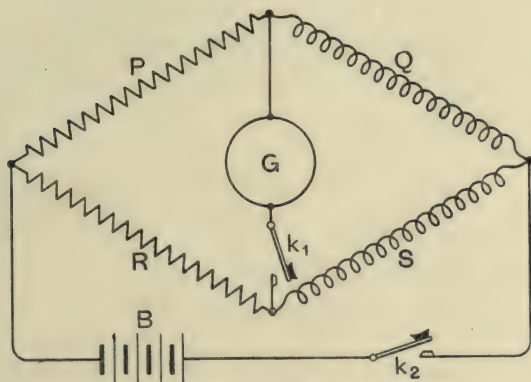


FIG. 29.

If the arms of a Wheatstone bridge arrangement (see Fig. 29) consists of four resistances,  $P$ ,  $Q$ ,  $R$ ,  $S$ , two of which, say  $S$  and  $Q$ , are inductive, then we may adjust the values of these four resistances so as to obtain a bridge balance, and, therefore, no deflection of the galvanometer, when the battery key is first pressed and then afterwards the galvanometer key. We shall call this condition the *steady balance* of the bridge, and to obtain this we must make  $P/Q = R/S$  or  $PS = QR$ .

Let  $I_S$  and  $I_Q$  be the currents in the arms  $S$  and  $Q$  when the bridge currents are all steady. Then, at that time, we have also  $I_S S = I_Q Q$ . If, then,  $L_S$  and  $L_Q$  are the inductances

of  $S$  and  $Q$ , then  $L_S I_S$  and  $L_Q I_Q$  are, as already shown (*see* § 15), the time integrals of the E.M.F.s of self-induction in these branches  $S$  and  $Q$ . It follows that, when the bridge is balanced for steady currents,

$$\frac{L_S I_S}{L_Q I_Q} = \frac{L_S Q}{L_Q S} = \frac{L_S P}{L_Q R}.$$

Suppose, then, that the galvanometer key is put down first and then afterwards the battery key, we shall obtain a “kick” on the galvanometer due to the difference between the time integrals of these E.M.F.s of self-induction in the  $S$  and  $Q$  arms, and this “kick” is proportional to this difference multiplied by a function of the resistances  $P$ ,  $Q$ ,  $R$ ,  $S$ ,  $G$  and  $B$  which form the bridge. Hence, if this “kick” is denoted by  $\theta$ , we have

$$L_S P - L_Q R = a\theta,$$

where  $a$  is some constant. Also, if we write  $T_S$  for  $L_S/S$  and  $T_Q$  for  $L_Q/Q$ , we can also write

$$T_S - T_Q = \beta\theta$$

where  $\beta$  is some other constant. For, since, when the bridge is balanced,  $PS=QR$ , we have obviously  $P=QR/S$  and  $R=PS/Q$ . Hence,  $L_S P = QRL_S/S$  and  $L_Q/R = PSL_Q/Q$ . Hence,

$$L_S P - L_Q R = \{L_S/S - L_Q/Q\}PS.$$

Accordingly, when the bridge is balanced the “kick” obtained on the galvanometer on closing the battery circuit after the galvanometer circuit is proportional to the difference of the time-constants of the circuits  $Q$  and  $S$ .

The above result may be generalised as follows:—If a Wheatstone bridge arrangement of resistances is balanced for steady currents so that no galvanometer deflection occurs when the galvanometer key is depressed after the battery key; but if the galvanometer gives a “kick” when the order of operation is reversed, then this galvanometer deflection is

proportional to the sum or difference of the time-constants of two of the circuits, according to the arrangement adopted.

We may consider, for example, four typical cases.

(i.) Two adjacent arms of the bridge have different inductances (*see* Fig. 29). Let P and R be non-inductive, and let S and Q be inductive and have inductances  $L_S$  and  $L_Q$ . Let  $L_S/S = T_S$  and  $L_Q/Q = T_Q$ . Then, as already shown, if  $PS = QS$ ,

$$T_Q - T_S = \alpha\theta,$$

where  $\theta$  is the "kick" on making the battery circuit after the galvanometer circuit.

Hence 
$$\frac{L_Q}{Q} - \frac{L_S}{S} = \alpha\theta.$$

Since  $PS = QR$ , we have

$$L_Q R - L_S P = \beta\theta.$$

Accordingly, if the "kick" is zero,

$$\frac{L_S}{L_Q} = \frac{R}{P}.$$

That is to say, if we adjust the four resistances so that  $PS = QR$ , and there is, therefore, no steady galvanometer deflection on closing the battery and galvanometer circuits, and if we also adjust the arms R and P so that there is also no "kick" on making the battery circuit when the bridge is balanced and the galvanometer key is down, then the ratio of the inductances  $L_S$  and  $L_Q$  is given by the ratio of the resistances R and P.

(ii.) Two adjacent arms of the bridge have different capacities (*see* Fig. 14, p. 146). Let  $C_Q$  and  $C_S$  be the capacities of condensers put in parallel with the arms Q and S of a bridge. Then the time-constant of the condenser across the arm Q is  $C_Q Q$ , and that across the arm S is  $C_S S$ . Hence, by this rule, if the bridge is balanced for steady currents, so that  $PS = QR$ , yet there will be a "kick,"  $\theta$ , when the battery key is closed after the galvanometer key, and by the rule we have

$$C_Q Q - C_S S = \alpha\theta.$$

But, since  $PS=RQ$ , we have also

$$\frac{C_Q - C_S}{R} = \beta\theta.$$

If  $Q$  and  $S$  are made very large, we finally approximate to the condition of De Sauty's arrangement, and then, when there is no "kick" we have

$$\frac{C_Q}{C_S} = \frac{R}{P}.$$

(iii.) One arm of the bridge inductive and an opposite arm having capacity (*see* Fig. 30). This is the arrangement

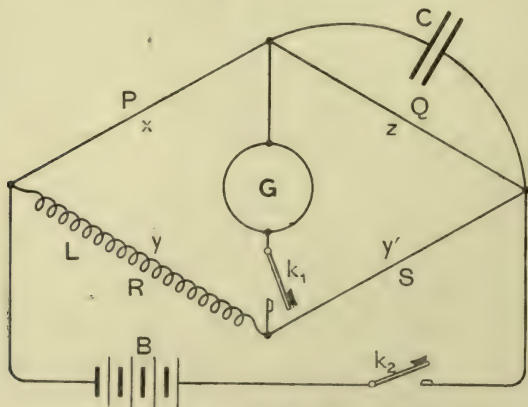


FIG. 30.

known as Maxwell's method of comparing an inductance and a capacity. Let the arm of the bridge  $R$  have an inductance  $L$ , and let the arm  $Q$  have a condenser of capacity  $C$  shunted across it. Then, if the bridge is balanced for steady currents so that  $PS=QR$ , and if we afterwards press the galvanometer key first and then the battery key, we obtain a "kick,"  $\theta$ , on the galvanometer, which is due to the difference of the time-constants of the arms  $R$  and  $Q$ , or

$$\frac{L}{R} - CQ = a\theta.$$

If, then, the value of  $C$  is adjusted so that there is no "kick," we have

$$L = CQR,$$

which is the rule given by Maxwell.

(iv.) Comparison of a self and mutual inductance (*see* Fig. 31). Let one arm of a bridge have an inductance  $L$ , and also let mutual induction exist between it and the battery circuit,  $M$  being the coefficient of mutual induction. Then, when the bridge is balanced for steady currents so that  $PS = QR$ , we

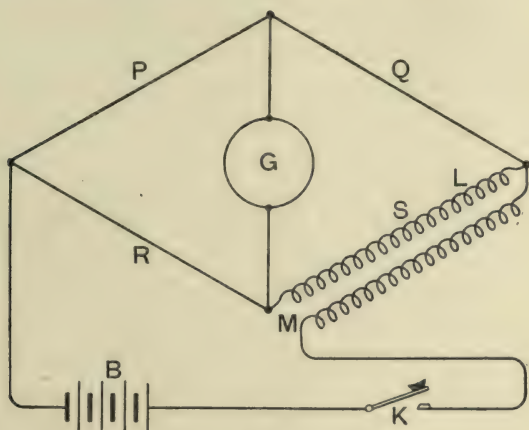


FIG. 31.

have, on making the battery circuit after the galvanometer circuit is closed, a "kick,"  $\theta$ , such that

$$\frac{M}{Q} - \frac{M+L}{S} = a\theta.$$

But, since  $PS = QR$ , therefore

$$MR - (M+L)P = \beta\theta;$$

also

$$L - \left(\frac{P-R}{P}\right)M = \gamma\theta;$$

and if either  $L$  or  $M$  is given the other is determined.

Many other illustrations of the same principle will be found explained and illustrated in a valuable Paper by Dr. W. E. Sumpner (see *Journal Inst. Elec. Eng.*, Vol. XVI., 1887, p. 344).

**§ 17. The Measurement of Inductance by Maxwell's Method.**—Maxwell gave (see "Treatise on Electricity and Magnetism," 2nd edition, Vol. II., p. 387, § 778) a method of measuring inductance by comparing, on a bridge, the inductive circuit with a condenser of known capacity. The coil of which the inductance is to be measured is made one arm of a Wheatstone bridge, and across the opposite arm a condenser of capacity  $C$  is connected. Let  $R$  be the inductive coil of which the inductance is  $L$ , and let  $P$ ,  $Q$  and  $S$  be the other arms of the bridge (see Fig. 28). Then let these resistances be adjusted so that  $PS=QR$ . Then let the galvanometer circuit be completed and let the battery key be put down. We obtain a "kick" on the galvanometer due to the difference of the time-constants of the arms  $Q$  and  $S$ . Let time be reckoned from the instant of closing the battery circuit, and let  $x$ ,  $y$ ,  $z$  be quantities of electricity which at any instant have from that time zero passed through  $P$ ,  $R$  and  $Q$ . Then  $x-z$  is the charge of the condenser.

$$\therefore \frac{x-z}{C} = Q \frac{dz}{dt}.$$

Suppose, next, that  $C$  is so adjusted, or the resistance arms so varied in magnitude, that whilst the relation  $PS=QR$  remains fulfilled, yet there is no "kick" of the galvanometer on making the battery circuit. Then no electricity passes through the galvanometer at all, and we have the conditions expressed by the following equations:—

$$R \frac{dy}{dt} + L \frac{d^2y}{dt^2} = P \frac{dx}{dt},$$

$$S \frac{dy}{dt} = Q \frac{dz}{dt}.$$

$$\text{Hence} \quad RQ\left(1 + \frac{L}{R} \frac{d}{dt}\right)z = SP\left(1 + QC \frac{d}{dt}\right)z;$$

$$\text{but} \quad RQ = SP. \quad \therefore L = RQC.$$

Thus  $L$  is determined in terms of a capacity and two resistances.

The practical difficulty of carrying out Maxwell's method is that hardly any laboratory possesses a stock of sliding condensers the capacity of which can be varied gradually. Hence, it generally needs much time and trouble to adjust the resistance arms  $P$ ,  $R$  and  $S$  so that, whilst the steady balance condition  $PS = QR$  is fulfilled, yet the resistances are so selected in comparison with the capacity  $C$  which is available that the conditions for no "kick" are also complied with.

It *can* be done, but it is usually a tedious process. Accordingly, modifications have been introduced with the object of saving time. One method, due to Dr. Sumpner (see *Jour. Inst. Elec. Eng.*, May 12, 1887; also *The Electrician*, Vol. XXXIII., p. 54), is to select a condenser which has a capacity  $C_1$  such that, when the bridge is balanced for steady currents, the "kick" of the galvanometer on making the battery circuit is in one direction—say, to the right—and equal to  $d_1$  divisions of the galvanometer scale. This capacity is now altered to another one,  $C_2$ , such that the "kick" is in the opposite direction and equal to  $d_2$  divisions of the scale. These deflections should not be very large, but be confined within those limits for which, with the galvanometer in use, "quantity passing" is proportional to "throw."

Then construct a diagram as follows:—Let a distance on a horizontal line,  $Oa$ , be taken to represent  $C_2$ , and a distance,  $Ob$ , to represent  $C_1$  (see Fig. 32). Then erect perpendiculars  $d_2$  and  $d_1$  at  $a$  and  $b$  to represent the galvanometer "throws" in opposite directions, and join the tops of these perpendiculars. This will cut the horizontal line at point  $c$ , and the corresponding capacity  $Oc$  is the capacity  $C$ , which would cause complete annulment of the "kick," and is, therefore; the value of  $C$  to be put into the formula  $L = RQC$ .

This device gets over the difficulty caused by the non-possession of a graded condenser, and enables us to obtain a measurement of the inductance without interfering with the resistance values of the bridge arms.

This method was re-stated by Mr. A. Russell in *The Electrician*, Vol. XXXIII., p. 5, 1894, and is an effective solution of the difficulties of this Maxwell method.

If only equal or large condensers are available, it should be remembered that we can construct capacities larger or smaller by joining them in parallel or in series. Thus, two  $\frac{1}{2}$  microfarad condensers joined in parallel give us a capacity

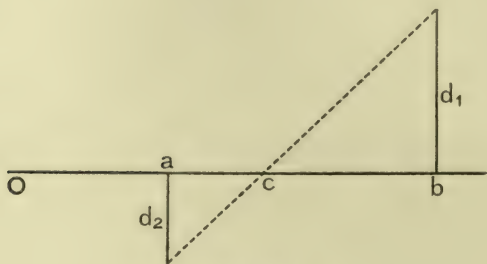


FIG. 32.

of 1 microfarad, and joined in series give us a capacity of  $\frac{1}{4}$  microfarad.

Two modifications of Maxwell's method have been given by Mr. Rimington (see *Phil. Mag.*, July, 1887) and Prof. Niven (see *Phil. Mag.*, Sept., 1887), which are very similar.

### § 18. Anderson's Method of Measuring Inductance.—

Amongst bridge methods for measuring inductance by the comparison of an inductance with a capacity, perhaps the best is that due to Prof. A. Anderson (see *Phil. Mag.*, 1891, Vol. XXXI., p. 329; or *The Electrician*, Vol. XXVII., p. 10). Anderson's method is a double process method like Maxwell's, but one operation does not upset the other. Hence,

there is no delay or waste of time in obtaining the required readings.

The inductive coil of which the inductance is to be measured is connected to a Wheatstone bridge in the usual manner, and its steady resistance measured. Let  $R$  be the resistance of this coil and let  $L$  be its inductance. Let the other bridge arms be, as usual,  $P$ ,  $Q$  and  $S$  (see Fig. 33). The usual adjustment of these four resistances is made so that no current flows through the galvanometer when the battery key  $k_2$  is first closed and then subsequently the

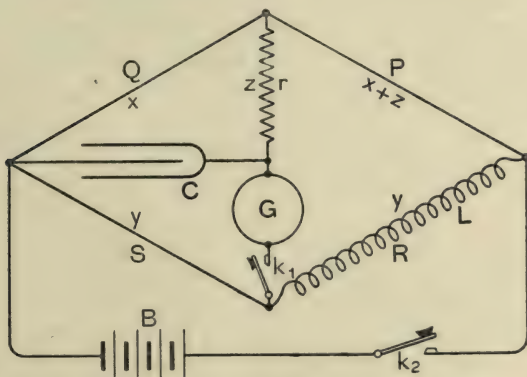


FIG. 33.

galvanometer key  $k_1$ . Then when this steady balance is obtained  $PS=QR$ . We next connect a resistance which can be varied into the galvanometer circuit. This can take the form of an ordinary plug resistance box. Let this resistance be denoted by  $r$ . We also connect a condenser  $C$  as shown in the diagram. When the steady balance is obtained no alteration in the value of  $r$  affects the galvanometer, but the value of  $r$  and the capacity  $C$  can be so adjusted as to annul the "kick" of the galvanometer on making or breaking the battery circuit when the galvanometer circuit is first closed.

When these values of  $r$  and  $C$  are determined, and also the values of  $P$ ,  $Q$  and  $S$ , which give a steady balance, we then have

$$L = C\{r(R + S) + RQ\}.$$

The proof of the formula is obtained as follows:—Let us suppose the bridge balanced for steady currents, and hence we have  $PS = QR$ , where  $R$  is the resistance of which the inductance  $L$  has to be determined. Then let  $x$ ,  $y$ ,  $z$  and  $x + z$  be the quantities of electricity which have, from the moment of closing the battery key, passed through  $Q$ ,  $R$  and  $S$ ,  $r$  and  $P$ . If  $r$  has been adjusted so that there is no “kick,” then no electricity flows through the galvanometer, and the quantity passing along  $R$  is obviously the same as that passing along  $S$ . Hence we have

$$Q \frac{dx}{dt} = \frac{z}{C} + r \frac{dz}{dt},$$

$$\frac{z}{C} = S \frac{dy}{dt},$$

$$r \frac{dz}{dt} + P \left( \frac{dx}{dt} + \frac{dy}{dt} \right) = R \frac{dy}{dt} + L \frac{d^2y}{dt^2}.$$

$$\therefore (r + P) \frac{dz}{dt} + \frac{P}{Q} \left( \frac{z}{C} + r \frac{dz}{dt} \right) = \frac{R}{S} \frac{z}{C} + \frac{L}{SC} \frac{dz}{dt}.$$

But, since  $P/Q = R/S$ , we have

$$r + P + \frac{P}{Q} r = \frac{L}{SC},$$

or

$$L = C\{r(R + S) + RQ\}.$$

If in the above method and formula we put  $r = 0$ , it reduces to Maxwell's method, and then

$$L = RQC.$$

Prof. Anderson shows, in the Paper above mentioned, that for greatest sensibility we should make  $Q$  and  $S$  large and  $P$  and  $r$  small. He gives an example of a measurement. The coil to be measured was balanced on a bridge, the arms of which were  $P = 10$  and  $Q = 100$ . The balancing arm was

$S=1,577$ , hence the value of the steady resistance was  $R=157.7$ . Then it was found that, if the condenser had a capacity  $C=1$  microfarad  $=10^{-6}$  farad, that  $r$  had to be made equal to 59 ohms. Accordingly the formula gives

$$L = \frac{1}{10^6} \{ 59(1,577 + 157.7) + 15,770 \} = 0.118 \text{ henry.}$$

As will be explained presently, the sensibility of this method can be enormously increased by employing a commutator (usually called a secohmmeter) to break and make the battery and galvanometer circuit, so that the galvanometer or bridge circuit is only closed at the time when the battery circuit is being broken, but not when the battery circuit is being opened.

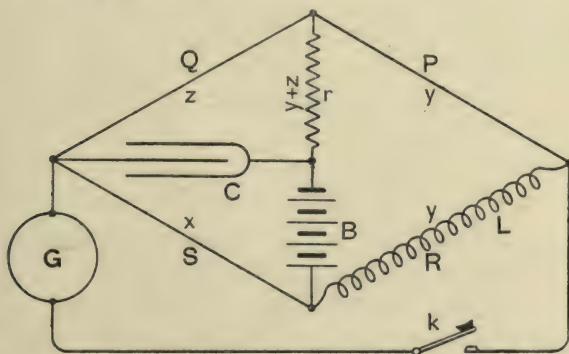


FIG. 34.

The above-described method can be varied by exchanging the places of the galvanometer and the battery. Prof. Stroud informs the Author that he has, for a long time past, employed this latter arrangement for the determination of inductance. The connections for the bridge and condenser, according to Prof. Stroud's plan, are as shown in Fig. 34. They are the same as in the Anderson method, but the galvanometer and battery change places.

Let us assume, as before, that the inductive resistance is  $R$  and its inductance is  $L$ , and that the other three non-inductive

arms of the bridge are P, Q and S. Let C be the capacity of a condenser connected between the junction of Q and S and the junction between the battery and a resistance  $r$  connecting it to the meeting point of Q and P. Then let  $x, y$  and  $z$  denote the quantities of electricity which have, from the closing of the battery circuit, passed through the branches S, R and P, and Q. Suppose the bridge balanced for steady currents so that  $PS = QR$ , and let C and  $r$  be so adjusted that there is no "kick" at the galvanometer on opening or closing the battery circuit. Then it is seen that the junction points of Q and S and R and P must remain always at the same potential.

Accordingly, we have the following equations:—

$$S \frac{dx}{dt} = L \frac{d^2y}{dt^2} + R \frac{dy}{dt}, \quad \dots \dots \dots (i.)$$

$$P \frac{dy}{dt} = Q \frac{dz}{dt}, \quad \dots \dots \dots (ii.)$$

and

$$\frac{x-z}{C} = Q \frac{dz}{dt} + r \frac{d(y+z)}{dt} \dots \dots \dots (iii.)$$

The equations (i.) and (ii.) are merely the expression of the fact that the junction points (QS) and (RP) are always at the same potential, and (iii.) is an expression for the potential difference of the terminals of the condenser.

Differentiate equations (ii.) and (iii.) with respect to  $t$ , and we have

$$P \frac{d^2y}{dt^2} = Q \frac{d^2z}{dt^2},$$

$$\frac{dx}{dt} - \frac{dz}{dt} = CQ \frac{d^2z}{dt^2} + Cr \left( \frac{d^2y}{dt^2} + \frac{d^2z}{dt^2} \right).$$

$$\text{Hence,} \quad \frac{dx}{dt} - \frac{dz}{dt} = C \left( Q + r + r \frac{Q}{P} \right) \frac{d^2z}{dt^2} \dots \dots \dots (iv.)$$

But from (i.) and (ii.) we have

$$S \frac{dx}{dt} = L \frac{Q}{P} \frac{d^2z}{dt^2} + R \frac{Q}{P} \frac{dz}{dt},$$

and, since  $PS=RQ$ , we can write this last equation thus :

$$S\left(\frac{dx}{dt} - \frac{dz}{dt}\right) = L\frac{Q}{P} \frac{d^2z}{dt^2} \quad \dots \quad (v.)$$

Therefore, from (iv.) and (v.) we have

$$\frac{LQ}{SP} = C\left(Q + r + r\frac{Q}{P}\right),$$

or

$$L = C\{r(R + S) + QR\},$$

and this is the same expression as found in the case of the Anderson method.

Accordingly, the exchange of the battery and galvanometer, as in the case of the ordinary bridge, makes no difference in the expression for the conditions of steady and initial balance.

It is a general principle which holds good of all Wheatstone bridge methods of testing, whether involving capacity and inductance as well as resistance in the various circuits, that, if two circuits are *conjugate*—that is, if the opening and closing of one circuit creates no current or change in the current in the other—then these circuits can be exchanged without affecting the result of the arrangement or the relation of the other circuit factors.

In this connection we may refer to an important Paper by Lord Raleigh “On the Sensitiveness of the Bridge Method in its Application to Periodic Electric Currents.” (See *Proc. Roy. Soc.*, Jan. 17, 1891; or *The Electrician*, Vol. XXVI., p. 791, 1891.) And to a Paper by Mr. O. Heaviside “On the Best Arrangement of Wheatstone’s Bridge for Measuring a given Resistance with a given Battery and Galvanometer” (*Phil. Mag.*, Vol. XLV., 1873, p. 114).

**§ 19. The Measurement of Inductance by the Secohmmeter.**—Profs. Ayrton and Perry published in 1887 (see *Journal Soc. Tel. Eng.* and *The Electrician*, Vol. XVI., p. 292) a Paper entitled “Modes of Measuring Coefficients of Self and Mutual Induction.” In this Paper they describe the use of a revolving circuit interrupter, which they call a

*secohmmeter*, which is used in connection with a Wheatstone bridge to periodically interrupt the battery circuit and short circuit the galvanometer, in such fashion that the latter is short-circuited whilst the battery circuit is broken, and unshort-circuited before, during and for a little while after the making of the battery circuit.

Let  $T$  stand for the time which elapses between the making of the battery circuit and the short-circuiting of the galvanometer. This time must be sufficiently long for the currents in all parts of the bridge circuits to become steady. Let  $x$  be the galvanometer current and  $i$  the current in an inductive circuit forming one arm of the bridge, of which  $L$  is the inductance. Then, in accordance with principles already explained, we have for  $x$  the value

$$x = L \frac{di}{dt} \frac{\delta}{\Delta},$$

where  $\delta$  and  $\Delta$  are two determinants formed of the bridge resistances (*see* Vol. I., Chap. II., p. 232, of this HANDBOOK).

Again,  $\int_0^T x dt$  is the time integral of the current through the galvanometer from the time of closing the battery circuit up to the time when the current  $i$  attains its steady value  $I$ . Hence

$$\int_0^T x dt = LI \frac{\delta}{\Delta},$$

and the impulse on the galvanometer is proportional to  $LI$ .

Next, suppose the inductance  $L$  removed, and the branch  $R$  to have its resistance increased by a small amount  $r$ , then  $Ir$  is the increase of fall of potential down  $R$  when the currents are steady, and this will produce a deflection on the galvanometer equal to a current,  $X$ , passing through it, such that  $X = rI \frac{\delta}{\Delta}$ .

Suppose, then, that, when  $R$  is inductive, the interrupter is rotated at a speed of  $N$  revolutions per second, the galvanometer impulses will be all in one direction, and will result in

the passage through the galvanometer of a quantity of electricity equal to  $NLI\frac{\delta}{\Delta}$  per second.

Next, suppose the inductance removed and the branch R to be non-inductive, but to have its resistance increased by a small amount  $r$  so as to be  $r + R$ . By what has already been explained it will be seen that the galvanometer is traversed by a current equal to  $frI\frac{\delta}{\Delta}$ , where  $f$  is the fraction of a whole revolution during which the galvanometer and battery circuits are *both* closed. Hence, if  $r$  is adjusted to make the galvanometer current equal in the two cases, we have

$$frI\frac{\delta}{\Delta} = NLI\frac{\delta}{\Delta},$$

or

$$L = \frac{fr}{N}.$$

In other words, the inductance  $L$  is determined in terms of a resistance  $r$  and a time  $1/N$ .

Ayrton and Perry show by a more elaborate analysis (see *Journal Soc. Tel. Eng. and Elec.*, Vol. XVI., p. 303, 1887) that the full expression for the inductance  $L$  is

$$L = r \left( T + \frac{\lambda}{g} - \frac{B}{A} \right),$$

where  $T$  is the fraction of time  $f/N$ ,  $\lambda$  and  $g$  are the inductances and resistance of the galvanometer, and  $B$  and  $A$  are expressions involving the resistances and inductances of the bridge circuits such that  $B/A$  is always a small quantity.

This method of using the secohmmeter may be called the absolute method. It is seldom used because of the difficulty of determining and adjusting the quantities  $f$  and  $r$ .

## § 20. Comparison of Inductances by the Secohmmeter.—

The usual method of employing the secohmmeter or battery-galvanometer interrupter for measuring inductance is by a comparison method. Profs. Ayrton and Perry pointed out, in the Paper already quoted, that when we are employing any

of the bridge methods for inductance measurements in which the test depends upon the creation of a "kick" or throw of the galvanometer by the break or make of the battery circuit, the bridge being otherwise balanced for steady currents, we can immensely increase the sensitiveness of the method by using a revolving interrupter which causes the galvanometer circuit to be "made" only during a "break" of the battery circuit, but not at other times. In this case the "kicks" are all in one direction, and run together into a steady current or galvanometer deflection, and we have a greatly increased power of making the adjustments on which the absence of a "kick" depends, and, therefore, of obtaining the necessary readings.

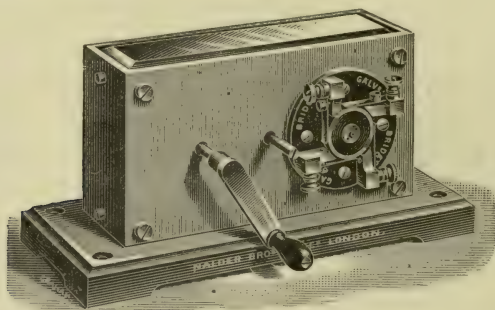


FIG. 35. — Ayrton and Perry Secohmmeter.

A double circuit interrupter of this kind continues to be called a secohmmeter. In one form it consists of an axle driven round by a winch and gearing. On that axle are keyed two barrels, which each consist of two insulated segments. Against each barrel four springs press in such fashion that as the barrel revolves it alternately reverses the connection between the springs. If we call one pair of springs  $x$  and  $y$  and the other pair  $a$  and  $b$ , the connection is first  $x$  to  $a$ ,  $y$  to  $b$ , and then  $x$  to  $b$  and  $y$  to  $a$  (see Fig. 35).

Accordingly, if the barrels are set on the axle so that this reversal takes place alternately, we can so arrange them that

the one put in the circuit of the battery is breaking the circuit at the moment when the one in the circuit of the galvanometer is not so doing.

To use this interrupter we require a variable standard of inductance, and this is joined up with the coil  $L_1R_1$  of which the inductance is to be measured on a bridge (see Fig. 36), the variable inductance being  $L_2R_2$ . We then adjust, in the first place, the steady balance, and make the ratio arms  $r_1r_2$  such that  $r_1/r_2 = R_1/R_2$ . Next the interrupter is rotated and the variable inductance  $L_2$  is adjusted so that the inductive deflection of the galvanometer vanishes as well as the steady

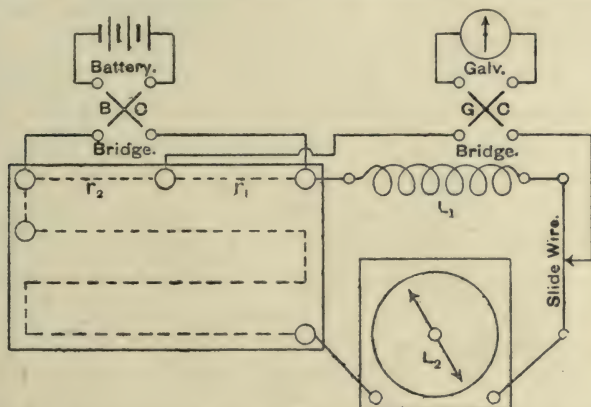


FIG. 36.—Arrangement of Bridge, Secohmmeter and Variable Inductance for Inductance Measurement.

deflection. When this is the case, we have the time constants of the circuits  $L_1R_1$ ,  $L_2R_2$  equal, viz. :—

$$\frac{L_1}{R_1} = \frac{L_2}{R_2};$$

but  $R_1/R_2 = r_1/r_2$ . Hence  $L_1/L_2 = r_1/r_2$ , or  $L_1 = L_2 \frac{r_1}{r_2}$ .

It will be noticed that in this method of using the interrupter we are not concerned with the absolute speed of commutation, but we do depend upon the accuracy of the standard of inductance. Moreover, we are limited by the

standard of inductance. Hence it is better to employ a method in which we are less dependent upon the instrument maker's accuracy.

§ 21. **The Practical Measurement of Inductance in the Laboratory.**—After a wide experience of the various methods which have been proposed for the measurement of inductance, the Author has found no method which is better adapted for the everyday purposes of the electrical laboratory than a combination of the Anderson method with the use of a battery and galvanometer circuit interrupter as proposed by Ayrton and Perry to increase its sensitiveness. The double interrupter, as made by most instrument makers, gives, however, a good deal of trouble with the contacts, and the Author has, therefore, in conjunction with Mr. W. C. Clinton, devised a model which can be thoroughly recommended for continuous work.

It consists of a  $\frac{1}{4}$  H.P. continuous current 100-volt shunt-wound electric motor fastened to a bedplate and provided with a starting resistance in the circuit of its armature. To this is connected, by a flexible coupling, a shaft having on it two insulated barrels. These barrels each consist of two parts like wheels, with four teeth set into one another but insulated from each other (*see* Fig. 37). Against these barrels four brass wire brushes press, one pair against the continuous flange of the barrels and one pair against the interrupted portion. The barrels are so set on the shaft that, when one pair of brushes pressing against one barrel are in electrical connection with each other through the barrel, the other pair on the other barrel is just being electrically disconnected.

The arrangement will be easily understood by a reference to Fig. 38, which represents each barrel surface laid out flat.

The rectangles marked I are the teeth of one wheel, and those marked A or B the teeth of the interconnected wheels. One pair of brushes rest on the flange portion of the barrels A or B, and the other pair on the teeth part.

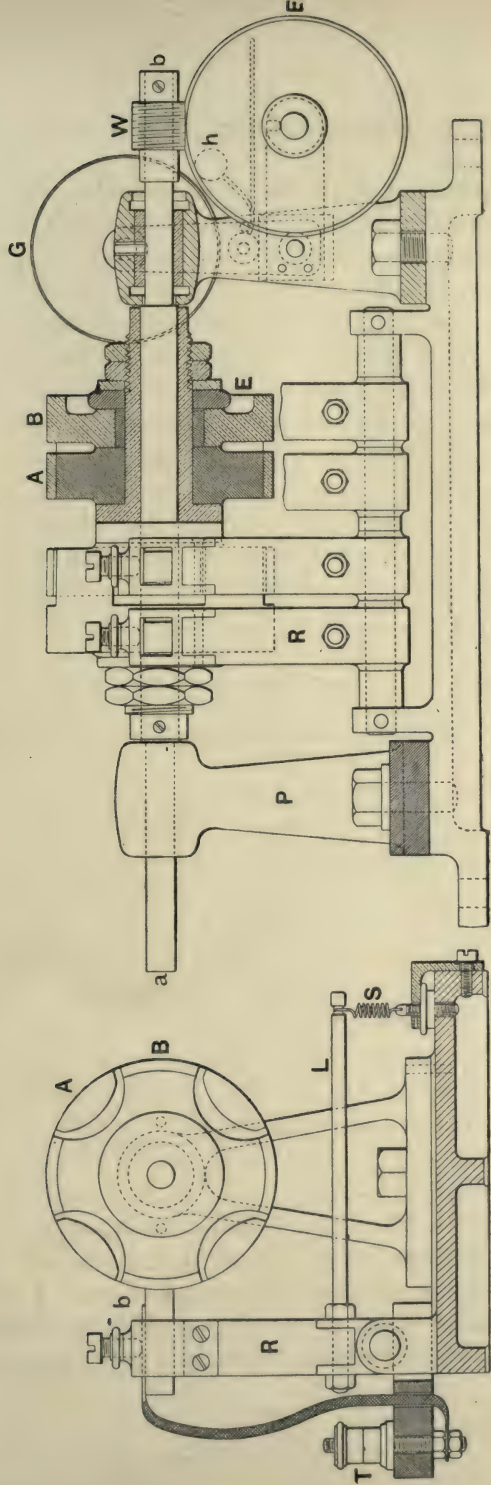


FIG. 37.—Double Circuit Contact-breaker, for Inductance Measurement. (Fleming and Clinton.)

The brushes are carried in suitable holders on insulating pillars. These brushes make better contact than the brass springs usually employed.

To employ this rotating commutator or motor interrupter, we connect one pair of the brushes in the battery circuit of a Wheatstone's bridge and the other pair in the galvanometer circuit, and complete the arrangements as for an Anderson method of measuring inductance with a condenser and galvanometer connected in as described in § 17 of this chapter. We then first obtain a steady balance on the bridge, and measure the resistance of the conductor whose inductance is to be determined. We then set the motor interrupter in

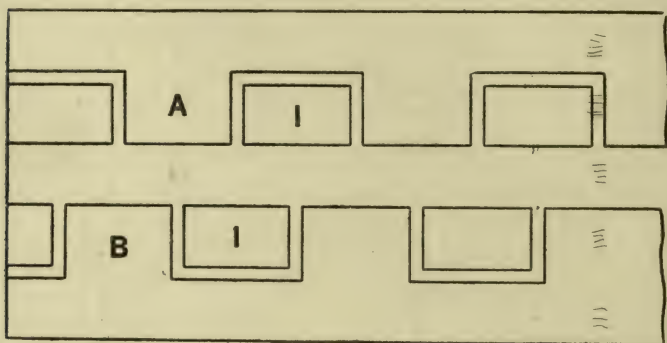


FIG. 38.

action. This causes all the electromotive impulses on the galvanometer due to the inductance of the bridge arm to run together into a steady deflection. This deflection is annulled by altering the capacity and the resistance in the galvanometer circuit; and we then calculate the inductance by the formula given in § 18.

The employment of this motor-interrupter in connection with the Anderson bridge method and a small air condenser enables us to measure an inductance of even a few microhenrys, whilst it is, of course, capable of dealing with inductances of any magnitude provided they are constant.

§ 22. **Determination of Inductance by Simple Periodic Currents.**—If the means of creating a simple periodic or simple sine curve alternating E.M.F. are at hand, we can determine the inductance of a coil of constant inductance by a method due to Joubert.

First send through the coil a continuous current and observe the potential difference of the ends of the coil with an electrostatic voltmeter and measure the current flowing through it. Then repeat the experiment, using the alternating E.M.F. The ammeter should be a Kelvin balance, or dynamometer, or hot-wire ammeter suitable for both continuous and alternating currents. Adjust the voltage so that the current is the same in both cases. Then, if  $A$  is this current and if  $V$  is the volt-fall down the coil with continuous current and  $V'$  that with alternating current, and if  $R$  is the resistance and  $L$  the inductance of the coil, we have

$$A = \frac{V}{R},$$

and also

$$A = \frac{V'}{\sqrt{R^2 + p^2 L^2}},$$

where  $p = 2\pi$  times the frequency of the alternating current.

Hence,

$$V^2(R^2 + p^2 L^2) = V'^2 R^2,$$

or

$$(V'^2 - V^2)R^2 = V^2 p^2 L^2.$$

$$\therefore L = \frac{R}{p} \frac{\sqrt{(V'^2 - V^2)}}{V^2}.$$

Or we may keep the volt-drop constant and observe the currents in each case. Then we have

$$A = \frac{V}{R},$$

$$A' = \frac{V}{\sqrt{R^2 + p^2 L^2}}$$

$$\therefore AR = A' \sqrt{R^2 + p^2 L^2},$$

or,

$$L = \frac{R}{p} \frac{\sqrt{A^2 - A'^2}}{A'}.$$

A variation on the above has been described by Mr. W. Brew (see *The Electrician*, Vol. XXV., 1890, p. 206).

The coil of which the inductance is to be measured is joined in series with another inductionless circuit and some form of ammeter, such as a hot-wire ammeter, suitable for measuring alternating current, but which has itself a negligible inductance. We first take a current reading with the inductive coil cut out of circuit.

Let  $r$  be the total resistance of the hot-wire ammeter and the non-inductive coil together, and let  $A$  be the current which passes when these are put across an alternating-current circuit of constant voltage,  $V$ . Then let the inductive coil be joined in series with the above, and a second reading,  $A'$ , be taken. Let  $L$  and  $R$  be the inductance and resistance of the coil to be tested. Then we have

$$A = V/r,$$

and

$$A' = V / \sqrt{(R+r)^2 + p^2 L^2}.$$

Hence

$$A^2 r^2 = A'^2 \{(R+r)^2 + p^2 L^2\},$$

or

$$A^2 r^2 - A'^2 (R+r)^2 = A'^2 p^2 L^2.$$

$$\therefore L = \frac{\sqrt{A^2 r^2 - A'^2 (R+r)^2}}{A' p}.$$

The above formula only holds good when the alternating voltage follows a simple sine law of variation, and when the coil (LR) is of constant inductance.

If the E.M.F. curve has not a simple sine form a correction has to be applied. It has been shown by Prof. H. F. Weber (*Ann. Phys. Chem.*, 63, 1, p. 366; or *Science Abstracts*, Vol. I., p. 407) that if the E.M.F. curve is not simply periodic, and if the amplitudes of the constituent simple sine curves into which it can be resolved are denoted by  $P_1, P_3, P_5$ , &c., then the inductance  $L$  of a coil of negligible resistance can be calculated from the formula

$$L = \frac{f \cdot V}{2\pi n A},$$

where  $n$  is the frequency,  $V$  the volt-drop down the coil, and  $A$  the current through it, and

$$f = \frac{\sqrt{P_1^2 + \frac{1}{9}P_3^2 + \frac{1}{25}P_5^2 + , \&c.}}{\sqrt{P_1^2 + P_3^2 + P_5^2 + , \&c.}}.$$

For a Ganz alternator of a certain type which had a rather peaky E.M.F. curve, Prof. Weber found the value of  $f$  to be 0.932. Hence the difference in the values of the inductance calculated with and without the factor is not great. The value of the true inductance in this case only differs 7 per cent. from that calculated on the assumption that the E.M.F. follows a simple sine law of variation.

The alternating-current test is, therefore, less liable to give erroneous results when dealing with inductances than when dealing with capacities if we neglect to take account of the form of the E.M.F. curve, but merely assume it to be a sine curve.

The late Prof. H. A. Rowland devised numerous methods for measuring inductance by means of alternating currents. (See *American Journal of Science*, December, 1897, 4, p. 429 ; also *The Electrician*, Vol. XL, p. 281.)

**§ 23. Mutual Inductance.**—If two electric circuits are in contiguity to each other, then, if one of them is traversed by a unit current, the number expressing the total magnetic flux due to that circuit which passes through or is linked with the other is called the *mutual inductance* or *coefficient of mutual induction* of the two circuits. It is denoted by the letter  $M$ . If we reckon the current in amperes and the flux in webers, then the mutual inductance is expressed in henrys. Thus two circuits, A and B, are said to have a mutual inductance of one henry when the passage of 1 ampere through one circuit causes a total flux of 1 weber to be linked with the other. The simplest illustration is that of a small short straight solenoid placed in the interior of an infinitely long

straight solenoid. Let the last-mentioned coil have a uniformly-distributed winding of  $N_1$  turns of wire per unit of length of the coil, and let the windings be traversed by a current of  $A$  amperes. Then the flux density  $B$  in the interior of this solenoid, assuming the space in and around the coil to have unit permeability, is expressed by

$$B = \frac{4\pi}{10} N_1 A = 1.25 N_1 A.$$

Hence, if the solenoid has a circular section and a mean radius  $R_1$ , the total flux  $Z$  in the interior passing any section of it is given by the equation

$$Z = \frac{4\pi}{10} N_1 A \pi R_1^2.$$

Suppose we insert in the interior a small straight solenoid of finite length having a mean radius  $R_2$  and a total winding of  $N_2$  turns. Then the flux which perforates this last coil is given by the expression

$$\frac{4\pi}{10} N A \pi R_2^2 N_2,$$

and the flux per C.G.S. unit of current in the first coil which is linked with the second circuit is expressed by

$$\begin{aligned} M &= 4\pi N_1 N_2 \pi R_2^2 \\ &= 4\pi^2 R_2^2 N_1 N_2. \quad . \quad . \quad . \quad . \quad . \quad (1) \end{aligned}$$

This, therefore, gives us the value of the mutual inductance of the two coils in centimetres if  $R_2$  is measured in those units and  $N_1$  is given in turns per centimetre.

Another simple case is that of a pair of wires wound together on a circular-sectioned ring. Let a ring be made of non-magnetic and non-conducting material, and let the mean radius of the ring be large compared with the radius of cross section. Let two identical wires be wound on it, making the same number of turns  $N$ , and laid on together. Let  $R$  be the mean radius of the ring and let  $r$  be the mean radius of the circular cross section of the endless solenoid so formed. Then the flux density produced in the interior of the solenoid by the

passage of a unit C.G.S. current through the coils is equal to

$$4\pi \frac{N}{2\pi R} = \frac{2N}{R}.$$

The total linkage of flux with the second coil is equal to the product of the last expression and  $\pi r^2 N$ . Hence the mutual inductance of the two coils is given by

$$M = \frac{2\pi r^2 N^2}{R}, \quad . . . . . (2)$$

and will be expressed in centimetres if  $r$  and  $R$  are measured in centimetres, or in henrys if the result is divided by  $10^9$ . So that we have

$$M = \frac{2\pi r^2 N^2}{R} \text{ centimetres,}$$

$$\text{or} \quad = \frac{2\pi r^2 N^2}{10^9 R} \text{ henrys.}$$

If the value of  $r$  is not small compared with  $R$ , then a correction is necessary. In this case we have to obtain first a value for the *mean flux density* over the cross section of the solenoid as it is not the same at all points. It can be shown (see Appendix, Note B, Vol. I., 3rd sec., "The Alternate Current Transformer" by J. A. Fleming\*) that in this case the total magnetic flux in the interior of the solenoid per unit C.G.S. current in its coil is given by

$$Z = 4\pi N \{R - \sqrt{R^2 - r^2}\}.$$

Hence the total flux linked with the other coil is

$$M = 4\pi N^2 \{R - \sqrt{R^2 - r^2}\}. \quad . . . . . (3)$$

The above expression gives us, therefore, the mutual inductance of the two coils.

When  $r/R$  is a small quantity, it is easy to show that

$$\{R - \sqrt{R^2 - r^2}\} = r^2/2R,$$

and then the expression (3) reduces to (2) above as it should do.

We have, therefore, in (3) an expression well adapted for numerical calculation which affords a means for constructing a standard of mutual inductance of known value.

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\* Published by "The Electrician" Printing and Publishing Co.

The calculation of the mutual inductance of circular coils or of finite solenoids in various positions is not an easy matter. Only a few cases are amenable to analysis. One important case is that of two circular coils of known mean radius, depth and thickness, wound in rectangular channels cut in the edge of a ring, the coils having their planes parallel and at a known distance apart. For the difficult calculations involved in this case, the reader is referred to Maxwell's "Treatise on Electricity and Magnetism," Vol. II., sec. 696, 2nd edition; also to Mascart and Joubert's "Treatise on Electricity and Magnetism" (Atkinson's translation), Vol. II., sec. 761. Maxwell gives the complete solution for the important case of the mutual inductance of two circular circuits, each of one turn of wire, the planes of the circuits being parallel, the radius of one circle being  $r$  and that of the other being  $r+x$ , and the distance between their planes being  $y$ , the controlling condition being that the distance  $\sqrt{x^2+y^2}$  must be small compared with  $r$ . In this case he shows that

$$M = 4\pi r \log \sqrt{\frac{8r}{x^2+y^2}} \left\{ 1 + \frac{1}{2} \frac{x}{r} + \frac{2^2+3y^2}{16r^2} - \frac{x^3+3xy^2}{32r^3} +, \&c. \right\} \\ + 4\pi r \left\{ -2 - \frac{1}{2} \frac{x}{r} + \frac{3x^2-y^2}{16r^2} - \frac{x^3-6xy^2}{48r^3} +, \&c. \right\}.$$

If  $x=0$  and  $y/r$  is very small—that is, if the circular currents are equal in size and very near together—we have

$$M = 4\pi r \left\{ \log \frac{8r}{y} - 2 \right\}$$

as the expression for the mutual inductance of two equal circular circuits extremely near together,  $y$  being the distance between their planes and small compared with the radius  $r$ .

**§ 24. Standards of Mutual Inductance.**—The simplest form of fixed standard of mutual inductance is that of a ring wound over with two insulated wires. The standard may be constructed as follows:—Let a wooden ring be made having a circular cross section like a large curtain ring.

The mean diameter may be about 30cm., or 12in., and the diameter of the circular cross-section about 2cm., or  $\frac{3}{4}$ in. This ring must have its mean diameter  $R$  and diameter of cross-section measured in eight places at angular intervals of 45deg., and the mean of these values taken. It should then be bound over uniformly with one layer of insulated wire formed of two insulated wires laid on in parallel, the circuits being separate. For this purpose silk-covered wire may be used, of such a gauge that the overall diameter of the covered wire is about 1mm. The two wires must make the same number of turns  $N$  and the turns must be carefully counted. The outside diameter of the circular solenoid so formed must be then measured, so as to obtain the mean radius of cross-section  $r$  of the circular solenoid. If then  $r$  and  $R$  have such values as 1 and 15, the mutual inductance of the two circuits is very approximately given by the formula

$$M = \frac{2\pi r^2 N^2}{R}.$$

Thus, if  $R=15$ cm. and  $r=1$  and  $N=500$ , we have

$$M = \frac{6.283 \times 250,000}{15} = 104,700\text{cm.}$$

$$= \frac{1}{16}\text{th millihenry nearly.}$$

Another form of standard of mutual inductance can be constructed of two circular coils, which are placed co-axially to one another, so that their diameters coincide, but their planes may, if required, be made inclined. One of these coils (*see* Fig. 39) is fixed in a vertical position, and the other one is slightly smaller, but can revolve round a diameter with its plane situated at any required angle to the fixed coil. For this purpose it is best to make the fixed circular coil in the form of two flat coils placed a small distance apart, and to place the movable coil so that its axis lies between the two fixed coils. By means of an arm attached to the movable coil, the angular position of its plane relatively to that of the fixed coil can be ascertained by inspection. The ends of

the fixed coil and the movable coil are brought to two separate pairs of terminals. If the movable coil is small compared with the fixed coil, then the coefficient of mutual inductance is proportional to the cosine of the angle between the planes of the coils. If, however, the movable coil is very nearly the same size as the fixed coil, then the mutual inductance is more nearly proportional to the complement of the angle between the plane of the coils than to the cosine.

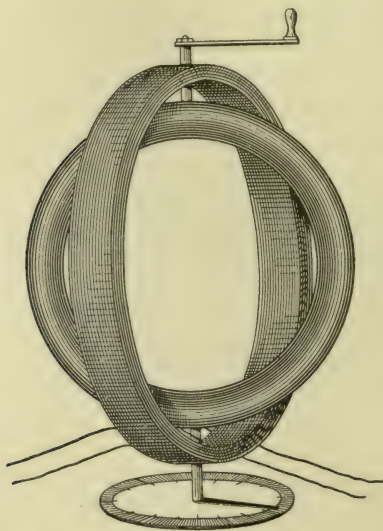


FIG. 39.—Variable Mutual Inductance.

The mutual inductance of two such coils in any relative position can be ascertained by comparing it with that of a fixed standard of mutual inductance made as already described. This comparison can be carried out as shown in the section below. Hence the scale of such a variable standard of inductance can be calibrated so as to show directly the mutual inductance of the two coils in any position in centimetres or other units.

§ 25. **The Measurement of the Coefficient of Mutual Inductance.**—There are many methods by which the coefficient of mutual inductance of two coils can be measured, but one of the most simple and effective of these is that due to Principal G. C. Foster, who described it first in 1887.\* In the following section the description of this method is taken nearly verbatim from the original Paper in which it was first described. The determination in absolute measure of the coefficient of mutual induction of two electric circuits by the employment of a calibrated ballistic galvanometer is a somewhat complicated matter necessarily occupying a good deal of time. The process may, however, be simplified if we have a variable condenser, of which the capacity is known. Let P and S be two coils whose mutual induction is required. Let

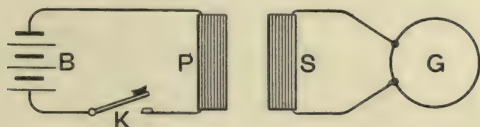


FIG. 40.

them be joined up as indicated in Fig. 40, the coil P being included in a circuit containing a battery, B, and a key, K, and the coil S being joined up in a second circuit including a galvanometer, G. Let the total resistance of the primary circuit, including the battery, be called  $R$ , and let the total resistance of the secondary circuit, including the galvanometer, be called  $r$ . Then on closing up or opening the key K a throw or deflection of the galvanometer will occur in consequence of its being traversed by a quantity of electricity,  $Q$ , which is given by the equation  $Q = M \frac{i}{r}$ , where  $M$  is the coefficient of mutual induction, or the mutual inductance between the coils P and S, and  $i$  is the strength of the current

\* See G. Carey Foster, F.R.S., "Note on a Method of Determining Coefficients of Mutual Induction," *Phil. Mag.*, Vol. XXIII., Series 5, p. 121, 1887.

in the primary circuit. In the next place, leaving the primary circuit unaltered, let the connections be made as in Fig. 41, where  $C$  is a condenser of known capacity represented by  $C$ , and  $R$  now represents a resistance,  $R$ , included between two points,  $A$  and  $D$ , on the primary circuit, these points being also closed by another circuit consisting of the galvanometer  $G$  and the condenser  $C$ . Then, on breaking and making the primary circuit by the key  $K$ , the galvanometer is traversed by a quantity of electricity,  $Q'$ , such that  $Q' = CRi$ . If the resistance between the points  $A$  and  $D$  is altered until it is found that the deflection of the galvanometer is the same in the two cases, then we have the equation  $M = CRr$ , and the mutual inductance is found in terms of the product

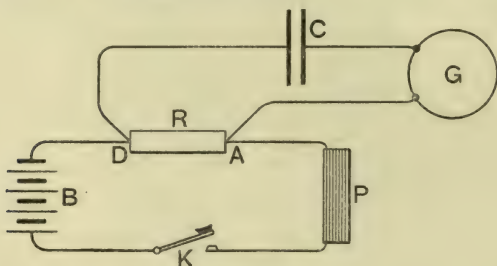


FIG. 41.

of a capacity and two resistances. This method of working has the obvious defect that it assumes that the current in the battery circuit is of exactly the same strength during each part of the experiment. If the current in one case were not the same, then the coefficient of mutual induction would be determined in the form

$$M = CRr \frac{i}{i'},$$

where  $i$  and  $i'$  are the currents in the two cases.

In order to avoid any measurements of a current, however, Principal Foster combined the two methods in a single arrangement of apparatus which makes the actual measurement a null method—that is to say, the observer simply

looks for the absence of any galvanometer deflection. The connections will be understood by reference to Fig. 42, where, as far as possible, the same reference letters are used as in the two previous diagrams.

It will be seen that the arrangement now resembles a Wheatstone's bridge, in which one arm of the bridge is replaced by a condenser,  $C$ , but in which the battery current only flows through one arm instead of two. The experimental observation made consists of adjusting the resistance  $T$ , which forms one arm of the bridge, until there is no movement of the galvanometer needle  $G$  on making or breaking the battery

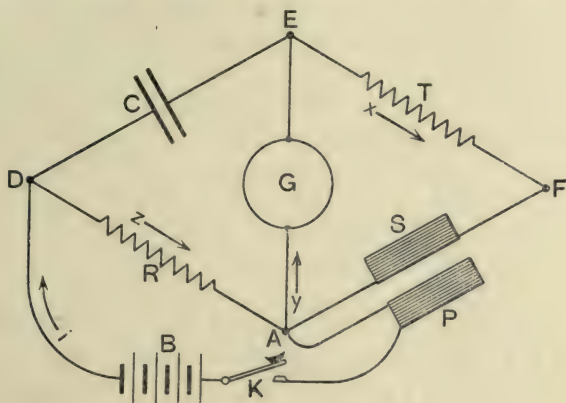


FIG. 42.

circuit by the key  $K$ . Let the resistance of the arms  $S$  and  $T$  together be represented by  $p$ , that of the galvanometer or bridge circuit by  $q$ , and that of the arm  $R$  by  $R$ . Also let the currents in those circuits at any instant be denoted by the letters  $x, y, z$ . Further, let the current in the battery and primary coil at any moment be denoted by  $i$ . The inductance of the coil  $S$  is denoted by  $L$ , and the potentials at the points  $A$  and  $E$  by  $A$  and  $E$  respectively. When the battery current has attained its steady value,  $I$ , it is evident that the currents  $x$  and  $y$  will both be zero, and that, therefore, the potential

at A will be equal to the potential at E, and also the charge in the condenser will be equal to CRI. If there has been no movement of the galvanometer needle, the average value of the current in the galvanometer circuit during the whole time of the establishment of the battery current must be zero. Consequently the total quantity of electricity conveyed by the current  $x$  must have been equal to the charge of the condenser. Accordingly we have the equation

$$\int_0^{\infty} xdt = \text{CRI}.$$

But if the average strength of the current  $y$  in the galvanometer circuit is zero, the average difference of potential between A and E must also be zero, and the effective E.M.F. in the conductor  $p$  is that due to the mutual inductance of the coils P and S only, for the integral value of the E.M.F. of self-induction must vanish. Hence we must have

$$\int_0^{\infty} xdt = \frac{M}{p} \int_0^{\infty} \frac{di}{dt} dt = \text{CRI},$$

or

$$M = CpR.$$

As an example, Principal Carey Foster gave the measurement of the mutual inductance of the primary and secondary circuits of an App's induction coil giving a 7in. spark. The resistance of the secondary circuit was 7.394 ohms at 16.5°C., and the resistance of the primary circuit was 0.278 ohms at the same temperature. The capacity of the condenser used ( $c$ ) was 4.926 mfd., and the resistance  $R$  was 0.58 ohms. The value of the product  $CpR = M$  was  $1.1896 \times 10^9$ . Hence  $M = 1.2$  henrys nearly.

The above method is applicable to a large range of measurements, provided that a condenser of suitable capacity is at hand, and it merely requires a very sensitive galvanometer capable of indicating the presence in a circuit of a very small transitory current. The method can be rendered more sensitive by interposing in the galvanometer and battery

circuit an alternated contact-breaker on the plan of the secohmmeter, and thus running together into a steady deflection the unilateral "throws" of the galvanometer as in the case of the inductance measurements.

§ 26. **The Comparison of Two Coefficients of Mutual Induction.**—A method was given by Prof. Clerk Maxwell in 1879\* for the comparison of the mutual inductance of two pairs of coils. Let these coefficients of mutual inductance be denoted by the letters  $M_1$  and  $M_2$ , and let the coils themselves be denoted by  $a, b, c, d$  (see Fig. 43). Then it is required

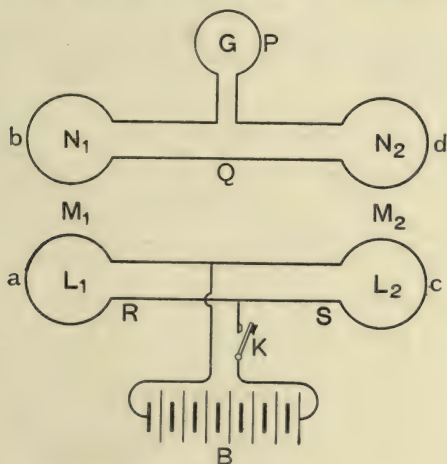


FIG. 43.

to compare the mutual inductance of the coils  $a$  and  $b$ , denoted by  $M_1$ , with that of  $c$  and  $d$ , denoted by  $M_2$ . The coils  $b$  and  $d$  are joined up in series with the galvanometer  $G$  and form one circuit, which we will call the secondary circuit. The coils  $a$  and  $c$  are joined up in parallel with a battery,  $B$ , and a key,  $K$ . The coils  $a$  and  $b$  and  $c$  and  $d$  are then placed in the position in which their relative coefficients of induction

\* This method was originally given in one of Prof. Maxwell's University lectures, and is reproduced in a Paper by the Author on "The Distribution of Currents in Networks of Conductors" (see *Phil. Mag.*, September, 1885, p. 221).

are required. Let  $R$  be the resistance of the coil  $a$  and  $S$  the resistance of the coil  $c$ , and let  $Q$  be the resistance of the circuit formed of the coils  $b$  and  $d$  and the galvanometer. Also let  $L_1, N_1, L_2, N_2$  be the inductances of the four coils  $a, b, c$  and  $d$ , and let  $P$  be that of the galvanometer. Call  $x$  the current in the coil  $a$  and  $y$  that in coil  $c$  and  $z$  that in coils  $b$  and  $d$  and the galvanometer. Then, if  $T$  is the electrokinetic energy of the system, we have

$$2T = L_1 x^2 + L_2 y^2 + (N_1 + N_2 + P)z^2 + 2M_1 xz + 2M_2 yz, \quad (1)$$

and the dissipation function  $F$  is given by

$$2F = Rx^2 + Sy^2 + Qz^2. \quad (2)$$

Then, if the E.M.F. of the battery is  $E$ , we have as the equation for the  $x$  circuit

$$E = \frac{d}{dt} \frac{dT}{dx} + \frac{1}{2} \frac{dF}{dx}, \quad (3)$$

and similar equations for the  $y$  and  $z$  circuits.

Accordingly, by performing the operation indicated by equation (3) on (1) and (2) we obtain the cycle equations

$$L_1 \dot{x} + M_1 \dot{z} + Rx = E, \quad (4)$$

$$L_2 \dot{y} + M_2 \dot{z} + Sy = -E, \quad (5)$$

$$M_1 \dot{x} - M_2 \dot{y} - (N_1 + N_2 + P)\dot{z} + Qz = 0, \quad (6)$$

where  $\dot{x}$  stands for  $\frac{dx}{dt}$  and similarly for  $\dot{y}$  and  $\dot{z}$ .\*

In the symbolisation adopted the battery current would be denoted by  $x - y$ , but if we substitute in the equations (4), (5) and (6)  $x + y$  for  $x$ , we transform them into a condition in which the battery current is represented by  $x$ . Making the change, we have

$$L_1(\dot{x} + \dot{y}) + M_1 \dot{z} + R(x + y) = E, \quad (7)$$

$$L_2 \dot{y} + M_2 \dot{z} + Sy = -E, \quad (8)$$

$$M_1(\dot{x} + \dot{y}) + M_2 \dot{y} - (N_1 + N_2 + P)\dot{z} + Qz = 0. \quad (9)$$

\* For the explanation of this method of dealing with problems in networks of conductors, the reader is referred to § 2, p. 194, of Vol. I. of this HANDBOOK.

Add together the equations (7) and (8) and put  $n$  for  $\frac{d}{dt}$  and rearrange, and we obtain

$$(L_1n + R)x + (L_1n + L_2n + R + S)y + (M_1 + M_2)nz = 0,$$

$$M_1nx + (M_1 + M_2)ny - (N_1 + N_2 + P)nz + Qz = 0.$$

Eliminating  $y$  from the two above equations we obtain an expression

$$z = \frac{\{(M_2L_1 - M_1L_2)n^2 - (M_1S - M_2R)n\}x}{\text{A denominator which does not concern us}}.$$

If, then,  $M_1$ ,  $M_2$ ,  $L_1$  and  $L_2$  are constants, the first term in the numerator is zero. Hence we can only have  $z = 0$ —that is, the absence of a deflection in the galvanometer—when the battery current varies—that is, when  $\frac{dx}{dt}$  or  $nx$  has a finite value—if we have  $M_1S - M_2R = 0$ .

Accordingly, if we adjust the resistances  $R$  and  $S$  of the circuits  $a$  and  $c$  so that the galvanometer shows no “throw” or deflection when the key  $K$  is closed or opened, then we have the proportionality

$$\frac{M_1}{M_2} = \frac{R}{S},$$

and the ratio  $M_1/M_2$  is determined.

The resistance of the coils  $a$  and  $c$  cannot be altered without changing the inductances  $L_1$  and  $L_2$ , but we can insert in series respectively with these two coils non-inductive resistance which can be varied. In the practical performance of the experiment we proceed, therefore, as follows: We connect in series with a battery and key the two primary coils of the pair to be tested, each coil having in series with it a resistance box consisting of a variable inductionless resistance. The experiment consists then in varying these resistances until on opening and closing the battery key the galvanometer shows no movement.

If  $R$  is the resistance of the coil  $a$  and  $S$  that of the coil  $c$ , and if  $R_1$  is the inductionless resistance in series with  $R$  and  $S_1$  that in series with  $S$ , then when the adjustment is exact, we have

$$\frac{M_1}{M_2} = \frac{R + R_1}{S + S_1}.$$

The galvanometer should be a sensitive mirror galvanometer. As in the other cases of inductance measurement, we can greatly increase the sensitiveness of the method by inserting the alternating interrupter or secohmmeter in the battery and galvanometer circuits, and so convert a mere impulse of the galvanometer one way or the other into a steady deflection which is annulled by a proper adjustment of the resistances.\*

**§ 27. Measurement of Small Inductances.**—The methods of inductance measurement already described are suitable for the determination of inductances as small as a few millihenrys, but when we come to deal with inductances which are best reckoned in microhenrys, they cease to be applicable, because in general the inductance of the connecting wires or of the coils of the Wheatstone bridge employed have an inductance of the same order. We can, however, evaluate these small inductances by means of an arrangement known as a Hughes inductance bridge. Prof. D. E. Hughes, in a remarkable Presidential Address to the Institution of Electrical Engineers,† showed that if a coil possessing inductance and resistance is balanced on a Wheatstone bridge having a telephone in place of a galvanometer, it is not possible to obtain silence in the telephone when the battery current is interrupted unless a pair of mutually inductive coils are inserted as shown in Fig. 43, one in the battery and one in the telephone circuit.

Let  $R$  be the inductive conductor and let its resistance be  $R$  in C.G.S. units and its inductance  $L$  in centimetres. Let it be joined up with other approximately inductionless

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\* This method of comparing mutual inductances has the melancholy interest attaching to it that it was given by Prof. Clerk Maxwell to his class at Cambridge on the last occasion on which he delivered a University lecture shortly before his lamented death.

† Prof. D. E. Hughes "On the Self-Induction of an Electric Current in Relation to the Nature and Form of its Conductor."—*Jour. Inst. Elec. Eng.*, Vol. XV., p. 6, January, 1886.

conductors, P, Q and S, to form a Wheatstone bridge. Let a telephone, T, be placed in the bridge circuit, and let a pair of mutually inductive coils,  $M_1$  and  $M_2$ , be inserted in the battery and bridge circuit as shown in Fig. 44. Then also let an interrupter, I, be placed in the battery circuit or the bridge circuit as required. First, let the interrupter be placed in the bridge circuit, then the resistances of the four bridge arms can be so adjusted that there is silence in the telephone. When this is the case, the currents in the bridge arms are steady and that in the bridge telephone circuit is zero. Next, let the interrupter be transferred to the battery circuit.

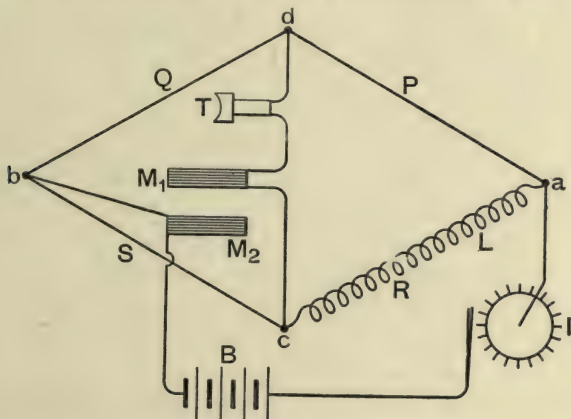


FIG. 44.

It is then found that the noise in the telephone reappears if one of the bridge arms, say R, is inductive, and it cannot be annulled except by introducing an opposing inductive action by means of the mutual induction of the coils  $M_1$ ,  $M_2$ . By suitably arranging these coils, however, we can obtain silence in the telephone, both when the interrupter is in the battery circuit and also when it is in the telephone or bridge circuit.

When this is the case, we have a relation between the resistances of the four conductors and also between the

inductance  $L$  of the arm  $R$ , and the mutual inductance  $M$  of the coils  $M_1, M_2$ , as follows:—

$$QR - SP = MLp^2,$$

also

$$L = M \frac{P + Q + R + S}{Q},$$

where  $p = 2\pi n$  and  $n$  is the frequency of the interrupter. The proof of these formulæ is not very simple, but it is given in the Author's treatise on the "Alternate Current Transformer," Vol. I., p. 286, 3rd edition.

The practical difficulty of carrying out the measurement of inductance by this method consists in securing a sufficiently high and regular interruption of the circuit, and also a small but known inductance between two coils. Prof. Hughes employed a simple toothed wheel and spring as interrupter, but Lord Rayleigh, who put the method to careful test (see *Phil. Mag.*, December, 1886), abandoned this appliance as untrustworthy, and substituted a reed blown by a regular blast of air. The tongue of the reed, which was an ordinary harmonium reed, was caused to make contact at each vibration with the rounded end of a finely cut screw, and by this means Lord Rayleigh constructed a vibrator which interrupted a circuit 1,050 times a second.

It is, however, possible to make an interrupter which will break a circuit 1,000 times a second by means of a small fan motor. On the shaft of a fan motor, which can make 1,200 revolutions a minute, or 20 per second, is placed a disc of *stabilit*. This material is a hard insulating material like *vulcanised fibre*, but less liable to warp. This disc can be about 9in. in diameter and  $\frac{1}{2}$ in. thick. In the circumferenec of this disc and quite near the edge are then drilled 50 holes  $\frac{1}{4}$ in. in diameter, and brass plugs are driven tightly in. The disc is then chucked in the lathe and the edge turned off so as to leave exposed the plugs, and if the turning off does not extend quite up to the pitch circle on which the plug holes are set, the parts of the plugs left will be keyed into the holes. In

this manner we secure a wheel with 50 metal segments let into its edge. Two light springs are then set to touch the edge of the disc, and when this last is in revolution these springs are connected and disconnected electrically 50 times per revolution, or 1,000 times per second.

The arrangements of the bridge circuits employed by Prof. Hughes were as follows:—The arms P, Q and S were segments of one and the same German silver wire 1 metre long, and having a total resistance of 4 ohms. The conductor R, of which the inductance is desired, is represented by the thick line in Fig. 45.

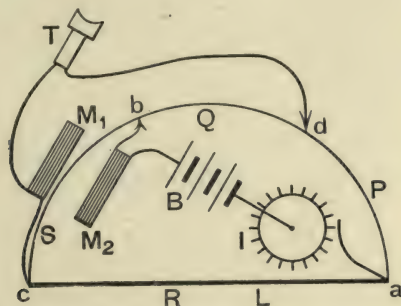


FIG. 45.

The variable mutual inductance was obtained by Lord Rayleigh by the use of a pair of flat circular coils, one of which could revolve round its diameter, this diameter being coincident with a diameter of the other coil. When the coils are nearly of the same size, the mutual induction is nearly proportional to the angle between the planes of the coils, at least within the limits of 40deg. and 140deg. between their planes. Lord Rayleigh determined by calculation the coefficient of mutual induction of two coils used for the graduation of the above variable inductance. It is possible to employ two circular coils having their planes parallel and to move them to and from each other in this position, and so vary their mutual inductance. We can determine experimentally the mutual

inductance at different distances by means of Principal G. C. Foster's method, as already described in a previous section. Having the value of the mutual inductance  $M$  we can obtain that of  $L$  in terms of it. Since, however,  $M$  is multiplied by a factor which must always be greater than unity, it is clear we can never measure an inductance which is numerically less than the known mutual inductance at our disposal. If the arms of the bridge  $P$ ,  $Q$ ,  $R$  and  $S$  are about equal in value, then the factor  $P+Q+R+S/Q$  is nearly equal to 4, and accordingly the method is adopted for the measurement of an inductance about four times greater than the mutual inductance used.

**TABLE I.**  
**Dielectric Constants.**  
**SOLIDS.**

Substance.	Dielectric Constant. Air = 1.	Authority.
Glass :		
Double extra dense flint.		
Density = 4.5 .....	9.896	J. Hopkinson
Light flint. Density = 3.2	6.72	Do.
Very light flint. Density		
= 2.87 .....	6.61	Do.
Hard crown. Density =		
2.485 .....	6.96	Do.
Sulphur .....	2.24	Faraday
	2.88	Wüllner
	3.84	Boltzmann
	4.00	Curie
	2.94	Blondlot
Ebonite .. .....	2.05	Rossetti
	3.15	Boltzmann
	2.21	Schiller
	2.86	Elsas
India rubber :		
Pure, brown .....	2.12	Schiller
Vulcanised, grey .....	2.69	Do.
Gutta-percha .....	2.462	Gordon
Paraffin .....	1.977	Gibson and Barclay
	2.32	Boltzmann
	2.29	Hopkinson
	1.99	Gordon
Shellac .....	2.95	Wüllner
	2.74	Gordon
	3.04	Winklemann
Mica .....	6.64	Klemencie
	8.00	Curie
	7.98	Bouty
	5.97	Elsas
Quartz :		
Along optic axis .....	4.55	Curie
Perpendicular to axis ...	4.49	Do.
Ice at - 23° .....	78.0	Bouty

It is particularly to be noted that the values of the dielectric constant depend upon the time of charging. In the case of the values given in the above Table, they are for the most part steady or low frequency values, but the investigators have not always stated the circumstances of the charge as regards time and temperature. For rapidly alternating charge the dielectric constants of glass and ice, for instance, are very much smaller than as given above.

**TABLE II.**  
**Dielectric Constants.**  
**LIQUIDS.**

Substance.	Dielectric. Constant. Air = 1.	Authority.
Water at 17°C. ....	80·88	Heerwagen
„ 25°C. ....	75·7	Rosa
„ 25°·8C. ....	78·87	Franke
Olive oil.....	3·16	Hopkinson
Castor oil .....	4·78	Do.
Turpentine .....	2·15	Silow
	2·23	Hopkinson
Petroleum .....	2·072	Silow
	2·07	Hopkinson
Ethylic alcohol at 25°C. .	25·7	Rosa
Ethylic ether .....	4·87	Doule
	4·8	Bouty
Acetic acid .....	9·7	Franke

The dielectric constant of a substance varies with the time of charge as well as with the temperature. Great variations therefore exist between the determinations of different observers.

The values given above are the low frequency or steady values of the dielectric constants.

**TABLE III.**

Dielectric Constants of some Liquids at 15°C., and of the same substance when frozen at -185°C.

(Fleming and Dewar.)

Substance.	Dielectric Constant.	
	At 15°C.	At -185°C.
Water .....	80	2.4 to 2.9
Formic acid .....	62	2.41
Glycerine .....	56	3.2
Methyl alcohol .....	34	3.13
Mononitrobenzol .....	32	2.6
Ethyl alcohol .....	25	3.1
Acetone .....	21.85	2.62
Ethyl nitrate.....	17.7	2.73
Amyl alcohol.....	16	2.14
Aniline .....	7.5	2.92
Castor oil .....	4.78	2.19
Ethylic ether.....	4.25	2.31

The low temperature values given in the third column were taken at a frequency of about 100 ~.



## CHAPTER III.

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### PHOTOMETRY.

§ 1. **Photometry.**—Photometry is the art and science of comparing sources of light in respect either of their luminous intensity or power of producing illumination or other qualities. It is therefore essentially dependent upon the properties of the human eye. It involves the possession of: (1) A system of photometric units and methods of measurement. (2) Standards of light and illumination. (3) Photometers, or instrumental means for effecting the necessary comparisons.

§ 2. **Photometric Units.**—There are at least six measurable quantities with which we are concerned in photometry which have to be distinguished and named. These are:—

- (1) *Luminous intensity*, commonly called *candle-power*.
- (2) *Luminous flux*, often, but erroneously, called *quantity of light*.
- (3) *Illumination*.
- (4) *Intrinsic brilliancy* or *brightness*.
- (5) *Quantity of light* or *time-flux*.
- (6) *Time-illumination*.\*

(1) *Luminous Intensity.*—Most countries have adopted and legalised some source of light as a standard of luminous intensity. This is taken as the unit, or standard light-producing agency, in terms of which all others are measured and described. The progress of photometric science has rendered

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\* See "Photometric Units and Magnitudes" (A. Blondel), *The Electrician*, September, 1894, Vol. XXXIII., p. 633.

many of these legal standards antiquated, and, although they may still have legislative authority behind them, they are yet not those in which the greatest confidence is now placed. Thus, for instance, the standard has been, and in some cases is, a *candle* of some kind, and, accordingly, luminous intensity is commonly stated in *candle-power*. In other cases, the standard is a special form of lamp, such as the Carcel or Hefner lamp, in which case luminous intensity is defined as *carcels* or *hefners*. These different standards are considered below from a practical point of view; meanwhile it may be said that there have been at various times a number of standards in actual use. Thus we have the *British candle*, the *German candle*, the *French candle* (*bougie decimale*), the *Carcel lamp*, the *Hefner lamp*, the *Pentane lamp*, all being names for lamps or candles of special kinds adopted by different countries.

Owing to this confusion, it has been proposed that the name for the unit of luminous intensity shall be one which has no reference to actual and national standards. One name which has been suggested for this is the *pyr*, and it has been proposed that luminous intensity should be measured in *pyrs* or *megapys* (=a million pyrs) according to its magnitude.\*

It has also been proposed that the unit of luminous intensity shall be one equivalent to that which is now called 10 candle-power, and that the name for this unit shall be the *lamp*.† The term "candle" or "candle-power" in photometry has, however, come to mean, not the light given by any particular cylinder of wax containing a wick, but an ideal and constant unit of luminous intensity representing a certain theoretical standard only imperfectly realised by the actual illuminant.

(2) *Luminous Flux*.—The next photometric quantity with which we are concerned is luminous flux, which is defined as

\* See A. Blondel, "Rapport sur les Unités photométriques," Congrès international des Electriciens, Genève, 1896.

† See J. A. Fleming, "On the Photometry of Electric Lamps," *Jour. Inst. Elec. Eng.*, Vol. XXXII., p. 164, 1903.

the flux or emanation of light from a source. It is commonly—but erroneously—called the quantity of light emitted. The practical unit is the flux which takes place through a unit solid angle emitted from a point source having a luminous intensity of one unit, say, one candle-power. Hence, since the solid angle surrounding a point is  $4\pi$ , the luminous flux proceeding from a light-source of intensity  $n$  units is  $4\pi n$  units of flux. The name which has been accepted for the unit of flux is the lumen; hence, the total flux from a source of light having a unit luminous intensity, say, 1 candle-power, is 12·6 lumens.

The value commonly called the “mean spherical candle-power” is equal to the total flux of light from a source divided by  $4\pi$ .

(3) *Illumination*.—The illumination at any point of a surface on which light falls is defined as the ratio of the luminous flux which falls on an element of the surface to the area of that element. This quality is sometimes called the brightness or luminosity of the surface. In England it is usually measured in *candle-feet*—i.e., the unit of illumination is that produced by one standard candle on a perfectly white surface at a uniform distance of one foot. At other times and places similar units, such as the *bougie-metre* or the *carcel-metre*, have been employed. In a system of units now adopted illumination is measured in units called a *lux*. This name was first proposed in 1889 at the International Congress of Electricians, in Paris, by Sir W. H. Preece.\* Hence, a *lux* may be defined as the illumination produced on a white surface of 1 square metre when a flux equal to one lumen uniformly distributed falls normally upon it. It is therefore also the illumination produced on a white surface at a uniform distance of one metre from a source of luminous intensity of one unit.

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\* Sir W. H. Preece, in 1883, proposed as a unit of illumination that given by one British standard candle placed at a distance of 12·7in. from a white surface. This is practically the same as the *carcel-metre*. (See *Proc. Roy Soc.*, Vol. XXXVI., p. 270.)

(4) *Intrinsic Brilliancy and Intrinsic Radiation.*—The intrinsic brilliancy of an illuminant is the quotient of the normal intensity of the source measured in candle-power or any other units by the surface of the source in square centimetres. The intrinsic radiation is the ratio of the flux of light from a source, measured in lumens, to the emitting surface. If we have a plain luminous surface, such as a white-hot sheet of metal, and if the intensity of the light emitted in any direction is numerically equal to the intensity along the normal, multiplied by the cosine of the angle between the selected direction and the normal, then the radiation is said to follow Lambert's law of emission.

The proof of Lambert's law is generally made to rest on the observation that if a solid incandescent sheet is held behind a hole in another plate, the intrinsic brilliancy is independent of the angular position of the hot plate behind it. Let  $I_n$  be the luminous intensity of the radiation in the direction of the normal  $n$ , and  $I_r$  that in any inclined direction  $r$ ; and let  $\theta$  be the angle between  $n$  and  $r$ . It is evident that if  $S$  is the area of the hole in the plate, and if the incandescent plane surface is inclined at an angle  $\theta$  to this plate, then the area of the radiating surface which radiates through the hole is equal to  $S/\cos \theta$ . Accordingly, if the intrinsic brilliancy of the surface seen is independent of the angle  $\theta$ , we must have the following relation satisfied :

$$I_r/S = I_n \div \frac{S}{\cos \theta},$$

or  $I_r = I_n \cos \theta$ , which is Lambert's law. Hence, if this law holds good, an incandescent sphere should appear to the eye as if it were a circular plane incandescent disc.

On the subject of Lambert's law much discussion has taken place. That the cosine law is not completely true for the brightness of an illuminated surface was shown by Bouguer. He found that when the surface was viewed at a very oblique angle the brightness was less than that given by Lambert's law. The reader may refer to a Paper by C. Wiener (see *Wied. Ann.* Vol. XLVII., p. 638, or *The Electrician* Vol. XXXIII., p. 549) for a discussion on "The Diffusion of Light by Dull Surfaces."

Wiener made experiments with surfaces of gypsum, and found that up to angles of 60deg. the brightness of an inclined

surface illuminated by parallel rays of light is usually somewhat smaller than that demanded by Lambert's law. At high angles of incidence the brightness is reduced to about 0.6 of that required by the above law. Some other valuable remarks will be found in a Paper by Prof. E. Lommel "On the Photometry of Diffused Reflection" (see *Wied. Ann.* Vol. XXXVI., p. 473, 1889; or *The Electrician*, Vol. XXXIII., p. 691, 1894).

Lommel shows that Lambert's law is approximately true for opaque glowing bodies such as incandescent plates, but not for self-luminous bodies transmitting light such as flames. Seeliger showed that it is not true for the surfaces of celestial bodies such as the sun or the planets.

Lommel deduces from first principles the following propositions:—

If an absolutely white body is illuminated by parallel rays at any inclination, the amount of light sent out in different directions from the surface—viz., its illuminating power—is proportional to the cosine of the angle of incidence. If an absolutely white body is equally illuminated from all directions, the quantity of light radiated by it in any given direction is proportional to the cosine of the angle of emergence.

These propositions enunciated for absolutely opaque white bodies also hold for coloured bodies whose absorption may be taken as feeble. Of the actual substances investigated by Lommel, only white marble was found approximately to follow the cosine law. Hence Lambert's law cannot be accepted as generally true, though for angles of emergence lying between 0deg. and 60deg. for dead-white surfaces, it is not very untrue. The same is true for the illumination produced on a surface held in a parallel beam of light. In the case of dead white surfaces such as plaster of paris, up to an angle of incidence of about 50deg., the cosine law holds good, but it is not even approximately true for glazed surfaces such as paper.

The term *intrinsic radiation* may be applied to the ratio of the flux of light emitted by a source to the whole surface of the source, and is reckoned in lumens per square centimetre.

There is in general no simple relation between intrinsic brilliancy and intrinsic radiation. If, however, the radiation follows Lambert's law, then the intrinsic brilliancy of  $n$  units per square centimetre corresponds to an intrinsic radiation of  $\pi n$  lumens per square centimetre. The proof is as follows :—

Consider a small area  $dS$  of the incandescent surface (see Fig. 1). Describe round it a hemisphere, and mark out by two closely adjacent lines of latitude a zonal element. Then all along this zone the element  $dS$  has the same apparent area.

Let  $r$  be the radius of the hemisphere, and let  $\alpha$  be the angle between the normal to the surface  $dS$ , and the direction of the ray considered. Let  $I_n$  be the luminous intensity of the element  $dS$  in the direction of the normal.

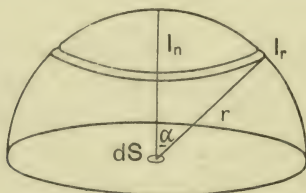


FIG. 1.

Then  $I_r = I_n \cos \alpha$  is the luminous intensity in the direction  $r$ . The area of the zone above mentioned is equal to  $2\pi r^2 \sin \alpha d\alpha$ , and the solid angle subtended by the zone is therefore  $2\pi \sin \alpha d\alpha$ . Hence the luminous flux on the zonal element is

$$I_n \cos \alpha 2\pi \sin \alpha d\alpha ;$$

and, if  $i_n$  is the normal intrinsic brilliancy,

$$i_n = I_n / dS.$$

Hence the luminous flux due to  $dS$  is given by

$$\begin{aligned} d\phi &= 2\pi dS \int_0^{\pi/2} i_n \cos \alpha \sin \alpha d\alpha \\ &= i_n \pi dS, \end{aligned}$$

or,

$$d\phi/dS = \pi i_n.$$

But  $d\phi/dS$  is the intrinsic radiation and  $i_n$  is the intrinsic brilliancy, and the former is therefore  $\pi$  times the latter.

(5) *Quantity of Light*.—The true meaning of the term quantity of light is that it is the name for the product

of the flux of light and its duration, and is therefore reckoned in *lumen-seconds* or *lumen-hours*, just as we measure quantity of electricity in ampere-seconds or ampere-hours. Since the flux is measured by  $4\pi$  times the intensity, the quantity of light emitted by a source is numerically equal to the product of the time, the intensity and  $4\pi$ .

(6) *Time-Illumination* is the product of the illumination of the surface and the time of exposure, and is therefore measured in *lux-seconds*. This quantity is important in photography, as the chemical action on a sensitive plate is proportional to time-illumination. We can, then, summarise these photometric quantities in a tabular form, thus :

Let  $s$  denote the surface of a radiating source of light, and  $S$  that of a receptive surface. Then we take the following symbols for the quantities named against them :

Physical Quantities.	Symbols.	Practical Units.
Luminous intensity..	$I$	The candle or other unit.
Luminous flux .....	$\Phi$	The lumen.
Illumination .....	$e$	The lux.
Intrinsic brilliancy..	$i$	Candles per square centimetre or similar units.
Intrinsic radiation...	$\phi$	Lumens per square centimetre.
Quantity of light ...	$Q$	The lumen-second.
Time-illumination...	$j$	The lux-second.

We shall give, later on, some examples of the employment of these units in photometric calculations.

**§ 3. Practical Standards of Light: Flame Standards.**—Standards of light are divided into two great classes :—

(1) Flame standards, comprising candle, lamp or gas flames.

(2) Incandescent standards.

The first class comprises all those practical standards which consist of a candle of some kind, or a lamp burning some liquid combustible by means of a wick, or a gas jet burning

some gaseous combustible. All these produce a flame dependent for existence on the surrounding atmosphere, and are therefore influenced greatly by its nature. Flame standards should comply with the following conditions :—

(i.) The combustible should have a constant and definite chemical composition, and be easily obtained in a state of known purity.

(ii.) It should be burnt under controlled and simple conditions easily defined.

(iii.) The unavoidable variations in the atmosphere should not create a sensible difference in the luminous intensity.

The second class comprises those standards of illumination which consist of an incandescent surface of metal or carbon, such as molten platinum or the carbon filament of a glow lamp. The conditions they should comply with are

(i.) The surface must be unalterable in nature or extent by repeated heating ;

(ii.) The conditions of use must be such that the material can be placed at a high and constant temperature easily controlled.

None of the standards of illumination actually used comply precisely with all these conditions. In Great Britain there are at present two legalised standards of light :

(1) The Parliamentary or Standard Sperm Candle ;

(2) The Harcourt Pentane 10-candle Lamp.

The last-mentioned standard of luminous intensity has been legalised in Great Britain, at any rate within the limits of the metropolis, by the Metropolitan Gas Referees, as the standard of comparison for London gas. Other flame standards, burning either liquid or gaseous fuel, which have been exhaustively investigated are as follows :—

(a) The Colza Oil or Carcel Standard, which remains the official standard for gas testing in France, and still preserves the form given to it by Dumas and Regnault.

(b) The various Pentane Lamps of Mr. A. G. Vernon Harcourt, well known and much used in Great Britain, one of which is now the official standard for London gas testing.

(c) The Amyl Acetate Lamp of Herr von Hefner-Alteneck, introduced in 1884, and extensively employed in Germany, where it is the legal standard.

Additional flame standards which have been suggested and more or less used are :

(d) The Argand Coal Gas Flame with Methven slit, the coal gas being sometimes enriched with pentane.

(e) The Benzene and Ether Flame, recommended by the Dutch Photometric Commission in 1893.

(f) The Acetylene Flame Standard of Charpentier.

(g) The Acetylene and Hydrogen Flame, two parts acetylene and one part hydrogen, burnt in pure oxygen, recommended by the American Institute of Electrical Engineers.

(h) The Ethylene Flame, consisting of pure ethylene burning in pure oxygen, suggested by M. A. Blondel.

(i) The Albo-carbon Lamp, burning naphthalene, proposed by M. Broca.

In spite of the fact that the colza oil lamp has maintained its position in France as the official standard of light for the greater part of the century, the uncertain composition of this combustible has prevented its adoption in other countries. The three flame standards which at present hold the field are :

(1) The 1-candle Pentane Reference Standard, introduced by Mr. A. G. Vernon Harcourt in 1877.

(2) The more recent 10-candle Pentane Lamp, by the same inventor, now adopted as the official working standard by the Gas Referees, brought out in 1898.

(3) The Amyl Acetate Lamp, introduced by Herr von Hefner-Alteneck, in 1884.

*Standard Candles.*—The standard sperm candle, unfortunately still the legal standard of light in Great Britain, is defined in the Metropolis Gas Act of 1860, and in the Gas Works Clauses Amendment Act, 1871, schedule A, parts I. and II., as follows :—

The candle is to be made of spermaceti tempered with beeswax, weighing six to the pound, and to consume 120 grains of spermaceti per hour; two candles being used together for the test, consuming 40 grains in 10 minutes.

The Metropolitan Gas Referees, who are a body of experts appointed to regulate the testing of London gas, in their regulations for the use of the sperm candle as a unit of light, directed that the variation in combustion of the standard candles should not fall outside the limits 114-126 grains per hour, or 38-42 grains for the two candles, in 10 minutes; corresponding with a variation in time between  $9\frac{1}{2}$  and  $10\frac{1}{2}$  minutes for the combustion of 40 grains for the two candles. The Gas Referees issued at one time an elaborate specification for the manufacture of the standard candle, defining the nature of the wick and the wax. This specification will be found in the "Gas Analyst's Manual," by Mr. Jacques Abady (Spon & Co., p. 484, Appendix J.). It is not necessary to reproduce it here, as the sperm candle is falling into disuse as a practical standard.\*

Many authorities on the subject of photometry have prescribed in detail the precautions to be taken in the use of the sperm candle as a standard of light. The fact that a more or less elaborate ritual has to be performed with them to secure anything like a uniformity in light-production is in itself proof that they comply very imperfectly with the conditions for a good flame standard. All experience goes to show that, if not used with attention to these details, variations in light to the extent of 15 to 20 per cent. may easily occur, and even with all precautions the light of the standard candles differs from one another to the extent of

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\* See also Dibdin's "Practical Photometry," pp. 87-96.

5 to 10 per cent. The material burnt is of indefinite composition. Some writers say that the spermaceti now obtained is not of the same composition as that used when the candle was first defined. The light is greatly influenced by the wick, the position in which it curls over, and its length when burning. The candle is, moreover, too small a unit of light for the purposes of electric photometry. If the metre is taken as the unit of length, then the illumination produced by a candle at a distance of 1 metre is also too small a unit of illumination.

The candle, however, has survived as a standard of light by reason of its portability and ease of use, and, above all, because it has had legislative authority at its back. Until lately it was the only legal unit of light, and hence all other sources were necessarily defined in terms of it. It has been extensively investigated and universally condemned. A long report was made to the American Institute of Electrical Engineers in 1896 on standards of light.\* The authors review the criticism on the British standard candle. Messrs. Harcourt, Keats and Methven were appointed a committee by the Board of Trade to report on the candle as a standard of light, and they found differences amounting to 15 per cent. in the average illumination produced, whilst two pairs showed a difference of as much as 22·7 per cent. Messrs. Heisch and Hartley in 1883, acting as a Committee on the Standard of Light for the Gas Institute, found differences varying from 1 to 16 per cent, the average being 7 per cent. Mr. Dibdin, acting for the Metropolitan Board of Works, found differences varying from 14·9 to 23·2 per cent., and variations of 5 to 8 per cent in the illuminating power of standard candles were quite common. The Dutch Photometric Commission found similar variations. The candle-power of the sperm candle varies with the position of the curl-over of the wick. Methven found that two candles

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\* See *Trans. Amer. Inst. of Elec. Eng.*, Vol. XIII., 1896. "On Standards of Light," by Prof. E. L. Nichol and Messrs. C. H. Sharp and C. P. Matthews.

which gave 2 candle-power with wicks at right angles to the photometer bar, gave  $3\frac{1}{2}$  per cent. less light when the wicks were pointing away from the photometer and 2 per cent. less light when pointed towards it. These numerous observations confirm the opinion that the British standard sperm candle is a very variable source of luminous intensity, and therefore photometric measurements made with this standard must be accepted with great caution.

The same remark applies to the candle standards of other countries. In Germany the official unit of light used to be a paraffin candle with 2 per cent. of stearine added, 10 candles weighing half a kilogramme. This candle was adopted on the recommendation of the German Association of Gas and Water Engineers, and was therefore called the *Vereinskerze*, or "Association Candle." An elaborate specification was issued by the Association, and the candle was only made and sold by them. The photometric measurement was to be made when the flame was 50mm. high, the rate of consumption of the paraffin being disregarded. In spite of these careful prescriptions, the use of this candle has been abandoned in Germany in favour of the Hefner lamp. The American Institute Committee found it was little better than the British candle.

A similar candle was at one time in use in France, called the *bougie de l'étoile*.

With such a record against it, the standard candle, whether British, German or French, can only be regarded as a very approximate unit of illuminating power, being, as one writer has observed, about on a par with the "barleycorn," three of which were said to go to the inch, as a unit of length. Hence the terms "candle" and "candle-power" have come to apply rather to an ideal constant unit of luminous intensity than to the light given out by any actual candle. There is, therefore, the same kind of relation between the term "candle" as a unit of luminous intensity and the light of any actual sperm candle as that between the theoretical

“ohm” or 10 C.G.S. units of resistance, and the actual mercury columns or wire coils used to represent it. Only in the case of the candle the real thing represents the ideal thing much more imperfectly.

If, by reason of the absence of any better standard, it becomes necessary to employ the sperm candle as a standard of light, the following precautions must be taken :—A sound candle is cut into two parts, and these, after having had the wick exposed, are held in a candle balance (*see* Fig. 2), by which the weight lost in burning is determined. The candles are then lighted, and, after the flames are well formed, they are allowed to burn for 10 minutes. They must be used in a place free from draughts, but with good ventilation. The balance has a 40-grain weight, and after balancing the lighted candles with shot, the 40-grain weight is put in the scale pan and the time noted in which the candles lose this weight. It should be 600 seconds. If it does not exceed 630, or fall below 570, then the light is assumed during that period to be  $\frac{1,200}{x}$  candles, where  $x$  is the time in seconds taken for the two candles to lose 40 grains in weight.

*Pentane Lamps.*—These standards owe their existence to Mr. A. G. Vernon Harcourt, who has evolved them during the last 30 years, their name being derived from the use of pentane as the standard fuel. Pentane is a very volatile and inflammable liquid, having the chemical composition  $C_5H_{12}$ , and is the distillate yielded by light American petroleum after three distillations respectively at 55°C, 50°C and 45°C, and subsequent treatment with strong sulphuric acid and caustic soda. The vapour of pentane is 2.5 times heavier than atmospheric air, and is as inflammable as ether. The specification for its preparation and testing is given in the London Gas Referee's Notification for 1901, as follows :—

*Preparation.*—Light American petroleum, such as is known as gasoline and used for making air-gas, is to be further rectified by three distillations, at 55°C., 50°C. and 45°C., in succession. The distillate at 45°C. is to be shaken up from time to time, during two periods of not less than three hours each,

with one-tenth its bulk of (1) strong sulphuric acid, (2) solution of caustic soda. After this treatment it is to be again distilled, and that portion is to

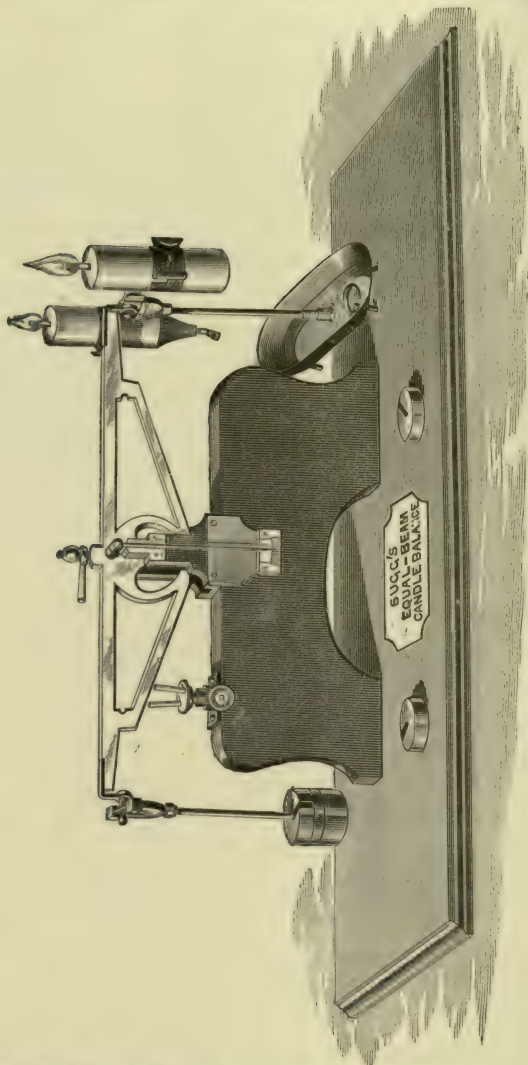


FIG. 2.—Sugg's Equi-beam Candle Balance.

be collected for use which comes over between the temperatures of  $25^{\circ}\text{C}$ . and  $40^{\circ}\text{C}$ . It will consist chiefly of pentane, together with small quantities

of lower and higher homologues, whose presence does not affect the light of the lamp.

*Testing.*—The density of the liquid pentane at 15°C. should not be less than 0·6235, nor more than 0·626, as compared with that of water of maximum density. The density of the pentane when gaseous, as compared with that of hydrogen at the same temperature and under the same pressure, may be taken. This is done most readily and exactly by Gay Lussac's method, under a pressure of about half an atmosphere and at temperatures between 25°C. and 35°C. The density of gaseous pentane should lie between 36 and 38.

Any admixture with pentane of hydrocarbons belonging to other groups, and having a higher photogenic value, such as benzine or amylene, must be avoided. Their presence may be detected by the following test: Bring into a stoppered 4oz. bottle of white glass 10c.c. of nitric acid, specific gravity 1·32 (made by diluting pure nitric acid with half its bulk of water); add 1c.c. of a dilute solution of potassium permanganate containing 0·1 gramme of permanganate in 200c.c. Pour into the bottle 50c.c. of the sample of pentane, and shake strongly during five successive periods of 20 seconds. If no hydrocarbons other than paraffins are present, the pink colour, though somewhat paler, will still be distinct; if there is an admixture of as much as  $\frac{1}{2}$  per cent. of amylene or benzine, the colour will have disappeared.

It is important to notice that these precautions as to testing cannot be dispensed with. Merely to write to a wholesale chemist for pentane, or something called pentane, and then use it in a Harcourt lamp, will not result in the reproduction of the standard of light. The pentane used in gas testing is prepared in bulk by the gas companies, and is then tested by the Referees and supplied in sealed cans to the gas-testing stations, which are under the control of Dr. F. Clowes, the chemical adviser of the London County Council.\*

Mr. Vernon Harcourt has devised at various times five forms of lamp for burning pentane, three of them being 1 c.p. standards and two of them 10 c.p. standards. The most important at the present time is the 1 c.p. standard, which was introduced by him to the British Association at Plymouth in 1877.† The burner of this lamp consists of a

\* Messrs. Wright & Co., of Precision Works, Page-street, Westminster, London, who supply the latest form of 10-candle pentane lamp, have undertaken to put on sale standardised pentane complying with the above specification.

† See *Proc. Brit. Assoc.*, Plymouth, 1877, p. 51; 1883, p. 426; 1885, and Bristol, 1898, p. 845.

brass tube, 4in. long and 1in. in diameter, having a brass plug  $\frac{1}{2}$ in. thick at the top with a hole bored in it  $\frac{1}{4}$ in. in diameter. Round the burner is placed a glass chimney 6in. high and 2in. in diameter, the top of which is level with the top of the burner. Air enters through holes in the gallery on which the chimney stands, and rises up round the flame. A piece of platinum wire 0.6mm. in diameter is supported by a bracket 63.5mm. above the top of the burner. The combustible used with this burner is a mixture of pentane vapour and air in the proportion of 3 cubic in. of pentane to 1 cubic ft. of air. This mixture is made in a gasholder in the proportion of 9 cubic in. of pentane and 3 cubic ft. of air, and, after standing, should have a volume, at a barometric pressure of 30in. and a temperature of 62°F., of 4 cubic ft., or, more exactly, between 4.02 cubic ft. and 4.1 cubic ft. This mixture is burned in the above jet at the rate of  $\frac{1}{2}$  cubic ft. per hour, or at a rate not exceeding the limits of 0.48 cubic ft. and 0.52 cubic ft. per hour. The air-gas passes through a small metre and governor on the way to the jet. The height of the flame is regulated by a delicate stopcock to be 2.5in. high, or just to touch the platinum wire. This adjustment needs care, and in doing it the observer's eye should be screened from the general mass of the flame and see only the tip. When these operations are performed, we have a yellow-white flame produced, which yields a light equal to the mean British standard candle, but is much more constant. It need hardly be said that this pentane lamp has to be used in a suitable position, with good ventilation but free from draughts, and there are certain corrections to be applied for variations in the atmospheric pressure and moisture and carbonic dioxide present in the air.

It will not be necessary here to describe in great detail all the operations of reproducing a standard of light with this pentane lamp. As a working standard the 10-candle lamp described below is more convenient, and those who employ

at any time the 1-candle reference standard will do well to consult Mr. Harcourt's original descriptions of it. After careful investigation, its use was recommended by a committee of the Board of Trade in 1881, and by the Standards of Light Committee of the British Association in 1888. This last Committee reported that the pentane standard fulfilled all the conditions required in a standard of light. They found that the light was not altered by slight variations in the specific gravity of the pentane, varying between 0.628 and 0.632. Out of 117 tests only one showed a variation of 1 per cent., and there were no larger variations. It has been demonstrated, therefore, that this standard affords a means of reproducing, with an accuracy of 1 per cent., a light which represents fairly the ideal mean British standard candle. The necessity for employing the gas-holder, meter, governor and other checking appliances, renders this lamp more suitable for a primary reference standard than a working standard. These last objections, however, have been removed in the latest form of Harcourt pentane lamp, which is the one mentioned as now adopted by the Gas Referees. A full description of this last lamp is given in the Notification of the Gas Referees for 1901, Appendix A, and also in a Paper by Dr. F. Clowes, superintending gas examiner of the London County Council, in the *Journal* of the Society of Chemical Industry, March 25, 1902, No. 5, Vol. XXI.

This 10-candle lamp (*see* Fig. 3) has a reservoir which contains pentane placed at the top of a hollow pillar. The reservoir has two openings closed by stopcocks, one to admit air, and the other as an exit for pentane vapour. The pentane vapour descends through an indiarubber tube by its own weight, being siphoned off from the space above the liquid pentane in the reservoir. It is led down into an Argand burner at the base of the pillar. Over this burner is a double metallic chimney. The air supplied to the centre of the burner is drawn up between two concentric chimney tubes and led down the pillar to the burner as shown in the

diagram. Hence, the arrangement forms a sort of regenerative burner. The chimney comes down to within a distance of 47mm. above the steatite ring burner, the proper gap being determined by a boxwood gauge. The chimney cuts off the top of the flame and there is a mica window in the

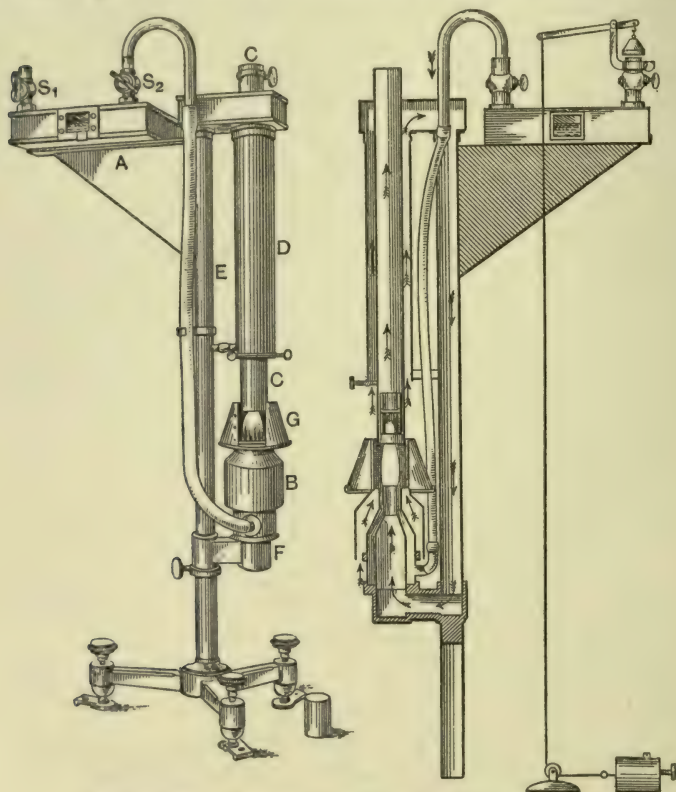


FIG. 3.—Vernon Harcourt 10-candle Pentane Lamp.

chimney, through which to observe the height of the tip of the flame. The flame is, moreover, surrounded by a conical metallic shield with an opening in it. This lamp is managed with great ease. All that is necessary is to put into the reservoir a pint of pentane, and then to open both stopcocks,

and after a few moments to light the jet of vapour at the burner, and regulate flow of air and vapour by the stopcocks until the tip of the flame is seen in the middle of the mica window. When so adjusted, the lamp gives a light ten times that of the 1-candle pentane standard, and is taken as the official standard of 10-candle light by the Gas Referees. It is necessary to adjust the height of the flame somewhat exactly, and to wait for the lamp to settle down to an uniform temperature before beginning observations.

The following is the official description of the lamp, given by the Gas Referees in the Notification for November, 1901:—

Mr. Harcourt's 10-candle pentane lamp is one in which air is saturated with pentane vapour, the air-gas so formed descending by its gravity to a steatite ring burner. The flame is drawn into a definite form and the top of it is hidden from view by a long brass chimney above the steatite burner. The chimney is surrounded by a larger brass tube, in which the air is warmed by the chimney, and so tends to rise. This makes a current which, descending through another tube, supplies air to the centre of the steatite ring. No glass chimney is required, and no exterior means have to be employed to drive the pentane vapour through the burner.

Fig. 3 (*see* page 246) shows the general appearance of the lamp. The saturator A is, at starting, about two-thirds filled with pentane. It should be replenished from time to time so that the height of liquid, as seen against the windows, may not fall below  $\frac{1}{2}$  in. The saturator A is connected with the burner B by means of a piece of wide indiarubber tube. The rate of flow of the gas can be regulated by the stopcock S, or by checking the ingress of air at S. For this latter purpose a metal cone, acting as a damper, is suspended by its apex from one end of a lever, to the other end of which is attached a thread for moving the cone up or down. The lever is supported by an upright arm clamped to the upper end of the stopcock immediately beneath the cone. From the top of the lamp the thread descends to a small pulley on the table, and thence passes horizontally to the end of a screw moving in a small block, by turning which the gas examiner can regulate the lamp without leaving his seat. It is best to so turn the stopcock S as to allow the flame to be definitely too high, but not to turn it full on, before letting down the regulating cone to its working position. Both stopcocks should be turned off when the lamp is not alight.

The chimney CC should be turned so that no light passing through the mica window near its base can fall upon the photoped. The lower end of this tube should, when the lamp is cold, be set 47mm. above the steatite ring burner. A cylindrical boxwood gauge, 47mm. in length, and 32mm. in diameter, is provided with the lamp to facilitate this adjustment. The exterior tube D communicates with the interior of the ring burner by means of the connecting box above the tube E and the bracket F, on which the

burner B is supported. A conical shade G is provided. This should be placed so that the whole surface of the flame beneath the tube C may be seen at the photoped through the opening.

The lamp should be adjusted by its levelling screws so that the tube E, as tested with a plumb-line, is vertical, and so that the upper surface of the steatite burner is 353mm. from the table. A gauge is provided to facilitate this latter measurement. The tube C is brought centrally over the burner by means of the three adjusting screws at the base of the tube D. This adjustment is facilitated by means of the boxwood gauge

When the lamp is in use the stopcocks are to be regulated so that the tip of the flame is about half-way between the bottom of the mica window and the cross-bar. A variation of  $\frac{1}{4}$  in. either way has no material influence upon the light of the flame. The saturator A should be placed upon the bracket as far from the central column as the stop at the end will allow. If it is found after the lamp has been lighted for a quarter of an hour, that the tendency of the flame is to become lower, the saturator may be placed a little nearer to the central column.

To prevent a gradual accumulation of dust in either the burner or the air-passage, a small cover of the size of the top of B, and shaped like the lid of a pill-box, should be kept upon the lamp when not in use.

This latest pattern of self-contained pentane lamp is altogether superior as a standard to the 1 c.p. pentane lamp with a wick, which was brought out some years ago to meet the requirements of a working standard.

For comparison with glow lamps, a 10 c.p. standard is a more convenient unit than 1 c.p., and, moreover, the earlier form of 1 c.p. lamp with a wick was more trouble to start in action and had other defects which are absent in the 10 c.p. standard.

An important point in connection with this 10-candle standard is that it requires and has no glass chimney. Other pentane lamps have been produced in which a glass chimney is employed, but this feature always introduces an element of uncertainty into the working of any standard of light. Moreover, the light from the top of the flame is cut off by the metal chimney, and this probably contributes to prevent the light emitted being influenced by normal variations of atmospheric pressure so much as is the case with open flame lamps. The light is, however, affected by the presence of water, vapour and carbonic dioxide in the air, as in the case of all other flame standards.

Several other forms of lamp burning pentane vapour have been introduced. Fig. 4 shows the form of one such lamp designed by Mr. Simmance.

*The Hefner Lamp.*—The third flame standard, which has come into very general use and is especially popular in Germany—no doubt on account of its German origin—is the so-called Hefner lamp, which was introduced by Herr von Hefner-Alteneck in 1884.\* This well-known lamp consists

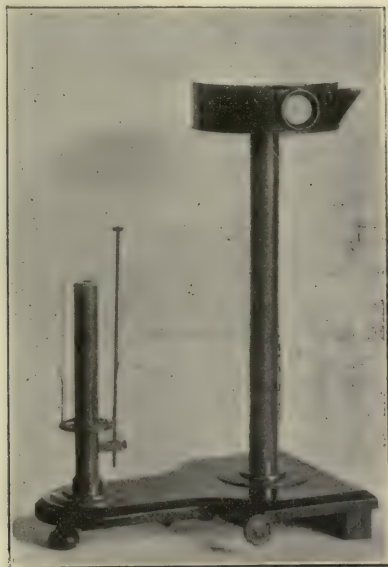
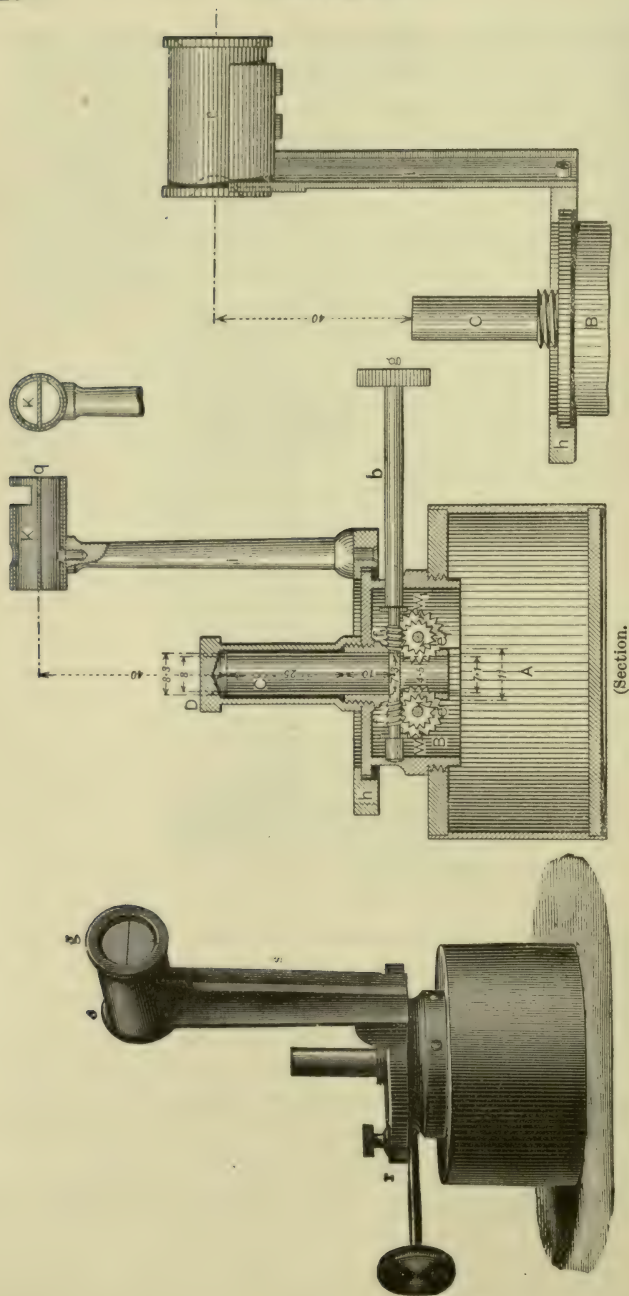


FIG. 4.—Simmance Pentane Lamp.

of a small metal body containing the combustible, and from out of it a metal tube made of German silver, containing the wick, rises (*see* Fig. 5). The tube is 8mm. inside diameter, and 8.3mm. outside diameter, and 25mm. high. The wick is formed of strands of cotton yarn. Separate threads to the number of 15 or 20 are laid together straight, not twisted, until the size of wick is sufficient to fill up the tube without squeezing. The exact number of strands is not of great

\* *Elektrotechnische Zeitschrift* Vol. III., p. 445, and Vol V., p. 20.



Perspective View of Hefner Lamp, showing  
Fiducial Mark on Sighting Screen *g*.

Fig. 5.  
Hefner Amyl Acetate Lamp,  
Reichsanstalt Pattern.

Sighting Arrangements for Adjusting  
Height of Flame.

consequence, and only affects the height of the flame. By means of a simple rack mechanism the wick is moved up and down, so as to alter the flame height, and by means of a small camera or *finder* placed on a pillar attached to the body the flame can be adjusted to be exactly of the standard height of 40mm. To do this the flame is turned up or down until the tip of the image of the flame as seen focussed by a lens on a ground glass screen just touches a line drawn on the screen.

The material burned in the lamp is amyl acetate,  $C_7H_{14}O_2$ . The quantity of the combustible in the lamp does not matter as long as all the lower ends of the wick are well immersed. The wick should be trimmed square at the top of the tube, and after filling the lamp it should be allowed to burn ten minutes before adjusting the flame and making the measurement. It was claimed by the inventor that the absolute purity of the amyl acetate is not of very great importance; but this has been lately denied, others asserting that it is essential to use chemically pure amyl acetate. The lamp is used without a chimney, and as the flame is very lambent or mobile, it must be carefully protected from draughts. The luminous intensity of this flame is less than that of a British standard candle. Measures of the ratio, however, made by different observers do not agree very well. The following table is taken from a preliminary report of a sub-committee of the American Institute of Electrical Engineers, on "Standards of Light," issued in 1896.\*

Table I.

Observer.		Ratio of the Hefner Unit to the British Candle.
Sharp .....	From comparison with standard candles corrected for rate of burning	0.872
Sharp .....	From observations on standard candles corrected for flame height	0.892
Sharp and Turnbull	From observations with the bolometer	0.941
Violle .....	.....	0.98
Reichsanstalt Investi- gations	Mean Value .....	0.876
Netherland Photo- metry Commission	.....	0.921
S. Schiele .....	Mean Value .....	0.881

\* See *Trans. Am. Inst. of Elec. Eng.*, Vol. XIII., 1896.

The variation in the value of the ratio is partly due to the uncertain value of the parliamentary candle, and to personal errors, and also to the different methods of comparison adopted, which affect the ratio in consequence of the difference in the quality of the two lights compared. The most probable value appears to be 0.88.\* Hence we may reasonably assume that luminous intensities expressed in Hefner units have to be multiplied by 0.88 to reduce them to their equivalent in British standard candles.

The chief objection that has always been raised in countries other than Germany to the use of the Hefner lamp as a standard is the reddish character of the light. In this respect it compares very unfavourably with the Harcourt 10-candle pentane lamp, the light of which is comparable in quality with that of a glow lamp working at about 3 watts per candle-power. In other words, the pentane flame is at a temperature nearer to that of the glow-lamp filament when in use. The employment of the Hefner lamp as a means of standardising glow lamps when used at the ordinary efficiencies gives rise to the difficulties of heterochromatic photometry, to which allusion will be made presently. Its use in arc-lamp photometry is out of the question.

**§ 4. Influence of the Surrounding Atmosphere on Flame Standards.**—The presence of water and vapour and carbonic dioxide in the air greatly affects all flame standards. This has been very carefully investigated by Liebenenthal.† He studied the effect of water vapour and carbon dioxide in the atmosphere on the luminous intensity of the Hefner lamp. If  $w$  represents the volume of water vapour in litres per

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\* This is the value taken in a specification for the supply of glow lamps, issued by the General Post Office. See a Paper by Sir W. H. Preece, F.R.S., on "Electric Glow Lamp Tests," *Proc. Brit. Assoc.*, Liverpool, 1896; or *The Electrician*, Vol. XXXVII., p. 738, 1896.

† *Phys. Soc. Abstracts*, Vol. I., Abs. 501; or *Elektrotechnische Zeitschrift*, 1895, Vol. XVI., p. 655; also *Zeitschrift für Instrumk.*, Vol. XV., p. 157, 1895.

cubic metre of dry air, then the light ( $L$ ) of the Hefner lamp is expressed by the following linear function :—

$$L = 1.049(1 - 0.0053w).$$

The formula holds good between 3 and 18 litres of water vapour per cubic metre. The light of the Hefner lamp decreases, therefore, about 0.5 per cent. per litre of water vapour per cubic metre of dry air, and has a value equal to unity when 8.8 litres of water vapour per cubic metre of dry air are present in the atmosphere, according to the regulations of the Berlin Reichsanstalt. Taking the average variations of moisture in the air, we find that this implies a variation of about 4 per cent. in the light between the wet and dry seasons of the year.

Again, if  $c$  represents the quantity of carbon dioxide present in the atmosphere in litres per cubic metre, then the luminous intensity  $L$  is expressed by the following formula :—

$$L = 1.012(1 - 0.0071c).$$

Also slight variations in the height of the flame have a great influence on the luminous intensity. If  $h$  is the height of the flame in millimetres, then the luminous intensity  $L$  is expressed by the following linear functions :—

$$L = (1 + 0.025(h - 40)),$$

$$L = (1 - 0.030(40 - h)),$$

according as  $h$  is above or below 40mm. A change of 1mm. in the height of the flame creates, therefore, a 3 per cent. change in the light.

Finally, variations in atmospheric pressure affect the light given by the lamp. Between 735mm. and 775mm. barometric pressure, the light variation may be expressed by the following formula :—

$$\Delta L = 0.00011(H - 760),$$

where  $\Delta L$  is the change in the value of the light corresponding to a barometric height of  $H$  mm. This represents a variation of 0.1 per cent. for 10mm.

Unless all these corrections are applied, the luminous intensity of the Hefner lamp is uncertain within limits greater than those which can easily be determined photometrically.

The effect of carbon dioxide in the atmosphere or the luminosity is important. A change of 1 litre per cubic metre of air—that is to say, a variation of 1 part in 1,000—affects the intensity of the light 0·7 per cent. The effect of variations in the hygrometric state of the air and of barometric pressure on the pentane flame, have been investigated by Liebhenthal and by Mr. Harcourt.\* The latter states, with reference to the 1-candle pentane standard, that the height of the cone of flame varies inversely as the barometric pressure, and he gives the following rule for the correction of standard height of flame. The standard height of flame for which the emitted light is equal to 1 candle is 63·5mm. at 30in. barometric pressure, and for every tenth of an inch above or below 30in. the flame must be set an equal number of fifths of a millimetre below or above 63·5mm. Hence, when the barometer stands at 30·5in., the height of flame to give one candle is 62·5mm.

Liebhenthal† examined the effect of water vapour on the Harcourt 1-candle lamp with wick, and found that its luminous intensity in terms of the Hefner unit (*see* below) was expressed by the formula

$$L = 1,232 (1 - 0\cdot0055w),$$

where  $w$  is the number of litres of water-vapour in each cubic metre of dry air. The formula holds good between 4 and 18 litres. Also he investigated the effect of atmospheric pressure, and states that the change in the illuminating power of the pentane lamp is expressed by the rule

$$\Delta L = 0\cdot00049(H - 760),$$

where  $\Delta L$  is the variation of light corresponding to a

\* See *Proc. Brit. Assoc.*, Aberdeen, 1885.

† *Elektrotechnische Zeitschrift*, Vol. III. p. 445, and Vol. V. p. 20.

barometric height of  $H$  mm. Thus an increase of 40mm. in pressure results in a variation of the light of 2 per cent.

These experiments were made with a form of portable 1-candle pentane lamp which was brought out some time ago, and sometimes called the Woodhouse and Rawson pattern, from the names of a firm who sold it. Hence it is quite clear that in badly-ventilated rooms, or rooms where many people are gathered together, carbon dioxide will be present to an extent which materially influences the light of the lamp.\*

§ 5. **Incandescence Standards.**—The only practical standards of this description which have been evolved are those in which either platinum or carbon, heated to a high temperature, is employed. M. J. Violle proposed in 1881 to define the unit of light as the light radiated normally from 1 square centimetre of platinum at its melting point. This platinum standard was adopted as an international standard at the Paris Congress of Electricians in 1884. The International Congress of Electricians in 1889 adopted the proposal that the practical unit of light should be  $\frac{1}{10}$ th part of the Violle platinum unit. This subdivision was called the *bougie decimale*, since the platinum unit had been shown to be equal to nearly 2 carcels, and the tenth part of the carcel is taken as the bougie. Objections have been raised to the platinum unit on several grounds. In the first place, a very large mass of expensive metal is necessary, and the practical difficulties in carrying out the photometric comparison with secondary standards were found to be considerable. An attempt was made at the Reichsanstalt in Berlin to reproduce the Violle standard, but apparently with no very great success, and the British Association Committee on the Standards of Light, in their report presented in 1888, stated

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\* For a series of curves, showing the variation of the Hefner lamp, with barometric pressure and moisture, see the *Electrical Review*, Vol. XLII., p. 759, 1898.

that they consider that this standard was not a practical standard of light, although they were prepared to accept it as the definition of a unit. The meaning of this decision is not very clear. Since that date, however, a long research was carried on at the Davy-Faraday Laboratory in 1899, by Mr. J. E. Petavel, who made the Violle unit the subject of a careful investigation.\* The first operation is to melt a large mass of pure platinum by means of the oxyhydrogen blow-pipe in a lime crucible. Mr. Petavel came to the conclusion that the essential conditions of success for the reproduction of a constant platinum light standard are :—(1) The platinum must be chemically pure; (2) the mass of it should not be less than 500 grammes; (3) the crucible must be made of pure lime; (4) the hydrogen burned must contain no hydrocarbons; (5) the gases should be burned in the ratio of 4 volumes of hydrogen to 3 of oxygen.

The process of recovering the standard consists in melting this mass of platinum under the above conditions. A water-cooled diaphragm screen is then placed over the molten metal, having in it an aperture 1 square centimetre in area. The light from the molten platinum is reflected to a photometer by a mirror, and the metal is then allowed to solidify. The temperature of the metal falls to the freezing point, and then remains practically constant until the solidification is completed. During this time of constant temperature the light emitted from the selected area is also practically constant. Full details of the operations are given in Mr. Petavel's Paper.

His inference from the whole of his work is that when carried out with the stated precautions, the probable variation in the light emitted by molten platinum, under the standard conditions, is not above 1 per cent., and he considers that with more perfect apparatus and with certain improvements, the accuracy of this standard would be increased.

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\* See *Proc. Roy. Soc.*, Vol. LXV., p. 469, J. E. Petavel, "An Experimental Research on some Standards of Light."

The diagram Fig. 6 shows one of the arrangements for reproducing the platinum standard of light. A mass of platinum, of not less than 16 oz. avoirdupois, is melted in a lime crucible by an oxyhydrogen blow-pipe, and as soon as the metal is melted the crucible is moved underneath a water-cooled diaphragm, having in it an aperture 1 square centimetre in area. The light coming through this aperture is reflected on to a photometer by means of a mirror, by which

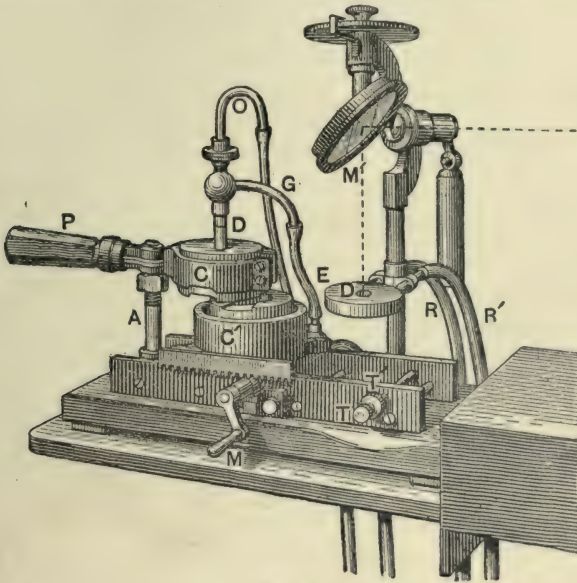


FIG. 6.

it can be compared with that of any secondary standard. Apart from the practical difficulties of its employment, the platinum standard realises in a very perfect form the conditions which are necessary for an incandescent standard of light, viz., pure material maintained at an absolutely constant temperature under definite conditions, the luminous radiation from a unit area of its surface constituting the unit of luminous intensity. It is, however, a standard of such a nature that it is not

likely to be set up anywhere except at a National Physical Laboratory.

It is not necessary to recapitulate here all the details of the various attempts that have been made by Siemens and others to replace the Violle platinum standard by one which can be more easily employed as a working standard. The most promising of these seemed to be at one time the method suggested by Lummer and Kurlbaum,\* which was as follows:—

The standard luminous intensity was to be the light emitted from 1 square centimetre of solid platinum when brought by an electric current to such a temperature that 10 per cent. of its radiation, as measured by a bolometer, could pass through a layer of water 2 centimetres in thickness, contained in a cell with quartz sides. For the details of this experiment the original Paper may be consulted. The apparatus was established in the Reichsanstalt at Berlin, and is used at present as a standard of reference for Hefner lamps. †

The only attempt to repeat this work in England has been (so the Author believes) made by Mr. J. E. Petavel, who set up the apparatus in the Davy-Faraday Laboratory. (See *Proc. Roy. Soc.*, Vol. LXV., p. 478.) His conclusion, however, was that the adjustments were very difficult, and, in addition, the spectral quality of the light was not satisfactory as a standard, being much less white than that of the Violle platinum standard.

Hence, he gives it as his opinion that the Lummer-Kurlbaum standard, in spite of the preference shown for it in Berlin, does not possess the qualities required in a primary standard of light.

*The Carbon Filament Electric Lamp Standard.*—The difficulties with flame standards which arise from variations in atmospheric pressure and moisture, and from the contamination of the air in badly-ventilated rooms by carbon dioxide,

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\* Lummer and Kurlbaum. *Elektrotechnische Zeitschrift*, Vol. XX., 1898.

† "The Reichsanstalt Unit of Light," by Lummer and Kurlbaum. *The Electrician*, Vol. XXXIV., pp. 37 and 77.

rendered it desirable to endeavour to devise a simple working standard which should be independent of atmospheric composition. Hence, many years ago, the Author's attention was called to the question of the use of a glow lamp as a standard of luminous intensity.

The first objection that of course arises is that the decay in light-giving power of a carbon-filament lamp, even when worked at a moderate efficiency, would render it perfectly valueless as a standard. A carbon-filament lamp changes in light-giving power, when used at constant voltage, for three reasons—viz., (1) From changes in electric resistance of the filament; (2) from changes in the nature of the surface of the filament; (3) from the deposit of carbon upon the interior of the bulb.

It is well known that the candle-power of new glow lamps of the majority of types increases for a short time after they have been put into use. This is due to a decrease in the resistance of the filament. The filament becomes more consolidated and probably denser, and therefore decreases in resistance. If, however, a good filament is run in a lamp, it reaches soon a condition in which a small further use will not much alter it. By that time, however, the glass bulb is somewhat blackened and the lamp will have lost candle-power. Experiments made by the Author several years ago showed, however, that if a filament which has been so employed is removed from the old bulb and put into a new clean bulb, the candle-power will again be brought back to a value near to its original value.\* It is possible, however, to prevent the blackening of a glow lamp by the following means:—The blackening is caused by the projection of carbon from the filament, and hence, other things being equal, proceeds most rapidly in small bulbs, because the carbon molecules then easily reach the glass. Suppose, however, that a filament is

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\* If the filament is a "treated" filament, then there will be a loss of candle-power by change in the surface quality, and this part will not be recovered by the re-bulbing.

mounted in a very large bulb the radius of which is much greater than the mean free path of the molecules at the pressure of the residual air. The chances of a molecule getting on to the glass are much reduced. Accordingly, about the year 1895, the Author requested Mr. E. Gimingham, superintendent of the Edison and Swan Electric Light Factory, to mount some 16 c.p. filaments in very large bulbs, and it was found that these lamps could be run for long periods without any of the usual blackening or loss of light. This observation led, therefore, to the following method of constructing a special form of carbon-filament lamp, to be used as a standard of light :—

Carbon filaments of the old Edison horseshoe shape were well selected and carefully treated, and were then mounted in ordinary bulbs and run as lamps for 50 hours at 5 per cent. above their normal voltage. Those filaments which then showed no defects were cut out of these slightly blackened bulbs and mounted in very large clear glass bulbs 6in. or 8in. in diameter (*see* Fig. 7). These lamps, if not very much used and not worked above a certain marked voltage, will remain practically constant for any length of time.

In 1896 a good many experiments were made on this plan by the Author at the Edison and Swan factory. A number of these large bulb lamps were prepared and carefully photometered against a pentane air gas standard, as giving the best means of fixing at that time a standard equal to 10 British standard candles. The working voltage of the lamps was carefully measured by means of a potentiometer and a Clark cell, and the candle-power, voltage and current recorded on each lamp, the candle-power being that taken in a horizontal direction and perpendicular to the plane of the filament when the axis of the flame was vertical.

A certain number of these lamps were set on one side and called primary standards, with the intention that they should be used only very occasionally for verifying the candle-power of others of the lamps, which were called

working or secondary standards. These secondary standards were to be employed to set the working lamps in the various photometer rooms.

Experience extending over six years showed that these large-bulb incandescence lamps can be employed with great convenience as secondary standards of light, and if carefully used and never worked above their marked volts or used for more than a few minutes at a time, they will enable a standard of light to be preserved with great constancy.

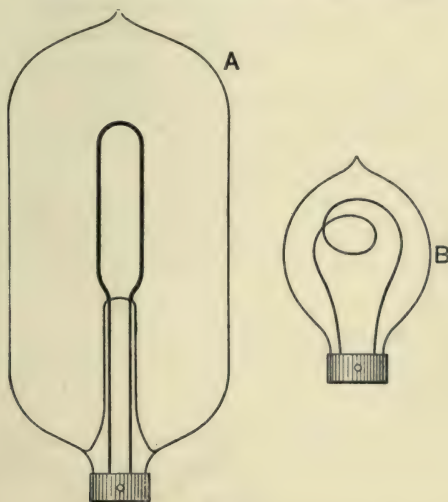


FIG. 7.

- A. Fleming-Ediswan Standard Photometric Glow-lamp.  
B. Ordinary Ediswan 16 c.p. Glow-lamp.

These large bulb lamps are now manufactured for sale by the Edison and Swan United Electric Light Company, and a certificate of their photometric value can be furnished if desired.

*Electric Arc Standard of Light.*—It was proposed by Mr. Swinburne, Prof. S. P. Thompson,\* M. Blondel†, and others

\* "Arc Lamps and their Mechanisms." By S. P. Thompson. *Journal of the Society of Arts*, London, March 6, 1889.

† "Sur un étalon secondaire pour la photométrie des Lampes à Arc." By M. A. Blondel, 1893.

that the light from 1 square millimetre of the crater of the arc lamp should be taken as a standard of luminous intensity. Measurements of the intrinsic brilliancy of the crater by different observers do not agree very well, and it cannot be said that the experimental work done so far holds out a promise that this source of light will fulfil all the requirements of a standard.

Measurements of the intrinsic brightness of the arc crater made by Mr. Trotter, M. Blondel (in 1893) and Mr. Petavel (in 1900) gave values for this constant respectively of 170, 158 and 147 candles per square millimetre. The actual incandescent area which forms the crater is, however, very small, and, according to a discovery made by Mr. Trotter, it very often exhibits a rapid rotatory movement, so that it is not so simple, physically speaking, as a surface of molten platinum. The intrinsic brilliancy of the crater is so great, and the quality of its light is so different from that of most secondary standards, that it is a matter of greater difficulty to compare this arc standard with a secondary standard than is the case with the Violle platinum standard. Hence, for all these reasons, an incandescent platinum standard will probably be preferred.

The conclusion, therefore, which may be drawn from the preceding facts is that there are five sources of light which can, in all probability, be regarded as sufficiently constant to enable them to be used as primary or secondary standards of luminous intensity with a degree of accuracy approximating to 1 per cent. or less. Two of these may be called primary or reference standards, and three working standards. The primary standards are:—

1. The Violle platinum incandescent standard.
2. The Vernon Harcourt pentane 1-candle flame standard.

And as practical working standards:—

3. The Hefner or amyl-acetate lamp.
4. The Vernon Harcourt self-contained 10-candle pentane lamp.
5. The Fleming large-bulb incandescence electric lamps.

The first two reference standards are most appropriate for use at a national standardising laboratory or a Government testing laboratory, and can be relied upon to preserve a selected standard or unit of light with an accuracy which is comparable with that of photometric measurements generally.

Of the three working standards, the amyl-acetate lamp is decidedly inferior to the other two in the quality of its light, and difficulties arise in using it, even to standardise glow lamps, whilst it is quite unsuitable for use with arc lamps. Moreover, experience shows that a 1-candle standard is not so generally useful as a 10-candle.

One objection which has been raised to the employment of the Violle standard as a primary standard is that it involves the use of a mirror to reflect the vertical ray from the molten platinum to the photometer being used, to compare it with a secondary flame standard, and hence there is possibility of error produced by the slight uncertainty attaching to the coefficient of the reflection of the mirror. The defect could be obviated by employing a photometer placed vertically over the molten platinum to compare the emitted light from it with a large-bulb glow-lamp standard made on the Author's plan, and this again could be compared with any required flame standard the ray from which must necessarily be horizontal.

One great advantage which the platinum unit possesses is that it is not only a unit of luminous intensity, but is also a unit of brightness or intrinsic brilliancy as well. According to Violle's measurements, the light radiated normally from 1 square centimetre of platinum at its melting point is equal to 18·5 British standard candles; hence its *brightness* in candles per square centimetre is represented by 18·5. The area of a Hefner flame is, according to Prof. L. Weber, 2·27 square centimetres, and the Violle unit is equal to 20·7, or nearly 21, Hefner units. Hence the relative brightness of the molten platinum and the Hefner flame is as

$\frac{20.7}{1}$  to  $\frac{1}{2.27}$ , or 47 : 1. On the same scale (Hefner's per square centimetre) the intrinsic brilliancy of the electric arc crater is 20,000.

§ 6. **Photometric Processes.**—The comparison of one source of light with another in respect of luminous intensity is effected by means of appliances called photometers. A photometer of a special kind may also be used to compare together the illumination on two surfaces, and a similar name has applied to any instrument for comparing together selected rays from two lights ; hence, photometers may be classified into

- A. *Intensity Photometers*, by means of which a comparison is made between the integral luminous intensity of two sources of light.
- B. *Spectro-Photometers*, in which rays selected of the same wave-length from the spectra of lights are compared together in respect of luminous intensity.
- C. *Illumination Photometers*, by means of which we measure the illumination in any locality in candle-feet, or some similar units.

The principle on which every intensity photometer is based is that of comparing together the illumination produced on two identically white surfaces by the two sources of light to be compared. Every simple ray of light has two qualities, which objectively are called its *wave-length* and *amplitude*, and subjectively (*i.e.*, considering the effect they produce on the eye) are called its *colour* and its *brightness*. A white surface is a surface which reflects any ray of light unaltered in colour. What we call a coloured surface is one which produces a selective effect upon a compound ray of light consisting of different kinds of simple rays, some of these being absorbed by the surface and some reflected, or all of them being partly absorbed and partly reflected. Disregarding the colour effect, and considering a ray of light

falling upon a purely white surface, it produces upon that surface a certain *illumination* or *brightness*. If two contiguous white surfaces are illuminated respectively by rays of light from two sources, these surfaces will, in general, appear differently coloured and differently bright. The brightness of the surface varies inversely as the square of the distance of the illuminant from that surface. This fact is sometimes stated to be a consequence of the rectilinear propagation of rays of light, but, in truth, it is deduced from the experimental observation that a white surface illuminated by one candle at a distance of 1ft. appears equally bright if illuminated by four candles at a distance of 2ft., or nine candles at a distance of 3ft. If, therefore, we can equalise the brightness upon two adjacent white surfaces illuminated by two sources of light, and if we can move these sources of light so as to alter their relative distances from the illuminated surface and make this appear equally bright, then the luminous intensities of these two lights are directly as the squares of their respective distances from the illuminated surface.

In general, there is a difference of colour as well as a difference of brightness, and the eye is perplexed in the endeavour to separate out the two effects. Any two sources of light are said to be *isochromatic* or *heterochromatic*, these terms being defined as follows:—If we form the spectra of the two lights, and equalise these spectra for one common ray or colour in regard to brightness, then, if the spectra are identically bright at all other corresponding points, the two lights are called isochromatic. If, on the other hand, the spectra, though equal in brightness at one point are unequal at other corresponding points, the two lights are called heterochromatic. Thus, for instance, if we equalise the spectra of an arc lamp and a candle in the yellow, then the candle is much brighter in the red and the arc light in the violet, when corresponding colours are compared together. It will be seen, therefore, that the eye has two powers—a power of colour

discrimination and a power of brightness discrimination, corresponding to the two principal qualities of a ray of light.

The difficulties of photometry arise chiefly from the difficulty of distinguishing between a difference in the colour and a difference in the brightness of two adjacent illuminated surfaces. This, however, can be done by practice, and is rendered easier by certain methods described below. Generally speaking, a photometric measurement for the comparison of luminous intensity consists in comparing together the brightness of two white surfaces, one illuminated solely by the light under test and the other by a standard light, and adjusting the distances of the sources until an equality in brightness or illumination of the surface is secured, without regard to any difference in colour.

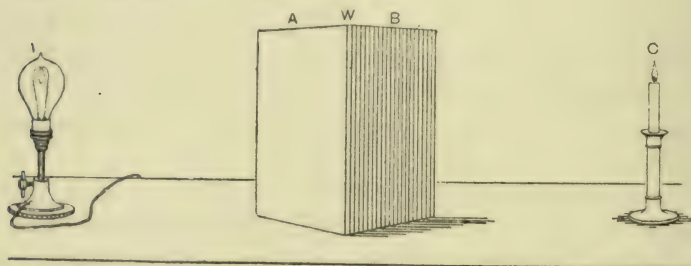


FIG. 8.

The typical and simple form of photometer is, therefore, the *Ritchie Wedge*, in which two adjacent sides of a white prism inclined at equal angles to the incident rays serve as the two surfaces, which are differently illuminated. The wedge is looked at in a direction which is perpendicular to the line joining the two sources of light, and is moved to and fro between them until the two inclined surfaces appear equally bright (*see* Fig. 8). To eliminate the effects of regular reflection the adjacent sides of this prism should be inclined at an angle between 60deg. and 70deg. The most suitable material for the wedge is plaster of paris.

It is a condition of sensitiveness that the edge of the wedge should be very sharp. If the eye has to travel far in going from one surface to the other, or to pass over a region not wholly illuminated by one light or the other, the difficulty in making a correct judgment as to the equality in their brightness is greatly increased. The eye is unable to make a numerical comparison between the degree of brightness of two surfaces, but with practice it is possible to estimate with an accuracy of one or two per cent. whether the two surfaces are equally bright or not, provided that the two lights are fairly isochromatic.

One of the difficulties connected with photometry is to procure a surface to be illuminated on separate parts by the two lights to be compared, which is, technically speaking, perfectly white—that is to say, which reflects all rays which fall upon it. There is a great difference between surfaces all of which are commonly called white surfaces. Thus, white cloth, white paper, white porcelain and newly-fallen snow all present very different appearances when examined together by the light of the sun. It is very difficult to obtain a white surface which is free from any tinge of yellow. Many so-called white surfaces, such as paper, are really a very faint buff colour. One of the best surfaces to employ for photometric purposes is a slab of plaster of paris, pressed magnesia, or magnesium carbonate, or else one of barium sulphate. Magnesium carbonate in certain physical states forms an impalpable powder, which can be easily compressed on to a metal plate by hydraulic pressure by means of a steel surface plate, and when this is done the magnesia surface so formed is a very dead white, with little or no regular reflecting power, even at very high incidences. In default of this, a suitable surface can be obtained by means of a slab of white chalk which is cut and rubbed smooth with glass paper. Or the best white drawing-paper with a hard-pressed surface can be painted over with several coats of chinese white, and this may then be employed as a photometric screen.

A great number of appliances have been devised for creating the two contiguous surfaces illuminated respectively by the two lights, and for achieving the equalisation in brightness from which we deduce the ratio of the luminous intensity of the sources compared. A broad classification of intensity photometers may be made as follows:—

(a) *Divided Screen Photometers*.—This class includes all those arrangements in which two portions of a semi-transparent or opaque screen, formed of paper, porcelain, ground glass or cardboard, is arranged so that one part is illuminated by one light and the other by another. One of the simplest devices for doing this is the arrangement of Bouguer, in which a screen of semi-transparent paper has an opaque partition placed at right angles to it in the middle. The lights to be compared are then placed on the two sides of this opaque partition, and are moved to and fro until the illumination on the two parts of the semi-transparent screen is equal. The lights must be so arranged that the direction of the opaque diaphragm bisects the angle made by joining the two sources of light to the point of intersection of the opaque diaphragm and the semi-transparent screen.

A modification of this photometer was made by Foucault, and in a modern form it is employed by the Metropolitan Gas Referees under the name of a *Photoped* for the photometric examination of gas light. In this appliance a small tube has one end covered with semi-transparent paper (see Fig. 9). In the tube a diaphragm, having a small rectangular aperture in it, is made to move to and from the semi-transparent screen by a rack motion. If two lights are placed a little away from one another, and the photoped is placed so that the tube axis bisects the angle formed by the lines joining the eye-piece with the two lights respectively, then each light casts upon the transparent screen an illuminated patch produced by the rays passing through the hole. By moving the diaphragm these two patches of light can be made to come just into contact. If, then, the distances of

the sources of light are varied, or the lights altered in intensity so as to make the illuminated patches appear equally bright, the intensities of the two lights are proportional to the squares of their distances from the screen.

(b) *Shadow Photometers*.—The second method of photometrical comparison is based upon the principle of equalising the intensity of the two shadows of a rod made by two lights,

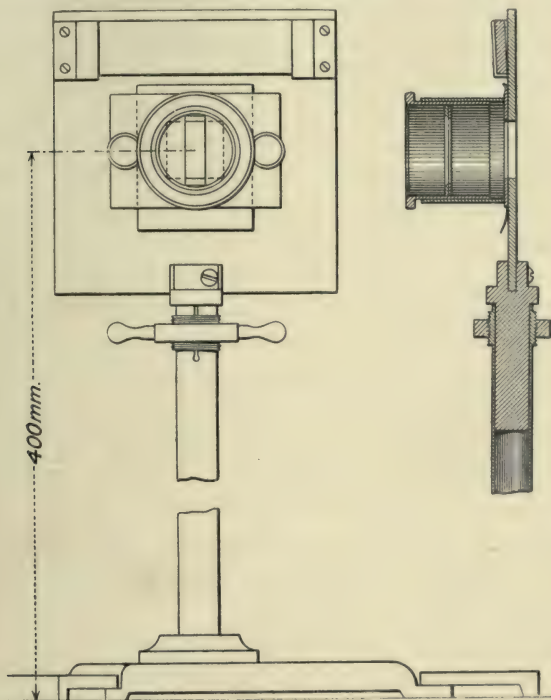


FIG 9.—The Metropolitan Gas Referees Photometer or Photoped.

the shadows being cast upon a white surface. This method was originally due to Lambert, but is generally ascribed to Rumford, and with certain modifications has been employed by Sir W. Abney and Profs. Ayrton and Perry.

If we place an opaque rod at a little distance from a white screen, and place two lights so as to cast two shadows of the

rod, it is possible to adjust the positions of the two lights so that the shadows just touch. Let the two lights be called A and B. Then the shadow cast by A is a place on the screen on which no light from A falls, but on which light from B does fall; and, vice versa, the shadow cast by B is a region illuminated only by the light of A. If the lights are isochromatic, then there is no difficulty in adjusting the positions of the lights so that the two shadows just touch and are of equal intensity. These two shadows, therefore, constitute regions on the screen illuminated by the two lights respectively, so that the illuminations are equal. The luminous intensities of the two lights are then proportional to the squares of their distances from the screen. If the experiment, however, is tried with heterochromatic lights, much greater difficulty will be found in pronouncing a judgment as to the equality in depth of the shadows, apart from their colour. If, for instance, we allow the shadow of a pencil to be thrown on a white card by the full moon, or by an arc lamp, and at the same time adjust a candle to throw another contiguous shadow, the shadow thrown by the moon or arc lamp would appear to be a deep orange colour, and the shadow thrown by the candle would appear to be a blue-grey. The observer will then have to endeavour to adjust the brightness or depth of these two shadows without regard to their colour difference. In doing this, great assistance is derived by employing a method of observation first put into practice by Sir W. Abney—viz., that of oscillating one of the lights. If, for example, the candle is moved to and fro so as to make the candle shadow wax and wane in intensity, it will be possible to find two positions for the candle, in one of which its shadow is decidedly brighter, and the other in which it is decidedly darker, than that of the arc light or moon. By gradually diminishing the amplitude of movement of the candle, it is possible to make a more accurate judgment of the position in which the two shadows are of equal brightness. When one of the lights is very much

brighter than the other, it may become necessary to reduce its brightness to a certain ratio. This can be done by the employment of either a double convex or a double concave lens, forming what is called a *dispersion photometer*. The diagram in Fig. 10 shows one method.

Let A be the position of the most intense source of light, and let  $a$  be the position of the other; let D be the distance of A from the photometer screen and  $d$  the distance of  $a$  from the screen; and let I and  $i$  be the intensities of these sources of light respectively. Let a concave lens, L, be placed between A and the screen and be so adjusted that the brightness or illumination on the screen due to the light A is equal to that produced by the light  $a$ . Then let  $\alpha$  be the percentage loss of illumination by the lens—that is to say, let the loss of illumination due to the light reflected or scattered by the lens surfaces and absorbed by the glass be such that a fraction equal to  $\frac{100-\alpha}{100}$  of the light falling on the lens passes through it.

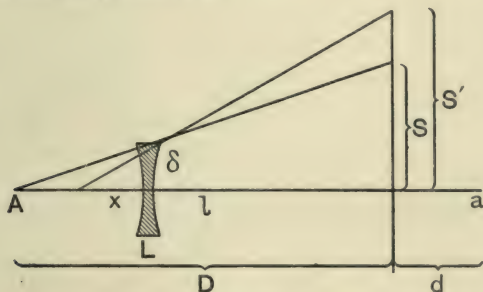


FIG. 10.

Furthermore, let  $f$  be the focal length of the lens,  $l$  its distance from the screen and  $x$  the distance of the focus conjugate to A from the lens. Then, if S is the area of the screen which would be covered by the light passing through an aperture equal to that of the lens if the lens were not there, and if S' is the area actually covered by the light passing through the lens, we have, obviously, the following equations:—If the lens were removed, the illumination on the screen produced by the light A would be  $A/D^2$ , but when the lens is interposed this becomes reduced to  $\frac{I}{D^2} \cdot \frac{S}{S'} \cdot \frac{100-\alpha}{100}$ . If this illumination balances that due to the light  $a$ , which is equal to  $i/d^2$ , we have

$$\frac{I}{D^2} \cdot \frac{S}{S'} \cdot \frac{100-\alpha}{100} = \frac{i}{d^2},$$

$$\text{or} \quad I = \frac{D^2 S'}{d^2 S} \cdot \frac{100}{100-\alpha}. \quad \dots \dots \dots (1)$$

If  $\delta$  is the diameter of the lens, it is evident that

$$\frac{S}{D^2} = \frac{\pi \delta^2}{4(D-l)^2}$$

and also that

$$\frac{S'}{(x+l)^2} = \frac{\pi \delta^2}{4x^2},$$

Hence,

$$\frac{S'}{S} = \frac{(x+l)^2}{x^2} \frac{(D-l)^2}{D^2}. \quad \dots \dots \dots (2)$$

Again, in virtue of the relation connecting the focal length of the lens  $f$  with the distances from it of the conjugate foci, we have

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{D-l}. \quad \dots \dots \dots (3)$$

If we eliminate  $x$  from the equations (1), (2) and (3), we obtain, after a simple reduction,

$$I = i \left( \frac{D(l+f) - l^2}{fd} \right)^2 \frac{100}{100 - a}.$$

The above expression can be written

$$\frac{I \left( \frac{fD}{fD + l(D-l)} \right)^2 \frac{100 - a}{100}}{D^2} = \frac{i}{d^2}.$$

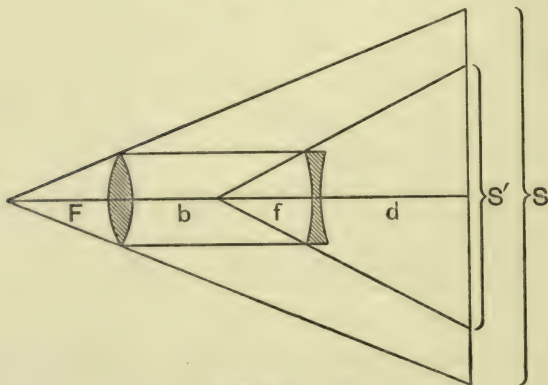


FIG. 11.

Hence we see that the effect of interposing the lens is as if the source of light of intensity  $I$  had been removed and one substituted of a less intensity, equal to  $\frac{I f^2 D^2 (100 - a)}{(fD + l(D-l))^2 100}$ , placed at the same position (A). The value of the coefficient  $a$  can be determined by experiment, and will be generally found to be about 12.

An arrangement which leads to a simpler expression for the resultant illumination is to employ a convex lens of focal length  $F$  and a concave lens of focal length  $f$  placed at any distance  $b$  from each other (see Fig. 11). The illuminant is placed at the principal focus of the convex lens, and a screen is placed at a distance  $d$  from the concave lens. Let  $\delta$  be the diameter of each lens. Then let  $S$  be the area of the screen which would be covered by the light passing through an aperture equal to the area of the convex

lens and  $S'$  be the area actually covered by the light traversing the concave lens when both lenses are in position. Also let  $a$  be the fraction of the light which passes through the first lens and  $\beta$  that which passes through the second expressed as percentage, so that  $a\beta$  is the percentage by which the original illumination is weakened, due to reflection and absorption by the lenses. Finally, let  $I$  be the intensity of the source of light. If, then, the lenses were removed, the illumination on the screen would be  $\frac{I}{(F+b+d)^2}$ .

When the lenses are in position as shown the actual illumination is

$$\frac{I}{(F+b+d)^2} \frac{S}{S'} a\beta.$$

Now

$$\frac{\pi \delta^2}{4S} = \frac{F^2}{(F+b+d)^2}$$

and

$$\frac{\pi \delta'^2}{4S'} = \frac{f^2}{(f+d)^2};$$

hence

$$\frac{S}{S'} = \frac{(F+b+d)^2}{(f+d)^2} \cdot \frac{f^2}{F^2}.$$

Therefore the illumination on the screen produced when the lenses are interposed is  $\frac{I}{(f+d)^2} : \frac{f^2}{F^2} a\beta$ . In other words, it is the same as if the illuminant had been moved from its actual position in the focus of the convex lens and placed at the focus of the concave lens, and at the same time had been altered in intensity in the ratio of  $a\beta f^2 : F^2$ . Hence, by a proper selection of lenses we can weaken the illumination in any required ratio.

The above method of weakening illumination by means of a lens has been much employed in the photometry of powerful arc lamps. Profs. Ayrton and Perry designed a diffusion photometer in which a double concave lens was employed to reduce the light.\* In experiments made at the South Foreland lighthouse a double convex lens was used for the same purpose, the general law being that the interposition of the lens weakens the illumination on the screen in the ratio of the area of the projection of the outline of the lens on the screen to the actual area covered by the beam emitted by the lens and corrected by a factor for loss of light by the lens.

(c) *Bunsen Disc Photometers*.—A third much-used photometric method consists in the employment of the principle first introduced by Bunsen in the grease-spot disc. If a piece of writing-paper has a portion of its surface treated with oil

\* See *Proc. Phys. Soc., London*, Vol. V., p. 109, "On a Simplified Dispersion Photometer."

or paraffin wax, this part becomes semi-transparent and constitutes what is known as the *grease-spot*. If a disc so prepared is placed between two lights and moved to and fro, the plane of the disc being perpendicular to the line joining the lights, the grease-spot being on this line, then it will be found that in one position the grease-spot will appear brighter than the rest of the paper when regarded from one side, and in another position it will appear darker as it is moved to and fro between the lights. If the lights are isochromatic, it is possible to find a position in which the grease-spot apparently vanishes. This is the case when the brightness on the two sides of the disc is exactly equal, for under these circumstances the light transmitted through the grease-spot is

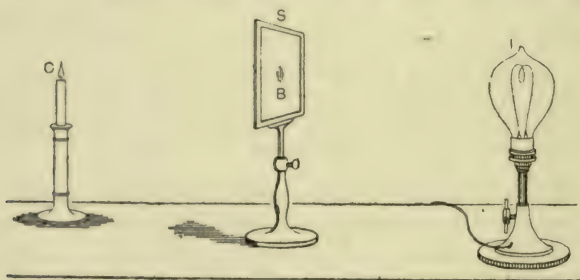


FIG. 12.

supplemented by the light reflected from the grease-spot, in such way that the total amount which reaches the eye is the same as that reflected from the portion of the paper which is opaque.

It is not necessary to give here the full theory of the Bunsen disc. The reader may refer for it to Palaz's "*Traité de Photométrie Industrielle*," p. 33, or to the English translation of the above work by Messrs. G. W. and M. R. Paterson, published by Sampson, Low, Marston & Co., London.

The problem may be treated in a limited manner as follows:—Let  $I_1$  and  $I_2$  be the two lights to be compared and let  $B$  be the disc. (See Fig. 12.) Let the distance of  $I_1$  from the disc be  $D_1$  and that of  $I_2$  be  $D_2$ . Let the outer or opaque portion of the disc be assumed to reflect all the light falling on it

which is equivalent to supposing it to be perfectly white, and let the "grease-spot" portion reflect  $1/r$ th part of the light and transmit  $1/t$ th part of the light incident upon it. Then, in the first case, let us imagine the disc set in the balancing position when from the side  $I_1$ . In this position the illumination of the inner and outer portions appears to be the same. Hence we have

$$\frac{I_1}{D_1^2} = \frac{I_1}{D_1^2} \frac{1}{r} + \frac{I_2}{D_2^2} \frac{1}{t},$$

or

$$\frac{I_1}{D_1^2} \left(1 - \frac{1}{r}\right) = \frac{I_2}{D_2^2} \frac{1}{t},$$

or

$$\frac{I_1}{I_2} = \frac{D_1^2}{D_2^2} \left( \frac{1 - \frac{1}{r}}{\frac{1}{t}} \right). \quad \dots \dots \dots (4)$$

Again, let the disc be viewed from the other side and set again in a new balancing position. We shall suppose that the coefficients of reflection and transparency of the grease-spot are slightly different on the two sides, which is equivalent to assuming that the disc is non-symmetrical. Let  $d_1$  and  $d_2$  be the distances of the disc from  $I_1$  and  $I_2$  when the second balance is obtained; then we have, as before,

$$\frac{I_1}{d_1^2} = \frac{I_1}{d_1^2} \frac{1}{r'} + \frac{I_2}{d_2^2} \frac{1}{t'}$$

$$\frac{I_1}{I_2} = \frac{d_1^2}{d_2^2} \left( \frac{1 - \frac{1}{r'}}{\frac{1}{t'}} \right), \quad \dots \dots \dots (5)$$

where  $r'$  and  $t'$  are the second coefficients of reflection and transparency respectively. Hence, combining equations (4) and (5), we have

$$\frac{I_1}{I_2} = \sqrt{\frac{D_1^2 d_1^2}{D_2^2 d_2^2} \left( \frac{1 - \frac{1}{r}}{\frac{1}{t}} \right) \left( \frac{1 - \frac{1}{r'}}{\frac{1}{t'}} \right)}.$$

Let us denote the ratio  $D_1^2/D_2^2$  by  $\rho$ , the ratio  $d_1^2/d_2^2$  by  $\rho'$ , and the function  $\sqrt{\frac{r-1}{rt} \frac{r'-1}{r't'}}$  by  $\beta$ ; then we have

$$\frac{I_1}{I_2} = \beta \sqrt{\rho \rho'}.$$

Hence the ratio of the luminous intensity is given by multiplying the geometrical mean of the photometer bar readings by a constant depending on the nature of the two surfaces of the disc.

We can also investigate from the above equations the conditions which must be fulfilled that the disc may have the maximum sensitiveness. With this object we have to express the fact that when the balance is obtained the difference in the appearance of the grease-spot, or inner, and the outer portions of the disc is as great as possible for a given small displacement of the disc. We need not trouble to obtain the general expression when the lights employed are of different intensities, but shall consider only the limited case when they are identical, and when the disc is therefore initially at the same distance,  $a$ , from either light.

Let  $I$  be the common intensity, then we have

$$\frac{I}{x^2} = \frac{I}{x^2} \left( \frac{1}{r} + \frac{1}{t} \right)$$

as the photometric equation when the balance is obtained. Next let the disc be displaced by a small amount,  $\delta x$ , and we have, as the equation for the difference in illumination of the inner and outer portion, the equation

$$\frac{I}{(x + \delta x)^2} - \left( \frac{I}{(x + \delta x)^2} \frac{1}{r} + \frac{I}{(x - \delta x)^2} \frac{1}{t} \right) = \delta\beta.$$

We have then to ascertain the conditions which give  $\frac{\delta\beta}{\delta x}$  the greatest value possible.

If we rearrange the above equation, neglecting squares and higher powers of  $\delta x$  in comparison with  $\delta x$  and  $x$ , and with reference to the original balancing equation, we have, as an expression for  $\delta\beta/\delta x$  the following :

$$\frac{\delta\beta}{\delta x} = \frac{I}{x^3} \left( 1 - \frac{1}{r} - \frac{1}{t} \right) \frac{1}{\delta x} - \frac{2I}{x^3} \left( 1 - \frac{1}{r} + \frac{1}{t} \right).$$

But from the original balancing equation we see that  $1 - \frac{1}{r} - \frac{1}{t} = 0$ . Hence we have as the value of  $\delta\beta/\delta x$  the expression

$$\frac{\delta\beta}{\delta x} = \frac{2I}{x^3} \left( 1 - \frac{1}{r} + \frac{1}{t} \right).$$

In order, therefore, that this may be as large as possible consistently with possibilities we must have  $r = \infty$  and  $t = 1$ , and then

$$\frac{\delta\beta}{\delta x} = \frac{4I}{x^3}.$$

Any other possible values of  $r$  and  $t$  makes the value of  $\delta\beta/\delta x$  less than this. Thus, for instance, if  $r = 2$  and  $t = 2$ , we have  $1 - \frac{1}{r} + \frac{1}{t} = 1$ ; and if  $r$  takes any value less than infinity, and  $t$  any value greater than unity, the value of  $\left( 1 - \frac{1}{r} + \frac{1}{t} \right)$  is less than 2.

Accordingly, the best results are obtained, and the Bunsen disc is most sensitive, when the grease-spot portion is perfectly transparent, and transmits all the light it receives and reflects none, whilst the surrounding part of the screen reflects all that falls upon it. We shall see below that this condition is perfectly realised in the case of the Lummer-Brodhun prism.

The above argument supplies an explanation of the fact that a star disc (described below), made with very thin and transparent paper, sandwiched between two sheets of white opaque paper pierced with stellate holes, is a more sensitive disc than the usual grease-spot made with paraffin; it more perfectly realises the ideal Bunsen disc.

For accurate work it is found necessary to be able to inspect both sides of the disc at once, as a very slight difference in the quality of the two lights prevents the

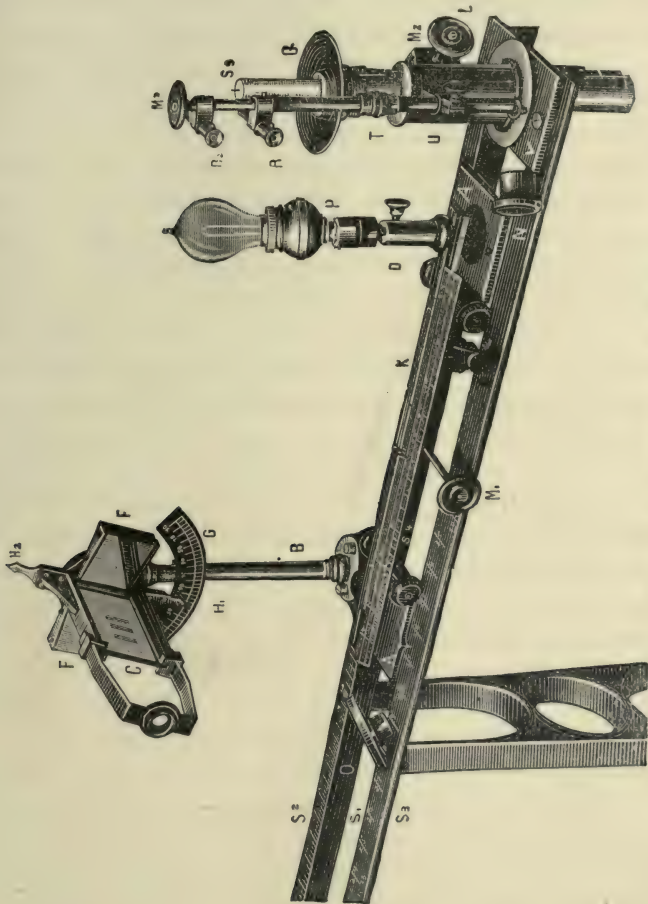


FIG. 13.—Bunsen Disc Photometer with Inclined Mirrors.

grease-spot from vanishing in any position. The observation consists, then, in fixing the position of the disc so that the appearance of the two sides is the same. This is most easily

achieved by means of a pair of inclined mirrors, FF (*see* Fig. 13), placed behind the disc, the observer's eye being in the plane of the disc and a little distance from it. Images are then seen in the two mirrors of the two opposite sides, and the box containing the disc and the mirrors is moved until the two images appear similar. The same result may be more perfectly obtained by means of a Krüss prism, in which, by means of total reflection, a view is obtained by one telescope of the two sides of the disc simultaneously.

Although the Bunsen disc has probably been more used for electro-technical photometry than any other device, it yet labours under many disadvantages. In the first place, the two images which are seen and compared cannot in general be made identical. If the two lights are even slightly heterochromatic, then the observer sees in one mirror a bluish spot on a yellow ground, and in the other mirror a yellowish spot on a bluish ground; and, if the two surfaces of the disc are not perfectly identical, it will be found very difficult to make these images in any way similar. To overcome these difficulties a disc was introduced by Leeson and Dibdin, called the "Star Disc," which is made as follows:—Two very thin pieces of white card have identical star-shaped holes cut out of them and between the two pieces of card is placed a thin piece of tissue paper, and the three are then pressed together. Using such a disc with the inclined mirrors, the observer sees two star images of different colours on two differently coloured backgrounds. Thus, in comparing an arc lamp with an incandescent lamp, on one side would be seen a slightly yellow star on a bluish background, and on the other side a bluish star on a yellow background. The observer has then to move the disc until these stars are equally distinct or sharp upon their respective backgrounds, without reference to colour.

(d) *Wedge Photometers.*—A fourth much-used principle of photometric comparison is the equalisation of the illumination on two white surfaces inclined at equal or unequal angles to

the line joining the two lights to be compared, placed on either side. In its simplest form this photometer is known as the Ritchie wedge. A prism of wood, having an angle of 60 degrees, has two adjacent surfaces covered with smooth dead-white paper or made dead-white in any other manner. It is essential that the angle of this wedge should be extremely sharp. If then the wedge is placed between two lights, so that the sides of the wedge make equal angles with the line joining the lights, and if the eye of an observer is placed so as to regard the wedge from a direction at right angles, then the wedge can be moved to and fro and a position found in which the two sides appear equally bright.

This principle was modified by Mr. A. P. Trotter as follows:—A piece of thin white card is bent in the middle so that the two sides form angles of 60 degrees, and a hole is cut in one of the sides, the edges of the hole being very clean and sharp. This folded card is placed between two lights to be compared, with its angle facing one of them and the two sides making equal angles with the line joining the lights. An observer then looking at one side sees through the hole the inside of the other leaf, and, if the distances of the lights are adjusted, it is possible to make the hole vanish. This occurs when the brightness of the two surfaces illuminated is the same. With regard to all these forms of wedge photometer, the same difficulties occur when the lights are heterochromatic.

(e) *Total Reflection Photometers*.—The fourth class of photometer includes those in which total reflection from prisms is employed to obtain two fields of light, the brightness of which is then compared. The best representative of this class is the Lummer-Brodhun photometer, and this, in its most recent form, is probably by far the most sensitive photometer yet devised for the comparison of isochromatic lights. The general construction of the Lummer-Brodhun photometer is as follows:—A metal box (*see* Fig. 14) contains a white screen, S, identical in whiteness on its two sides. This is obtained by

employing a thin slab of plaster of paris or by coating a metal plate with compressed magnesia or sulphate of barium and obtaining thereby an exceedingly dead-white smooth surface. The two lights to be compared are placed on either side of this disc and illuminated. By means of two totally reflecting prisms,  $P_1$  and  $P_2$ , part of the light, scattered through the two sides of the disc, is reflected into a compound prism, PQ, which is really a very superior form of Bunsen disc. This compound prism is made as follows:—A cube of glass is cut into two parts diagonally so as to form two right-angle

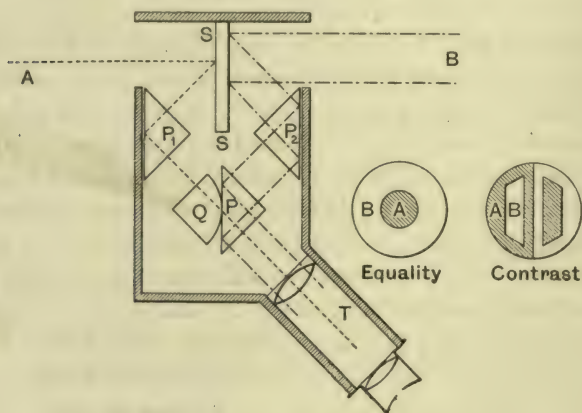


FIG. 14.—Lummer-Brodhun Photometer.

S Magnesia Screen.	Q P Lummer-Brodhun Prisms.
$P_1 P_2$ Totally Reflecting Prisms	T Telescope.

prisms. One of these prisms has its hypotheneuse surface truncated or bevelled off; the first prism has then its whole hypotheneuse surface optically worked, and the second one partially so, to make optical contact between the two over a certain circular area. This compound prism is fixed in a box, as shown, and the apparatus completed by a telescope. A perspective view of the arrangement with the box lid removed is shown in Fig. 15. On looking through the telescope the

observer sees a circular patch of light surrounded by another zone of light. The first is produced by the light which has been transmitted through the centre of the compound prism, and the outer zone is produced by light which has been totally reflected from the hypotheneuse surface of one of the prisms. The two lights are then adjusted as to distance until the field of view becomes perfectly uniform.

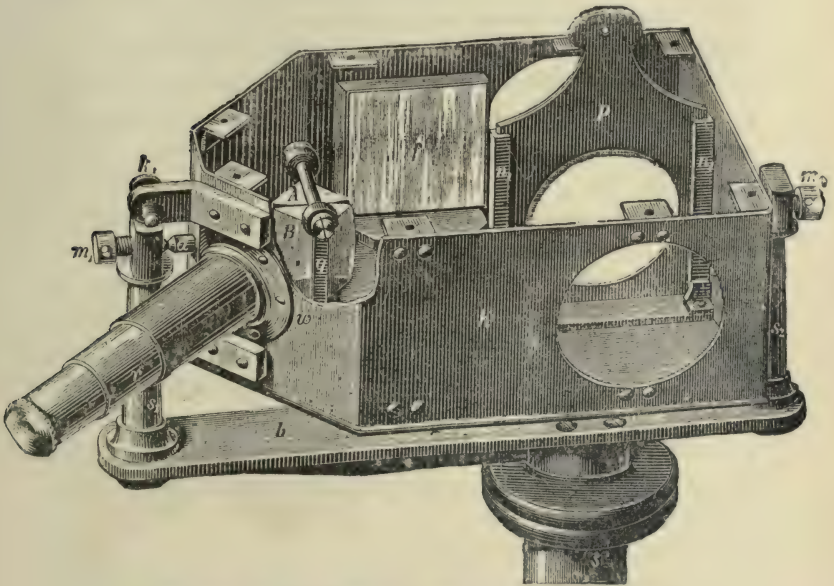


FIG. 15.—Lummer-Brodhun Photometer (top removed).

In another form of Lummer-Brodhun prism, the hypotheneuse surface of one of the prisms is cut out by a sand-blast into a pattern, as shown in the diagram marked "Contrast" in Fig. 14, and the remaining portion is polished to come into optical contact with the plane hypotheneuse surface of the complementary prism. The observer then sees in the telescope a field of view as shown in Fig. 14, and by moving the lights either the whole field can be equalised, or else

an equal contrast produced between the isolated patches of light on their respective backgrounds, which constitute the two parts of the whole field of view.

The principle of abolishing the total reflection at the hypotheneuse of a right-angle prism had been employed previously by W. Swan in a photometer.\* In Swan's prism photometer two right-angle prisms, produced by cutting a cube of glass obliquely across, were placed with their hypotheneuse surfaces nearly touching, and in the centre a small drop of Canada balsam was placed to cement the two prisms together. This compound prism was used as a Bunsen disc, the central portion transmitting completely, and the circumferential portions reflecting completely, rays incident on the common surface at the proper angle. The employment of Canada balsam is not, however, permissible for an accurate incident, as it has not the same refractive index as glass, and there is, therefore, a certain amount of light lost in transmission. A photometer on the same principle has been devised by the Author. In this instrument a pair of similar right-angle prisms, P and Q, are first prepared, and the hypotheneuse surface of one of them is cut down to a lower level half-way across the prism. The two hypotheneuse surfaces are then placed together, as shown in Fig. 16, the contact surfaces being optically worked so as to come into perfect contact. This compound prism is contained in a small brass box having two dead-white screens, S S, placed at angles of 45 degrees to the prism surfaces, and the box is fixed to the end of a small telescope. One of the lights is placed in the line of the telescope and the other at right angles, and the distances of these lights are altered until an observer, looking into the field of view of the telescope, sees the two parts of exactly equal brightness. This form of photometer is very useful in arc light photometry, as it is possible to affix to

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\* See a Paper by Prof. C. G. Knott on "Swan's Prism Photometer," *Phil. Mag.*, Vol. XLIX., January, 1900. Also reply by Messrs. Lummer and Brodhun, Vol. XLIX., June, 1900.

the telescope a long rod carrying a small incandescent lamp in such fashion that, by moving this lamp to and from the prism, a comparison can be made between the incandescent lamp and a ray proceeding from an arc lamp in any direction. The whole arrangement appears as in Fig. 16.

(f) *Polarisation Photometers*.—The equalisation of two rays of light from two different sources may be accomplished by means of Polarisation Prisms. If two Nicol prisms are crossed, and a ray of light sent through the two prisms, the intensity of the emergent ray varies as the square of the cosine of the angle between the principal planes of the prisms. By means of a divided circle this angle can be read

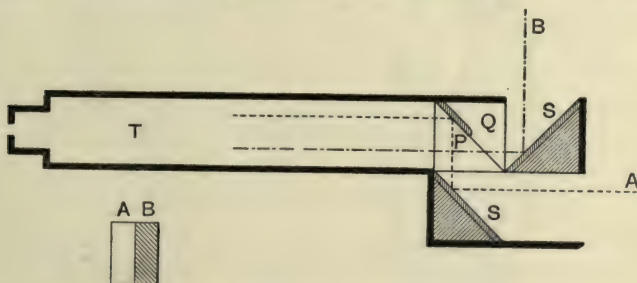


FIG. 16.—Total Reflection Photometer (Fleming).

P Q Right-angle Prisms.  
S S White Diffusing Screens.  
T Eye-tube.

A B Field of View, the two parts  
illuminated respectively by  
light A and light B.

off. Employing two totally reflecting prisms and a pair of such Nicol prisms, placed at the middle point of one of two lights to be compared, it is possible, by altering the angle between the Nicol prisms, to equalise the intensity of the rays emerging from the totally reflecting prisms, and, therefore, to compare their intensity. Polariscopes depending on this principle have been devised by Arago, Zollner, Wild, Salomans, Pickering and Nichols.

(g) *Rotating Disc Photometer*.—A photometric method, which was first suggested by Mr. Fox-Talbot in 1834, and has been very much employed by Sir W. Abney and others,

and which presents many advantages in certain investigations, is the employment of a rotating disc having sectors or windows cut out of it, the aperture of which can be varied. If a disc has an angular sector cut out of it, having an angle  $\alpha$ , then if this disc is caused to rotate rapidly and a ray of light is passed transversely to the disc so as to pass through the sector when it is in the proper position, but to be eclipsed during the remainder of the rotation, the emerging ray of light has an intensity compared with the incident of light in the ratio  $\alpha$  to  $360 - \alpha$ . The validity of this conclusion has been disputed at various times, but it seems established on

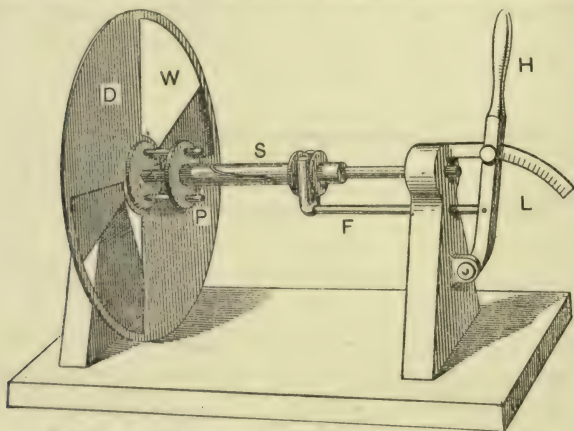


FIG. 17.—Fox-Talbot-Abney Variable Aperture Disc.

satisfactory grounds. This method for weakening the intensity of a ray of light in any required ratio was originally suggested by Mr. Fox-Talbot,\* but was re-invented by Napoli, Guthrie and Sir W. Abney. Various devices have been employed for altering the aperture of the sector as the disc is running. The simplest plan is to have two discs cut out of the sectors, which are superimposed the one upon the other (see Fig. 17). One of these discs is fixed

\* The rotating disc photometer was first described in 1834 by Mr. Fox-Talbot, noted for his inventions in connection with photography. His Paper will be found in the *Phil. Mag.* for 1834, Vol. V., p. 331.

to a shaft, and the other is fixed to a sleeve moving on the shaft, this sleeve being driven by the shaft through the intermediation of a pin fixed in the shaft, which works in a helical slot cut in the sleeve. If, therefore, the sleeve is pushed to and fro on the shaft parallel to itself, it displaces one disc relatively to the other through an angle which completely opens or completely closes the common apertures. A disc of this kind can be driven by a small electric motor (*see* Fig. 14), and by means of a fork working in a groove in the sleeve the aperture of the sector can be varied whilst the discs are running from nothing up to the full aperture. It is, therefore, possible to vary a ray of light in intensity and reduce it from anything we please to one-half its original intensity and complete extinction. If two such rotating sectors are interposed in the paths of two rays of light proceeding from two sources at the same distance, the rays, after passing the sectors, may be made to be equal, and this equality judged by the employment of a Foucault photometer or any other arrangement.

The question has sometimes been raised whether the Fox-Talbot disc reduces the intensity of the rays falling on it in the ratio of the total open sectorial angle to 360 degrees. An experiment confirming the affirmative opinion was tried at the Pender Laboratory, University College, London. Two incandescence lamps were placed on either side of a Lummer-Brodhun photometer, and a Fox-Talbot disc interposed between one lamp and the photometer. One lamp was then moved to various distances and the photometric balance obtained by adjusting the apertures in the Talbot disc. The following readings were made:—

Angular apertures of Talbot disc opening expressed as a percentage of full aperture.	Distance on the photometer bar at which the movable lamp was balanced, reckoned from the photometer.
97 per cent.	80in.
44    "	120in.
24    "	160in.
15    "	204in.

If the illumination on the photometer disc when the lamp is at 80 inches is taken as unity, then the illuminations when the lamp was at 120, 160, and 204 inches respectively were 44.4, 25 and 15.4 per cent. of that when it was at 80 inches. These calculated values of the illumination are very nearly in the ratio of the apertures of the disc, thus showing that the luminous intensity of a ray is reduced by the disc in the proportion of the open sector aperture.

(h) *Absorption Photometers*.—Photometers have been constructed depending upon the equalisation of two fields of illumination by the interposition of a variable thickness of an absorbing medium, such as a glass wedge made of slightly tinted glass resembling smoky quartz. These have been employed in astronomical photometry, but are not very useful for electro-technical purposes.

(i) *Diffusion Photometers*.—The best known form of this photometer is due to Elster and to Joly. In it two cubical blocks of semi-transparent paraffin wax are separated by a thin metal sheet, such as a sheet of tinfoil. The light from each source enters each translucent block and diffuses itself, and when the compound block is viewed from a position at right angles to the line of the incident light, the blocks are seen to be illuminated internally. The lights are then moved until the appearance of each half of the block is the same.

The Joly photometer is easily made, and is said to be exceedingly sensitive.

(j) *Physical Photometers*.—Under this title may be mentioned all those various forms of photometer in which an attempt has been made to abolish the use of the human eye as the working appliance, and reduce the comparison of two lights to a measurement of their relative effect in producing some physical effect, such as altering the resistance of selenium, affecting a radiometer, producing expansion of chlorine, or causing chemical combination of hydrogen and chlorine. None of these devices has been very successful. Since the object of photometry is to compare together sources

of light with regard to their power of enabling us to *see*, it is difficult to imagine that anything else can be a substitute for the human eye in testing the relative value of two lights for visual purposes.

In comparing together two isochromatic lights, the Lummer-Brodhun photometer, or a good star-disc or Bunsen photometer, will be found to be the best appliance to employ.

In a Paper by Messrs. Barr and Philips, "On the Brightness of Light: Its Nature and Measurement," in *The Electrician*, Vol. XXXII., p. 525, will be found a diagram showing the relative accuracy of various forms of photometer. Much, however, depends upon the observer. In all the above forms of intensity photometer in which two surfaces or fields are compared, which may be unequally coloured as well as unequally bright, the accuracy of the measurement essentially depends upon the skill of the observer in distinguishing between the difference in brightness and the difference in colour. The greatest difficulties occur in this respect in heterochromatic photometry, and accordingly some forms of photometer especially designed for this purpose must be noticed.

(*k*) *Flicker Photometers*.—A very interesting discovery was made by Prof. O. N. Rood in 1893, which led to the construction of what is known as a flicker photometer.\* If two surfaces illuminated by two heterochromatic lights are presented alternately and very quickly to the eye, the colour difference blends together in the persistence of vision into a common tint. If, however, the surfaces differ in brightness, a curious flickering appearance is presented which vanishes entirely when the surfaces are equally bright. Hence, this absence of flicker constitutes a means for determining the equality in brightness of two surfaces, apart from their colour difference. Prof. Rood employed an arrangement consisting of a Ritchie wedge covered with white paper and placed

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\* Prof. O. N. Rood, *American Journal of Science*, Vol. XLVI., p. 173, 1893, "On a Photometric Method which is Independent of Colour."

between the lights to be compared. By means of a vibrating lens or prism the eye of an observer obtains alternate and rapid glimpses of the two sides of the wedge, thus producing the flicker until it was annulled by a proper disposition of the lights. A better arrangement is the following one, which is a slight modification of an arrangement due to Prof. F. P. Whitman.\*

On to the shaft of a small fan motor is placed a cardboard fan cut out in the shape of a Maltese cross, the open sectors being equal to the solid sectors (*see* Fig. 18). This fan is placed in front of a sheet of cardboard of the same kind as that used for making the fan, the place of the fan being inclined at an angle of 60 degrees to the background. This arrangement is fixed on a photometer bench so that the two lights to be compared are placed on either side of the line joining the two lights and must bisect the angle between the planes of the cardboard background and the cardboard fan. An eye-tube is then placed in front of the arrangement so that when the motor rotates the light reflected down the tube comes alternately from one source and the other. When this is done, if the fan is rotated not too quickly, but about three revolutions a second, then a flickering will be seen on looking through the tube, unless the distance of the two lights is adjusted to produce equal brightness on the screens. The fan may be driven by clockwork, and fan, screen, and clockwork included in one box, with an eye-tube, as shown in Fig. 18. It is possible with this arrangement to compare together with great ease an arc lamp and a candle, or even two incandescent lamps, one in a red bulb and the other in a green one. The colour difficulty entirely vanishes. This is probably due to the fact that the eye requires a slightly longer time to be influenced by a colour sensation than by a

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\* Prof. F. P. Whitman, *Physical Review*, Vol. III., p. 241, 1896, "On the Photometry of Differently Coloured Lights and on the Flicker Photometer." *See also* Prof. O. N. Rood, *Science*, Vol. VII., p. 757, 1898; also Vol. VIII., p. 11; also *Science Abstracts*, Vol. II., p. 10.

brightness sensation. Hence the difficulty involved in comparing two illuminated surfaces different in colour as well as

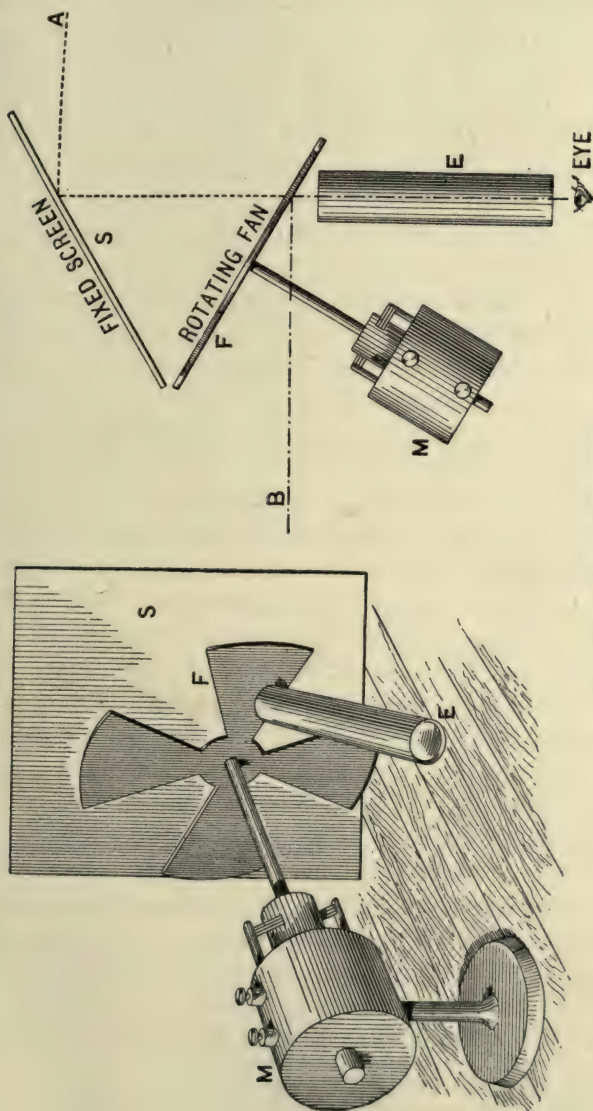


FIG. 18.—Flicker Photometer.

M Motor carrying White Maltese Cross Fan, F. S Fixed Screen. E Eye-tube. A and B Two Sources of Light.

different in brightness vanishes when the two surfaces are seen, not simultaneously but alternately.

(1) *Selected Ray Photometers*.—The difficulties of estimating an equality in brightness in the case of two illuminations of different colour have been met in some cases by selecting particular rays for comparison. In comparing an arc lamp with a candle or other standard lamp it was usual at one time to place a piece of red or green glass before the eye, and so select a narrow range of spectrum, which was made the basis of comparison, and arc lamp luminous intensities were then expressed in "red candles" and "green candles." But these terms have no scientific meaning at all, and the results were perfectly valueless. A. Crova has, however, established on a more scientific basis this method of selected ray photometry. His method depends on the fact discovered by him (*Comptes Rendus*, Vol. XCIII., p. 572) that the integral brightness of two fairly white lights are in the ratio of the brightness of the rays in them having a wave-length  $0.582\mu$  ( $\mu = 0.001$  millimetres). Hence, if by means of an absorbing medium we select from the two heterochromatic lights rays approximately of this wave-length and determine their relative intensity, we have, according to Crova, a figure which gives us the relative luminous intensity of the two lights. For this purpose Crova employed a solution consisting of sublimated anhydrous ferric chloride 22.321 grammes, crystallised nickelous chloride 27.191 grammes, dissolved in distilled water and the volume brought up to 100 cubic centimetres at  $15^{\circ}\text{C}$ .<sup>\*</sup> This solution is placed in a glass trough, and transmits radiation of a wave-length lying between  $0.630\mu$  and  $0.534\mu$ , having a well-marked maximum at  $0.582\mu$ . If this trough is held in front of the eye when making a photometric comparison on an arc lamp and a candle, and if the distance of the illuminants are adjusted so that the two parts of the photometer disc seen through this solution are of equal brightness,

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<sup>\*</sup> *Comptes Rendus*, Vol. CIX., No. 16, p. 627, October 15th, or *The Electrician*, Vol. XXXIII., p. 754, or Palaz's "Traité de Photométrie Industrielle," p. 82.

then, according to Crova, the luminous intensities of the two lights are inversely as the square of their distances from the screen. Crova's method, however, must be applied with some discrimination. It is not applicable in those cases in which the range of the spectra is very different. Thus, for instance, if an incandescent lamp is worked at 5 watts per candle, so that the filament is barely red-hot, and another smaller lamp is worked at  $2\frac{1}{2}$  watts per candle, so that the filament is highly incandescent, the spectra of these two sources of light would have a very different range, and Crova's method could not be applied to these two sources of light to determine their relative luminous intensities. It is applicable only in cases where the range of the spectra of the two lights is not very different.

The principle involved may be explained a little more at length thus :— Suppose we form the spectrum of any source of light and divide the length of the visible spectrum into small elements of length. Let  $\lambda_1$  and  $\lambda_2$  be the extreme limits of the spectrum, and let  $\delta\lambda$  be the width of any elementary slice of the spectrum. If we denote by  $b$  the brightness of this slice, then the whole spectrum—that is, the whole original ray—has a luminous intensity denoted by

$$\int_{\lambda_1}^{\lambda_2} b d\lambda,$$

provided we take as our unit of length the width of the spectrum. Hence, if we delineate by any means a brightness or luminosity curve for the spectrum, the ordinates of which are proportional to the brightness of the spectrum at each point, then the integration of this curve will give us a number representing the integral luminous intensity of that ray.

Suppose that a curve of brightness is delineated in this manner for the spectra of two sources of light. It will then be found that the ratio of the areas included by the brightness curve—in other words, the relative integral brightness of the lights—are in the same proportion as the ordinates of the brightness curve which correspond to a wave-length of  $0.582\mu$ . According to Prof. E. L. Nicholls, this wave-length is nearer to  $0.600\mu$ . If, therefore, we make one observation of the relative brightness or luminous intensity of rays having this wave length, which are selected from the two sources of light, we have a number which gives us the relative integral luminous intensity of the two lights.

Another way of putting the matter would be to say that, as the spectrum of a source of light increases in brightness, the integral luminous intensity increases in the same ratio as the intensity of the single ray whose wave-length is  $0.600\mu$ . It is clear, however, that this is merely an empirical rule

which is approximately true for sources of light which can be described as white light. The rule could not be applied in the comparison of two lights, one of which was decidedly a red light and the other decidedly a blue light.

The only satisfactory comparison to make between two sources of light of different spectral composition is to examine them ray by ray by means of a spectro-photometer and compare their relative intensities.

B. *Spectro-Photometers*.—A spectro-photometer consists of some arrangement by which two spectra can be formed from the light proceeding from the two sources to be compared, these spectra being formed side by side either vertically or horizontally. Some arrangement is then required for weakening the light of one of these spectra relatively to the other, so as to make any particular ray in one spectrum match the corresponding ray in the other in brightness. This is sometimes achieved by the employment of crossed Nicol prisms, by means of which the intensity of the light forming one of the spectra can be varied from a maximum value to zero. An excellent form of spectro-photometer has been devised by Prof. E. L. Nichols.\* In this instrument two spectra are formed vertically side by side by means of two totally reflecting prisms, which direct the light from two sources placed at the extremities of a photometer bench into a direct vision spectroscope. Each prism covers one-half of the slit of the spectroscope. The spectro-photometer is made to move between the two sources like a Bunsen disc photometer carriage, and the illumination on the totally reflecting prisms therefore varies inversely as the square of the distances of the two lights respectively. The observer can therefore equalise in brightness the identical rays in each spectrum by moving the spectroscope. By this simple arrangement, the spectral comparison of two lights becomes a very simple matter, and a curve can be drawn showing the distribution of brightness in the spectrum of any light

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\* "Note on a New Photometer," by Dr. E. L. Nichols. *Trans. Inst. Am. Elec. Eng.*, Vol. VII., 1890; also *American Journal of Science*, Vol. XXXVI., p. 332.

relatively to a certain standard spectrum. Thus, for instance, the spectrum of the electric arc, or of the sun, may be compared with that of an incandescent electric lamp working at 3 watts per candle, taken as a standard; that is to say, the brightness of each portion of the spectrum of the light of the last mentioned lamp is taken as unity, and a number is found representing the relative brightness of the corresponding ray in the spectrum of the other sources of light.

The diagram Fig. 19, shows the arrangement of the apparatus. In front of the slit of a direct vision spectroscope

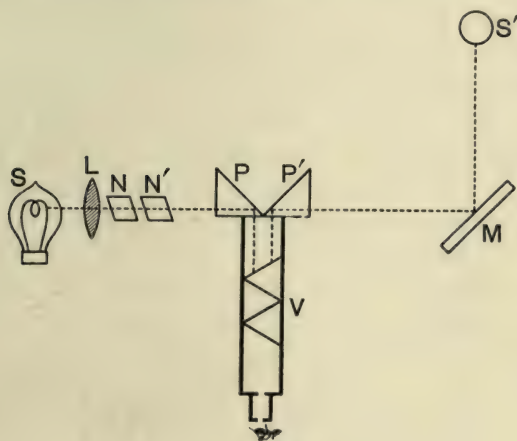


FIG. 19.

are placed two right-angle total reflection prisms,  $PP'$ , each one covering half the slit. As a source of light an Edison 16 c.p. lamp was adopted by Prof. Nichols. The rays from this lamp  $S$  were made parallel by means of a lens,  $L$ , and then passed through a pair of Nicol prisms  $NN'$  by means of which the intensity of the light from the lamp  $S$  could be reduced in any required degree by rotating one Nicol prism relatively to the other. This ray of light was then reflected by one total reflecting prism into the spectroscope and formed a continuous spectrum, the brightness of which could be

adjusted at any point by rotating one of the Nicol prisms. The source of light  $S'$  to be compared was allowed to cast light on to a block of magnesium carbonate,  $M$ , placed so as to irregularly reflect rays into the second total reflection prism in front of the slit. In this manner two spectra were formed, one over the other, one of which could be altered in intensity at any point. The experimental measurement consisted in adjusting the brightness of any one part of the spectrum of the light  $S'$  until it matched the corresponding ray in the spectrum of the light  $S$ . In plotting out these results the brightness of the spectrum of the Edison glow lamp was taken as unity at every part of its spectrum, and the brightness of the corresponding ray in the spectrum of the other light was then expressed by means of an ordinate drawn to scale. In this manner a number of curves could be drawn representing the relative brightness in the different portions of the spectrum of various sources of light, such as the arc light, daylight, or limelight, and some of these results are delineated in Fig. 20.

The position of the principal dark lines in the solar spectrum is shown at the foot of the diagram, and a horizontal line drawn above the spectrum represents the intensity of the light of the comparison lamp at every point in the spectrum. The ordinates of the other curves represent the relative brightness of the other lights at the same point. It will be seen, for instance, that the light from an arc lamp, when so adjusted as to have the same brightness at a point near the line  $D$  in the solar spectrum, is less bright in the red and much brighter in the violet than that of an incandescent lamp, and that there is a very sudden increase of brightness in the neighbourhood of the solar line  $G$ , due to the violet band in the arc lamp. Here the relative intensity of the arc lamp spectrum rises abruptly from about three to about 20 times that of the incandescent lamp. In the same manner a number of interesting experiments were made by Profs. Nichols and Franklin on the spectral character

of the light emitted from a glow lamp, the filament of which was placed at various temperatures. Thus a glow lamp, the normal candle-power of which was 16, was worked at various voltages so that it gave respectively 4, 10, 16, 22 and 28 c.p. The brightness of the light of this spectrum

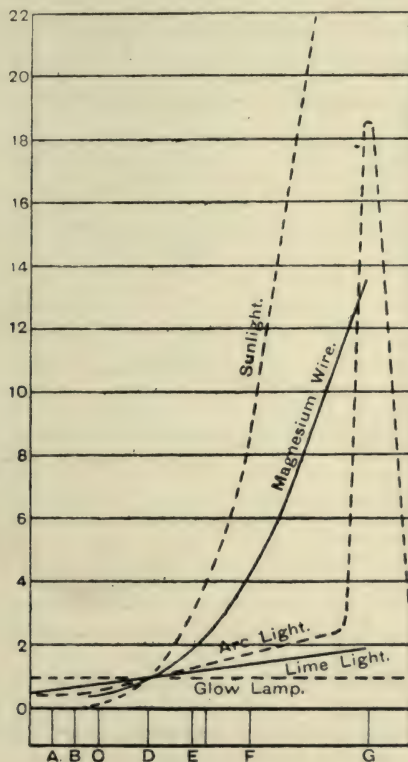


FIG. 20.

when giving 16 c.p. was everywhere taken as unity, and the curves in the diagram Fig. 21 show the relative intensities of the corresponding rays at the different temperatures. The vertical dotted line in the diagram shows the wave-length for which the relative intensities are in the same ratio as the

candle-powers of the lamp, measured by a Bunsen photometer. This point is very near the wave-length  $0.600\mu$ . Hence, at this point, the intensity of the light of the spectrum alters when the temperature of the filament changes in the same ratio as the candle-power of the lamp. This ray, however, does not divide the spectrum into two portions which are equal either in total luminous intensity or in total energy. Hence, if we imagine four lamps made with carbon filaments

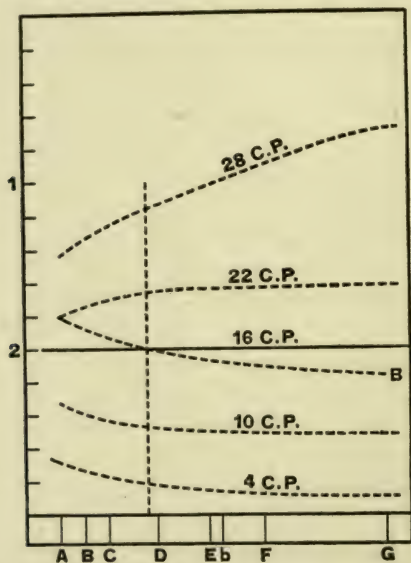


FIG. 21.—Curves showing the Relative Brightness along the Spectrum of an Incandescent Lamp. Curves marked 4 c.p., 10 c.p., 16 c.p., 22 c.p., 28 c.p. refer to the same Lamp worked at various temperatures. Curve B refers to a Low-voltage Lamp at normal candle-power.

which are brought respectively to the temperatures corresponding to 4 candles, 10 candles, 22 candles, 28 candles, for the 16-candle lamp, and if these filaments were made of such lengths that in each case the total light given out would be, as measured by a Bunsen photometer, declared to be 16 c.p., the lamps would differ in total energy of radiation and in total luminosity, as represented by the area of their luminosity

curves. It follows therefore that the candle-power, as determined by means of the Bunsen photometer, or any equivalent photometer, affords neither the correct measure of the light-giving energy nor of the total luminosity of the sources of light, the direction of the error being always such as to favour sources of a low degree of incandescence when compared with those of a high degree of incandescence.

The discrepancy between candle-power and luminosity with increasing incandescence is less marked than that between candle-power and energy of radiation, because luminosity depends very largely upon the less refrangible rays of the spectrum.\* This effect is really a consequence of the Purkinje phenomena in the eye, in that the sensation of violet increases less fast with the external stimulus than the sensation of red; hence, for purposes of producing artificial light, it is in one sense more economical to spend the energy in the production of the less refrangible rays than of the more refrangible, because the increase in the brightness sensation in the first case is far greater than in the second case for a corresponding expenditure of energy.

(C.) *Illumination Photometers*.—In some cases we have to determine the illumination on a surface or in a region reckoned in candle-feet or similar units, without regard to the distances or disposition or luminous intensity of the lights producing this illumination. Thus, for instance, in a street we may desire to ascertain the illumination due to the street lamps in various places, taken either on the roadway surface or on a plane at any height above it. This is done by means of an appliance called an Illumination Photometer. Instruments of this kind have been devised by Prof. L. Weber, Sir W. H. Preece and Mr. Trotter.† The illumination photometer of

\* See Profs. E. L. Nichols and W. S. Franklin "On a Spectro-photometric Comparison of Sources of Artificial Illumination," *American Journal of Science*, Vol. XXXVIII, August, 1889.

† See Mr. A. P. Trotter, "On the Distribution and Measurement of Illumination," *Proc. Inst. Civil Eng.*, 1892, Vol. CX., part 4; also "A Portable Photometer," by Sir W. H. Preece and Mr. Trotter, *The Electrician*, Vol. XXXV., p. 671; also "On a New Standard of Illumination and the Measurement of Light," Sir W. H. Preece, *Proc. Roy. Soc.*, Vol. XXXVI., p. 270.

Sir W. H. Preece and Mr. Trotter consists of a wooden box having on the upper surface a flat white card disc with a star-shaped hole in it. This screen is illuminated on the upper surface by the external illuminants. Looking through the hole, the observer sees the surface of a sheet of white card, which is itself illuminated by the light of a small glow-lamp contained in the box. The card or reflecting surface

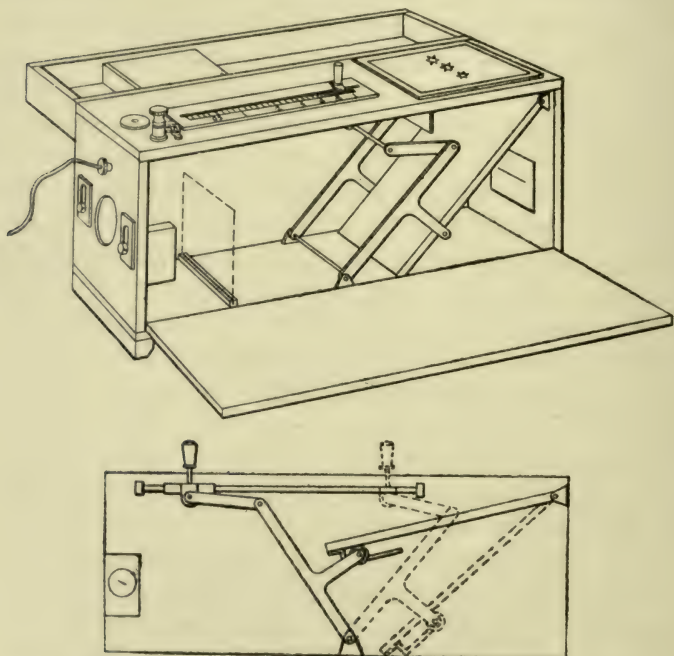


FIG. 22.—Preece and Trotter Illumination Photometer.

(see Fig. 22) is tilted at various angles by means of a system of levers, and its brightness is therefore varied, and hence the amount of illumination which it exhibits is in contrast to that of the upper card or photometer disc. By means of an ingenious cam the inventors were able to calibrate the instrument so as to read off at once the illumination on

this upper surface in candle-feet. The experimental measurement consisted in altering the angle of inclination of the inclined card reflector until, on looking at the photometric disc on the upper surface, the star disappeared. The observer therefore matched the illumination on the upper surface of this last-mentioned card by varying the illumination on the surface of the card underneath, which was inclined at various angles to the rays from the lamp so as to alter its brightness. With this arrangement Mr. Trotter made a large number of experimental observations on the distribution of illumination in London streets, and recorded the results in the form of curves and contour lines.\*

Another form of illumination photometer is that devised by Prof. L. Weber, which consists of a horizontal tube A (*see* Fig. 23). In this is placed a movable disc of opal glass which can be moved parallel to the axis and its position determined by a scale. At one of the extremities of the tube A is a second tube, B, fixed at right angles, provided also with a similar translucent glass disc. Inside this last tube is a totally-reflecting prism of the Lummer-Brodhun type. A standard lamp is placed at the extremity of the horizontal tube so as to illuminate one of the opal glass discs to a degree which is determined by its distance from this standard lamp. The illumination to be compared falls upon the disc at the end of the other tube, and the illuminations of these two semi-transparent discs are then compared by means of the Lummer-Brodhun prism. The light proceeding from the underneath side of one disc can be weakened by the interposition of a pair of crossed Nicol prisms which are placed in the tube, which provides an alternative method to moving the semi-transparent disc in the horizontal tube. In this manner light passing through the Lummer-Brodhun prism can be made to be identical with that reflected from the hypotheneuse side of one portion of the prism. The Weber

\* "The Distribution and Measurement of Illumination" by A. P. Trotter. *Proc. Inst. Civil Eng.*, Vol. CX., Part 4, 1891-92.

photometer can be used like a Bunsen photometer to determine the ratio of the luminous intensity of two lights or that proceeding from an arc light at any angle. The chief merit of the instrument, however, lies in its ability to measure the diffused light in a space so illuminated, in a simple and easy manner, for which purpose its portability is a great advantage. It reads the degree of illumination on any surface within an

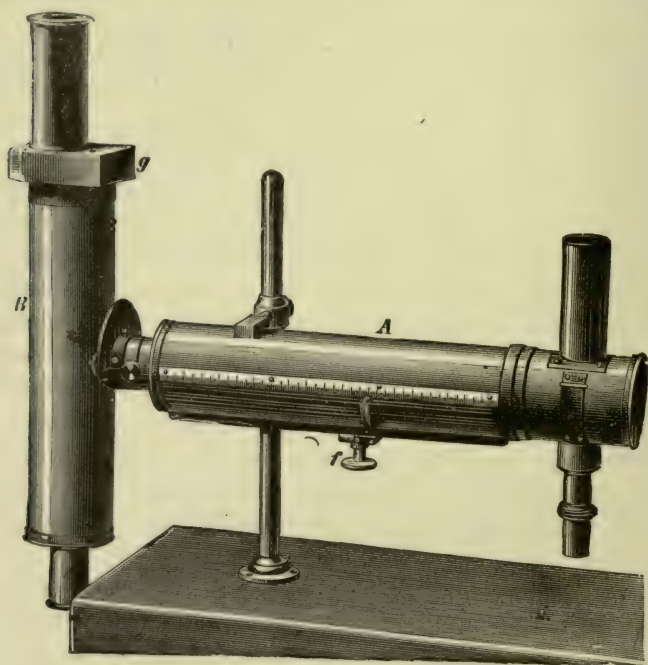


FIG. 23.—Weber's Illumination Photometer.

illuminated space directly in bougie-metres or any other unit. The comparison light is generally either a standard candle or a benzene lamp with flame control, but owing to the confined space in which the flame is placed, errors are liable to arise from variation of the flame standard, unless care is taken to calibrate it at intervals against some reliable

external standard. The instrument is generally provided with several opal glass discs of various grades of transparency, which are used in measuring the illuminations of different degrees.

§ 7. **The Measurement of Luminous Efficiency.**—The energy of every source of light is radiated partly in the form of radiation which affects the eye as light and partly in the form of radiation which does not affect the eye, and is called invisible radiation. The *luminous efficiency* of any source of light is the ratio between the energy represented by the luminous or eye-affecting radiation and that of the total radiation. In all artificial sources of light this efficiency is a very small number. It can be determined by several methods. One method adopted by Mr. Ernest Merritt\* to determine the luminous efficiency of an incandescent lamp was to place the lamp in a glass calorimeter, through which a constant flow of water was maintained, the temperature of the water entering and leaving being determined by means of thermometers. When the lamp was rendered incandescent, the water was discharged at a higher temperature than that at which it entered, and the difference of temperature, combined with the known flow of water through the calorimeter, enabled the amount of energy removed from the lamp to be determined, proper correction for rate of loss of heat by radiation being applied, also correction being applied for the absorption of the light by the water and the incomplete opacity of the latter for heat rays. In this manner the total expenditure of energy in the lamp in producing invisible radiation was determined. The energy of total radiation was determined in the same way by employing a calorimeter vessel made of copper. The difference between these two observations gave the energy radiated by the lamp as light, and the ratio of this to the total radiation fixed the luminous efficiency of the lamp. The result was found to be rather

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\* *American Journal of Science*, Vol. XXXVII., p. 167.

under 5 per cent. when the lamp was working at normal candle-power. When the 16 c.p. lamp employed in these experiments was worked respectively at 4, 10, 16, 22 and 28 candles, the ratio of candle-power to total power absorbed, and the ratio of luminous radiation to total radiation, were as shown in the following table :—

State of Incandescence equivalent to that of an Edison 16 c.p. lamp, giving a light of	Relative Efficiency referred to that of a 16 c.p. lamp as unity.	
	Ratio of Candle-Power to Watts.	Ratio of Luminous Radiation to Total Radiation.
4 Candles .....	0·394	0·211
10   " .....	0·729	0·632
16   " .....	1·000	1·000
22   " .....	1·218	1·368
28   " .....	1·391	1·589

In carrying out a measurement of luminous efficiency by the above method we have to measure the weight,  $w$ , of water which flows through the calorimeter, entering at a temperature  $T_1$  and leaving it at a slightly lower temperature  $T_2$ . The heat removed is then  $w(T_2 - T_1)$ , leaving out of account a small correction for the variation in the specific heat of water. The heat lost by radiation at the temperature  $\frac{1}{2}(T_2 + T_1)$  has to be determined by a cooling curve. The calorimeter filled with water at a slightly higher temperature is allowed to cool and its temperature is taken at various intervals of time close together. From this we can determine the rate at which the calorimeter loses heat by radiation and convection. If the weight of water at any moment in the calorimeter is  $W$ , and if it cools through an interval of temperature  $d\theta$  in a small time  $dt$ , the rate of loss of heat is  $W \frac{d\theta}{dt}$ . Hence, if the water enters the calorimeter at a temperature  $T$  and leaves it at a slightly lower temperature  $T - \delta T$ , and if in a time  $\delta t$  a mass of water equal to  $w$  flows through the calorimeter, the rate at which heat is

being removed by the flow of water is  $w \frac{\delta T}{\delta t}$ , and the rate at which it is being removed by cooling is  $W \left( \frac{dT}{dt} \right)_T$ , where the last symbol signifies the rate at which the calorimeter loses temperature at the temperature  $T$ . Hence the total rate at which heat is being removed is equal to

$$W \frac{\delta T}{\delta t} + w \left( \frac{dT}{dt} \right)_T.$$

This must, therefore, be equal to the rate at which heat is being given to the water. If we employ, in the first place, a copper calorimeter, and place in it an incandescent lamp taking a current of  $A$  amperes at a voltage  $V$ , then the rate at which energy is given to the lamp is  $AV$  watts or  $AV$  joules per second, or  $10^1 AV$  ergs per second. Since  $42 \cdot 10^6$  ergs are the equivalent of one gramme-degree Centigrade of heat, the whole energy given to the lamp per second, reckoned in heat units or calories, is  $\frac{10}{42} AV = 0.24AV$ .

If, then, we make the measurements of weight of water and temperature as described, in a copper calorimeter, we ought to find that

$$\frac{10}{42} AV = W \frac{\delta T}{\delta t} + w \left( \frac{dT}{dt} \right)_T,$$

where  $W$  and  $w$  are measured in grammes and  $\delta T$  and  $dT$  in degrees Centigrade. If this relation is satisfied, it shows that the arrangements are in order.

We then make a similar experiment with a glass calorimeter, and we find that the quantity  $W \frac{\delta T}{\delta t} + w \left( \frac{dT}{dt} \right)_T$  is now less than before. The difference is due to the energy which escapes as light. Some light rays are, however, absorbed by the water, and an experiment has therefore to be made, photometering the light of the lamp both in the glass calorimeter when immersed in water, and again when outside, to determine this loss of light. The loss may amount to 20 to 25 per cent.

Let us suppose that the measurement of the light of the lamp shows it to be  $I$  when outside and  $I'$  when inside the calorimeter, then  $I - I'$  is loss in candle-power due to the absorption by the water and the glass. If, then, the readings as above described are taken, the luminous efficiency ( $E$ ) of the lamp is given by the expression

$$E = \frac{\left\{ 0.24AV - \left( W \frac{\delta T}{\delta t} + w \left( \frac{dT}{dt} \right)_T \right) \right\} I}{0.24AV I'},$$

where the temperature changes  $\delta T$  and  $dT$  refer to the experiment with the glass calorimeter.

Another method by which the luminous efficiency can be determined is to employ a thermopile and galvanometer to measure the total radiation emitted by the lamp. By the interposition of a glass cell containing a solution of alum, the whole of the invisible radiation can be cut off, and a second measurement made, which, when corrected for the loss in luminous intensity by the absorption of light by the cell, gives a deflection on the galvanometer which is a measure of the luminous radiation. The ratio obtained by Mr. Merritt in his researches when employing this last method agreed well with that obtained by the calorimeter method.

Similar experiments made with an arc lamp by Mr. H. Nankano, making observations on the rays emitted at different angles below the horizon, gave the following figures for the ratio of luminous to total radiation for an open arc lamp:—

Angle below the horizon.	Efficiency = $\frac{\text{Luminous Radiation}}{\text{Total Radiation}}$ .
0°	0.0548
10°	0.0901
20°	0.1228
30°	0.1506
40°	0.1552
50°	0.1059
60°	0.0676
63°	0.0942

The integration of the above values gave a mean luminous efficiency of the arc lamp in question of 10.4 per cent.

§ 8. **Photometer Rooms and Photometers.**—In the comparison of two illuminants by means of a photometer, it is, of course, necessary that all external light should be excluded. Hence the operations have to be conducted in a special room or chamber, which is called a photometer room, or photometric gallery. In many cases this takes the form of a long box blackened in the interior, having the two lights to be compared at the two extremities, and some photometer arrangement sliding on a bar, called the photometer bar, between them. An arrangement of this kind is called a closed photometer, but it possesses many defects and should be avoided when possible. Generally speaking, it is assumed that a photometer can be fitted up on any shelf by hanging in front and behind black velveteen curtains. An arrangement of this kind is open to the objection that rays of light are reflected obliquely from either the blackened interior of the box or from the velveteen curtains, and so reach the photometer by paths which are other than a straight line drawn from the illuminant to the photometer disc. The principle on which all intensity photometry is conducted is that no light must reach the photometer disc except that which comes in straight lines from the illuminant, because it is only under these circumstances that the brightness on the photometer disc varies as the square of the distance of the illuminant. If there is anything of the nature of stray reflection from the sides of the photometer gallery or box, or from any curtains, then the law of the squares is invalidated. No matter how dead-black a surface may be, it reflects a considerable amount of light at oblique incidence. This can be seen by holding in the hand a small piece of the blackest cloth or velvet, and allowing rays of light from a lamp to fall upon it and reach the eye. At very oblique incidences it will be seen that there is a considerable amount of reflected light; hence it is desirable that the walls or sides of the photometer gallery should be at some distance from the lights compared and from the photometer itself.

An arrangement of two or three baffle screens, consisting of large blackened cards with holes in them, may with advantage be placed with their planes perpendicular to the photometer bar. The best arrangement for electric incandescent lamp photometry is to equip a room, which should not be less than 20ft. in length, 6ft. or 7ft. in width, and 8ft. or 9ft. high, with the necessary appliances, the walls of this room being painted dead-black. If the standard light is to be a flame standard, then it is absolutely essential that the greatest attention should be paid to the ventilation of this room, otherwise great errors may occur owing to the accumulation in it of moisture and carbonic acid, which vitiate the atmosphere and reduce the illuminating power of the standard. The room ought to be ventilated by air drawn in from the outside, which, as far as possible, is kept at a constant temperature. This air should be filtered so as to free it from dust or vapours, if the photometer room is in a large town or in connection with a factory, and the air should be drawn through the room by means of a small fan driven by an electric motor, so as to make a slow, steady current of air through the room, but not to produce a draught. It is essential that a continual current of fresh air should move upwards round the flame standard if it is to be preserved at its normal intensity. All stray light should, of course, be excluded from the room, but the comfort of the operator is greatly increased if the room is well-ventilated and also kept at a constant temperature. This latter desideratum is most easily achieved if the room can be electrically heated, as then the operator can always preserve a temperature at or very near to 15°C. This is the more necessary if electrical instruments, and especially standard cells, are used in the room. It is desirable that the photometer room should be electrically lighted, so that the operator has complete control over the illumination.

The general arrangements of the room should then be as follows (*see* Fig. 24): Down the centre should run a wooden

railway, consisting of a pair of beams carried on stout supports on which can travel easily wooden slabs or tables

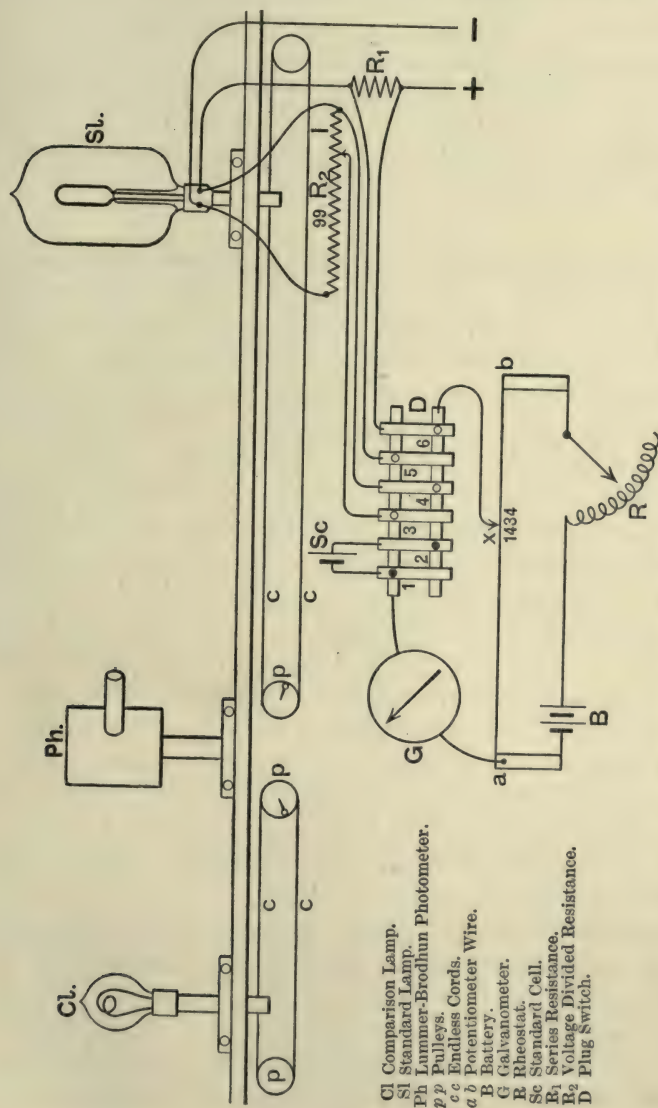


Fig. 24.—Arrangements of Standard Photometer Bench and Potentiometer.

holding the lamp to be tested, the standard lamp, and the photometer, the height of this railway being such as to bring the photometer telescope or eye-piece to a level convenient for the eyes of persons of ordinary height when standing. Underneath this railway or gangway the various resistances can be placed which are employed to control the currents used, and on a narrow shelf or table in front of it can be placed the necessary electrical instruments for reading the currents and voltages.

**§ 9. The Photometry of Incandescence Electric Lamps.—**

The arrangements which the Author has found, after a long experience, to be most suitable for the photometry of glow lamps are as follows:—The photometer employed is the Lummer-Brodhun contrast photometer with totally-reflecting prisms at the sides of the photometer box. This photometer is kept fixed in one position on the railway of the photometer room, and has on one side the lamp to be compared and on the other side the standard. These two lamps are carried on sliding tables which are moved to and fro by means of endless cords running over pulleys (*see* Fig. 24). On the left-hand side of the table is an incandescent lamp which is called the comparison lamp, and by means of appropriate resistances this lamp can be adjusted accurately for voltage, and, therefore, candle-power. This electromotive force must be provided from large secondary batteries, if possible, so as to secure steadiness. If for any reason it is impossible to do this, then the comparison lamp and lamp to be tested must be worked in parallel off the supply mains coming straight from a single dynamo set apart for the work.

The sliding table on the right-hand side carries a socket in which can be placed either a standard lamp or the lamp to be tested. This socket is carried on an universal joint, so that the lamp can be placed with its axis in any required direction (*see* Fig. 25). Under the railway are placed the resistances and the controlling handles for regulating the

current and the voltage of the lamp to be tested or the standard lamp, and for this purpose from the socket holding the above lamp there come two pairs of leads, one the current leads and the other the potential leads.

For measuring the electrical quantities no instrument is so satisfactory as the direct-reading potentiometer. Out of a large experience the Author can say that no ammeter or voltmeter yet made is sufficiently accurate for electric lamp

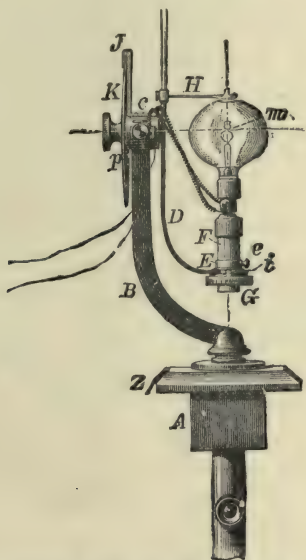


FIG. 25.—Universal Joint for Carrying the Incandescent Lamp on the Photometer Bench.

photometry. It was with this object that the Author introduced, as far back as 1888, the direct-reading potentiometer, set by a Clark cell, which has been since transformed into a very perfect instrument for commercial work by Messrs. Crompton & Co. and other firms of instrument makers. Employing a Crompton potentiometer, readings of the current and voltage of incandescent lamps can be taken quite as quickly in the photometer room as by the use of any ordinary

ammeter or voltmeter, and with an accuracy which is far greater. The only point to which attention need be drawn is that, the temperature variation of the Clark cell being considerable, it is better to use a Weston cadmium cell, or Helmholtz calomel cell, in place of the Clark cell.

The operation of making measurements by means of the Author's large bulb standard glow lamps is as follows:—A standard lamp is selected, giving, say, 16 candles at 96 volts in a certain direction. This standard lamp is placed in the testing socket with its axis upright, and set at a distance of 4ft. from the photometer disc, so as to produce on the disc an illumination of one candle-foot. The distance of the comparison lamp is then varied until the photometric balance is obtained. The standard lamp is then removed from the testing socket, and the lamp to be tested placed therein, and its distance varied until a photometric balance is again obtained. From the relative distances of the tested lamp and the standard the luminous intensity of the former is determined in terms of the latter; for the candle-power of the lamp under test is then numerically equal to the square root of its distance in feet from the photometer disc. The railway bar can, of course, be calibrated to show at once candle-power. It will be seen that this process is a form of *double weighing*. The exact candle-power of the comparison lamp does not matter so long as it remains constant during the experiment. In some forms of photometer bench, as in the Reichsanstalt pattern (see Fig. 26), the Lummer-Brodhun photometer is shifted between the two lights, which are kept fixed. There is, however, a decided advantage in keeping the illumination on the photometer screen constant.

In making a series of tests of incandescent lamps it is desirable to check the setting of the comparison lamp by means of a large bulb standard at intervals, just as the setting of the potentiometer is checked at intervals by means of the standard cell. In making photometric examinations of incandescent lamps it is, of course, necessary to take the

candle-power in different directions. In order to eliminate the variation in candle-power which exists in different hori-

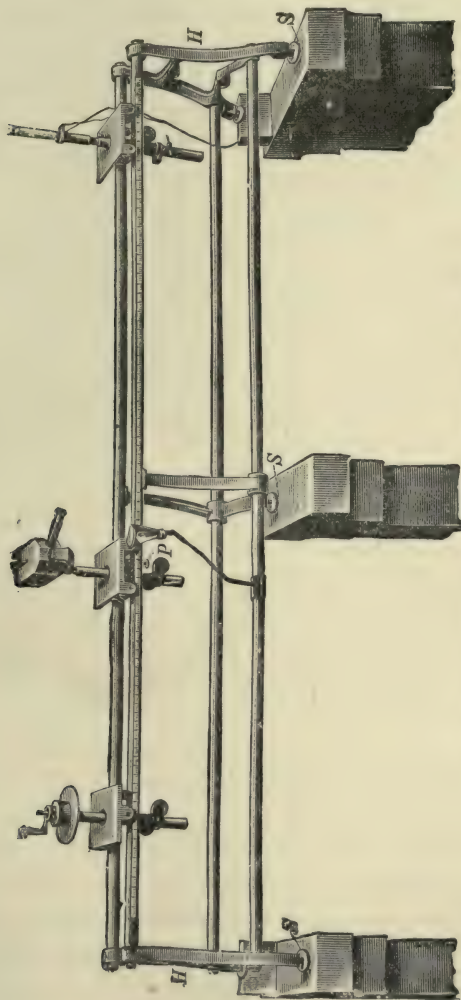


FIG. 26.—Reichsanstalt pattern of Photometer Bench.

zontal directions when the axis of the tested lamp is vertical, a committee of the American Institute of Electrical Engineers

recommended that the lamp under test should be revolved on its vertical axis at the rate of 180 revolutions per minute. This is not difficult with stiff filaments, but with the long high-voltage filaments now used there is a risk of breaking the filament or forcing it against the bulb of the lamp if the speed of revolution exceeds about 3 per second, and this is hardly sufficient to eliminate all flickering. In any case the maximum horizontal and minimum horizontal candle-power should be taken, and also the candle-power in the direction of the axis of the lamp. For certain purposes it may be necessary to take the mean spherical candle-power. The usual process is to be content with taking the maximum candle-power in a horizontal direction, but since by far the larger number of lamps are hung head downwards in use, this value alone does not give sufficient information as to the performance of the lamp, and the candle-power in the above-stated three directions should always be furnished. There can be no question, however, that the only method of rating incandescent lamps is to measure the mean spherical candle-power and deduce from it the total flux of light per watt.

In the actual photometric measurements it is desirable to oscillate or move one of the lamps on a plan recommended by Sir W. de W. Abney. If the lamp under test is moved to and from the photometer in gradually diminishing arcs, it is easier to determine the exact position of balance than if this is not done. One advantage of the above-described method is that the comparison lamp can be adjusted to work as nearly as possible at the same watts per candle as the lamps under test. This can be easily arranged when using the Lummer-Brodhun photometer, as it is very sensitive to small differences in the spectral quality of lights compared.

The measurements which have to be made in the case of every incandescent lamp, in order to determine its value as an energy transforming device, are :—

1. The current, taken when the lamp is working at its marked volts;

2. The candle-power in the three principal directions—*i.e.*, the maximum and minimum and mean candle-powers or luminous intensities in a horizontal direction when the axis of the lamp is vertical; and
3. The mean spherical as well as the mean horizontal candle-power.

From the current and voltage we calculate the total power taken by the lamp in watts, and from this figure and the observed maximum candle-power the watts per candle-power. This last number is generally called, though not quite correctly, the efficiency of the lamp, lamps being called *high-efficiency lamps* if they take less than 3 watts per candle and *low-efficiency lamps* if they take more than 3 watts per candle.\* If the light is very unsymmetrically distributed, then it becomes necessary to take the mean spherical candle-power and to calculate out the mean spherical watts per candle-power. The method of determining this mean spherical candle-power when the distribution of light is symmetrical with regard to the vertical axis is given in the following section. In addition to these measurements, some tests should always be applied to determine the *ageing* of the lamp, or the deterioration in its light-giving quality when worked at constant voltage. The changes in the illuminating power of incandescent lamps, under the above circumstances, take place in consequence of

1. Changes in the resistance or surface of the filament.
2. The deposit of carbon upon the interior of the glass bulb, called the blackening of the lamp.

In the case of most incandescent lamps, it is found that these, when run at the marked voltage, begin by increasing in candle-power. This is due to the diminution in resistance of the filament by the initial incandescence; the candle-power

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\* Properly speaking, this so-called efficiency should be reckoned in *lumens per watt*, as the scientific expression for the flux of light for a given expenditure of power in the lamp.

therefore rises up to a maximum, but, owing to the blackening of the bulb and loss of surface of the carbon filament, it then gradually falls again, and a curve of candle-power taken over the whole life of the lamp would be of the general form shown in Fig. 27.

It is customary to consider that the efficient working life of the lamp is over when the candle-power has fallen 20 per cent. below its initial value ; hence, if we draw a horizontal line across the candle-power diagram at this height, the practical value of the lamp is measured by the area of that part of the curve which is included between the initial ordinate and the ordinate passing through that point on the

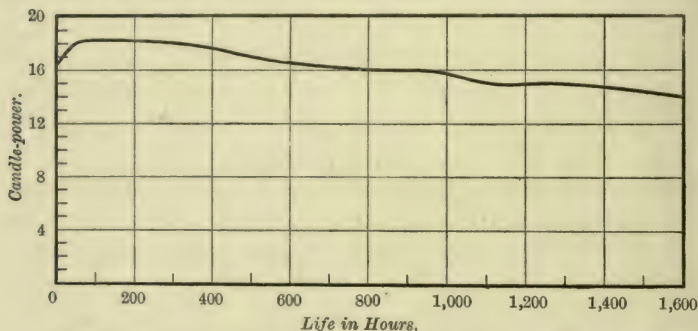


FIG. 27.—Typical Curve of Variation of Candle Power with Age, of a 16 c.p. 100-volt Incandescent Lamp.

candle-power curve where it is intersected by the above-mentioned horizontal line (see Fig. 27). The commercial value of the lamp may be considered to be fairly given by the quotient of those effective lamp-hours or candle-power-hours represented by the above-mentioned area, by the cost of the lamp.

Any test of an incandescent lamp, to be valuable, should include an ageing test of the above kind, but such an ageing test applied to a single lamp is not of any value. The only numerical result of any importance is one which enunciates the results of such tests upon a fairly selected number of

samples of a given batch. Thus, for instance, if 100 lamps are purchased from a maker, and if 50 of these lamps are taken fairly at random from the batch and photometered and measured as above, the commercial value of this type of lamp may be fairly estimated by the mean value of the tests made for watts per candle-power and current and effective candle-power-hours, and in addition by the deviation of the first three mentioned quantities from the mean value. As incandescent lamps of any given nominal candle-power and voltage should be, as far as possible, identical, it is necessary to state how far the observed values differ from the marked value and from the mean value. Thus, for instance, of the 50 sample lamps selected, if we suppose the marked candle-power to be 16 at 100 volts, the marked current to be 0·56 amperes, and the marked watts per candle-power to be 3·5, then the results of the measurements of the 50 lamps will give values which not only differ from the mean of the 50, but differ from the marked value. These results can be set out graphically as follows:—Take a straight line on which is marked off lengths proportional to the candle-power of the various lamps measured from a common origin. This will give the number of points distributed along the line in proximity to one another. Take the centre of gravity of all these points, which is equivalent to taking the mean value of the candle-power, and mark that point on the line by a cross. Mark also by a transverse mark the position corresponding to the nominal candle-power—viz., 16—and the diagram then represents to the eye, in a simple and explicit manner, how far the actual performance of the lamp agrees or disagrees with its professed value. The results as to watts per candle and current may be set down in the same manner; all these measurements being made, of course, at the marked voltage.

The ageing of the lamp can, of course, only be indicated by a curve of candle-power corresponding to various periods of its life. Thus the candle-power may be taken at the end of 100, 200, 300 hours, &c., and the candle-power curve

delineated. The practical difficulty of these ageing tests is that they take a prolonged period of time, and no attempt to hurry them by running the lamp above its marked volts can be relied upon as giving an indication of the performance of the lamp in ordinary use. The examination of a batch of lamps should also include tests applied to a certain selected number to see how far the lamps will bear variation in voltage to the extent of 10 per cent. above the marked volts

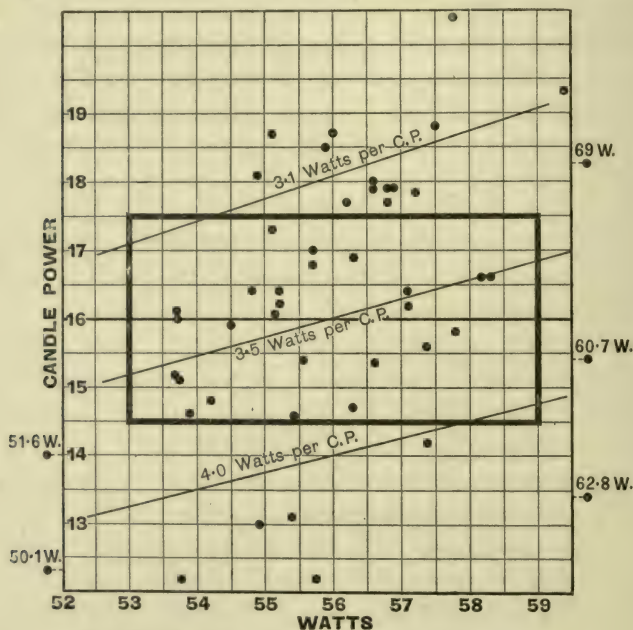


FIG. 28.—Target Diagram (Badly-Classified Lamps).

without breaking. This is to supply an indication of the probable behaviour of the lamps when worked on public supply circuits in which the pressure is very irregular.

The appraisalment of any particular type of incandescent lamp can only be achieved by tests in which the statistical method is applied to a sufficient number of lamps to eliminate the individual variations. At the same time an important

fact to ascertain is how far the lamps made by any maker, and marked as of the same kind, are really the same in actual value. If, for instance, we are purchasing 16 candle-power 200-volt lamps with an efficiency of 3·5 watts per candle-power, we require to know how far the lamps purchased do, on the whole, comply with these statements. In dealing with the evaluation of lamps it is now customary in the United States to employ the method called the *target diagram*.

If, for instance, we desire to arrive at a correct judgment as to the real value of any particular type of lamp, it is

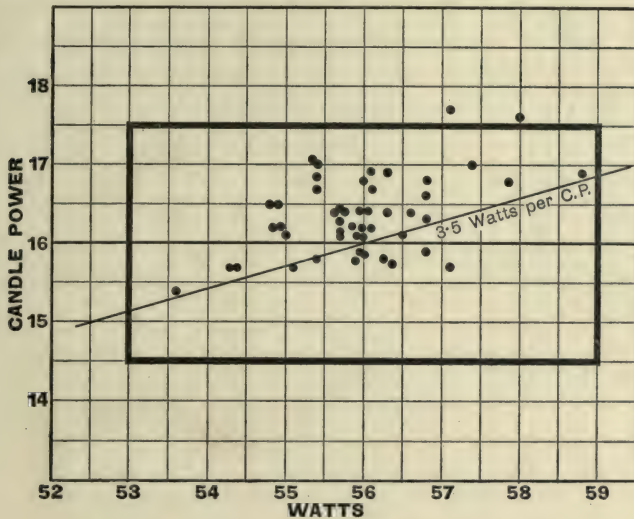


FIG. 29.—Target Diagram (Well-Classified Lamps).

necessary, in the first place, to secure a fair sample in sufficient numbers. It is no use to write to the makers for samples, because they may, of course, supply picked specimens. The best method to adopt is to instruct several persons to purchase independently small batches of lamps of the required kinds, say two dozen, without allowing the manufacturer to know that they are for test. When 100 lamps of the same make and kind have in this manner been

purchased, they may be looked through for defective lamps and wasters, and a fair sample of 50 tested for current and horizontal candle-power at the marked voltage. These observations are then set out on squared paper as follows:—Let



FIG. 30.—Life Curves (Badly-Classified Lamps).

us suppose the lamps are nominally 16 c.p. 56-watt lamps, then on the squared paper vertical distances are taken for candle-power and horizontal distances for wattage, and a dot made on the diagram corresponding in position to the

observed candle-power and watts of each lamp. A rectangular target is then drawn (*see* Fig. 28), of which the horizontal length corresponds with the length taken for 6 watts and the vertical height with the length taken for 3 c.p., and the centre of the rectangle made to coincide with the point corresponding to 16 c.p. and 56 watts. We can then draw inclined lines corresponding to 3.0, 3.5 and 4 watts per candle-

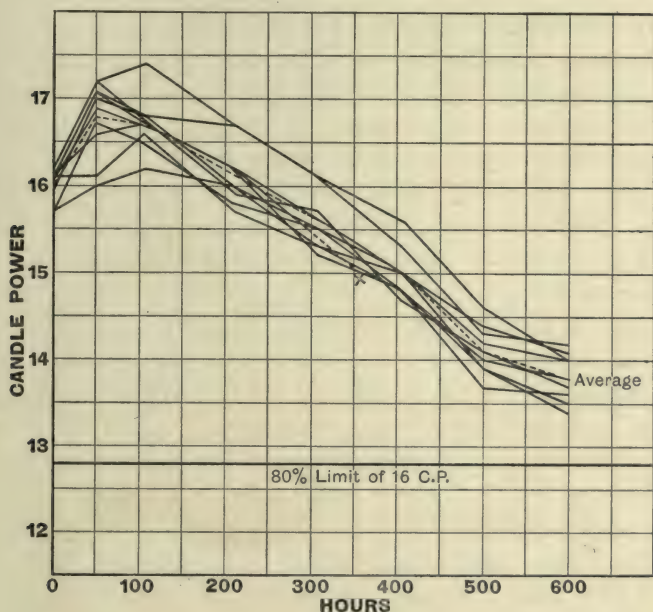


FIG. 31.—Life Curves (Well Classified Lamps).

power, and can count the number of dots which fall within the target, and estimate their distribution with respect to the bull's-eye represented by the point 16 c.p. 56 watts.

When such a diagram is made for well-classified lamps it will be found that 90 per cent. of the dots fall within the target, but for badly-classified lamps a very large percentage, or even the whole of the dots, may fall outside (*see* Figs. 28

and 29). The target test therefore decides how far the lamps are what they profess to be, and gives a discriminating figure.

In addition to this, a series of life curves should be plotted on the same sheet of paper for a considerable number of lamps, to ascertain if the life behaviour of various lamps is alike or very different. Well-made lamps should give life curves which are not very different (*see* Figs. 30 and 31).

### § 10. The Determination of Mean Spherical Candle-power.

—Although it may be necessary for many purposes to know the candle-power of a lamp in given directions, the most important characteristic constant is the *mean spherical luminous intensity* or *mean spherical candle-power*.

If we suppose a lamp rotating on its vertical axis to be placed in the centre of a spherical room, the walls of which are painted white, then an inspection of the room when the lamp was in operation would show that the illumination on the walls of the sphere was distributed uniformly along the lines of latitude, but was non-uniform in passing from one latitude to another. In other words, it is symmetrical with what may be called the polar axis of the room. The illumination at any point on the wall is measured by the quotient of luminous intensity of the lamp in that direction by the square of the radius  $R$  of the spherical room. In other words, it is equal to the quotient of the flux of light in that direction by the element of surface considered. We may therefore imagine the lamp replaced by an illuminant which sends out its light equal in all directions, the intensity or luminous flux being so adjusted that the whole flux of light from this uniform source is equal to the whole flux of light from the actual lamp. The constant luminous intensity of this imaginary source would then be considered to be equal to the *mean spherical luminous intensity* of the lamp. Let  $I_0$  be this mean spherical luminous intensity; then this quantity is defined by the consideration that  $4\pi R^2$  multiplied by  $I_0$  is equal to the sum of the products of

the actual luminous intensity  $I$  in any direction and the area of an elementary zone of the sphere corresponding to that direction. If the light emitted by the lamp is symmetrical with respect to a vertical axis, we can obtain the mean spherical candle-power (M.S.C.P.) by the following geometrical construction, which is due to M. Rousseau\* :—

Let  $A$  be the source of light, and let the polar curve of candle-power  $VDB$  (see Fig. 32), as determined experimentally, be set off round it. This curve is one the radii of which are proportional to the candle-power in that direction. Let  $GEF$  be a semicircle just touching this polar curve at  $O$ . On the other side of the line  $FG$  and on the base  $FG$  describe a rectangle  $FKHG$ , of which the side  $KF$  is equal to the maximum radius vector  $AO$  of the polar curve.

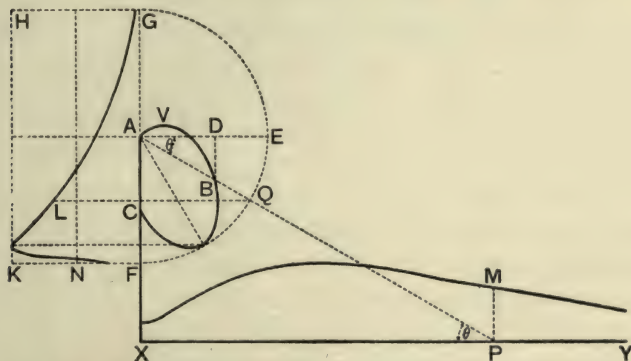


FIG. 32.—Polar Curve of Luminous Intensity and Rousseau Curve.

Draw the horizontal line through  $A$ , and draw a line,  $BC$ , horizontally through  $B$  where  $AB$  is any radius of the polar curve. On  $BC$  produced set off  $CL$  equal to  $AB$ . If other radii of the polar curve are drawn and the same construction followed out, then the extremities of all the lines similar to  $CL$  will define a curve  $GLF$ , called the Rousseau curve, which has the property that its mean ordinate  $FN$  is the "mean spherical candle-power of the lamp."

The proof of this construction is as follows :—Let the angle  $DAB$  be denoted by  $\theta$ . Let  $I$  be the luminous intensity of the lamp in the direction  $AB$ . Hence  $AB = CL = I$ . Let the radius  $AF$  of the semicircle be denoted by  $R$ . Then, if we consider the angle  $\theta$  to increase by an increment  $\delta\theta$ , the corresponding increment of the arc of the semicircle is  $Rd\theta$ . If the whole diagram be supposed to revolve round  $GF$  as an axis, then the area of the zone swept out by this elementary arc is  $2\pi R^2 \cos \theta d\theta$  and the whole light

\* See *Comptes Rendus*, "Des Essais Photométriques à l'Exposition d'Anvers, 1885."

falling upon it is  $2\pi R^2 I \cos \theta d\theta$ . If we call  $I_0$  the mean spherical luminous intensity, then we have  $I_0$  defined by the relation

$$4\pi R^2 I_0 = 2\pi R^2 \int_0^\pi I \cos \theta d\theta,$$

or

$$2RI_0 = \int_0^\pi R d\theta \cdot \cos \theta \cdot I.$$

The right-hand side of this last equation is the area of the Rousseau curve, and the left-hand side is the area of the rectangle GF . FN. Hence FN represents to scale the mean spherical luminous intensity.

To determine the *polar curve of luminous intensity* for any source of light of which the candle-power is symmetrical with respect to any axis, various devices are used. Thus, for arc lamp photometry photometers have been invented which can be tilted up at different angles like a telescope, and directed to an arc lamp suspended at a suitable height, the arc being capable of being raised or lowered. None of these devices, however, are very convenient, because they involve constantly re-measuring the distance from the arc. Accordingly, after a large experience of this matter, the Author has devised the following arrangement, which is exceedingly convenient:—The arc lamp to be photometered is suspended from a wooden frame or gallows at a convenient height. This frame has two hollow trunnions (*see* Fig 33) on which swings a rectangular frame capable of being tilted at any angle, the exact position of the frame with reference to the horizon being indicated by a scale of degrees and a pointer. This frame carries three plane mirrors placed at angles of 45 degrees, which catch the ray proceeding from the arc in any inclined direction and reflect it down one of the hollow trunnions. As the electric arc is in constant motion, producing a variation of the light in any direction, it is best not to make a comparison between the selected ray and any other fixed standard illuminant, but to make a comparison between this ray and the ray sent off from the arc in some other direction. For this purpose three other plane mirrors are placed at angles of 45 degrees which reflect a ray coming off from the arc, say, at 45 degrees below the horizon and

conduct it round so as to be in the same line, but oppositely directed to the inclined ray. We have then, as it were, drawn off from the arc two rays, one in a horizontal direction, and the other in the same vertical plane but inclined at an angle  $\theta$  to this horizontal ray, and have brought these rays round into positions in which they can be compared together on any

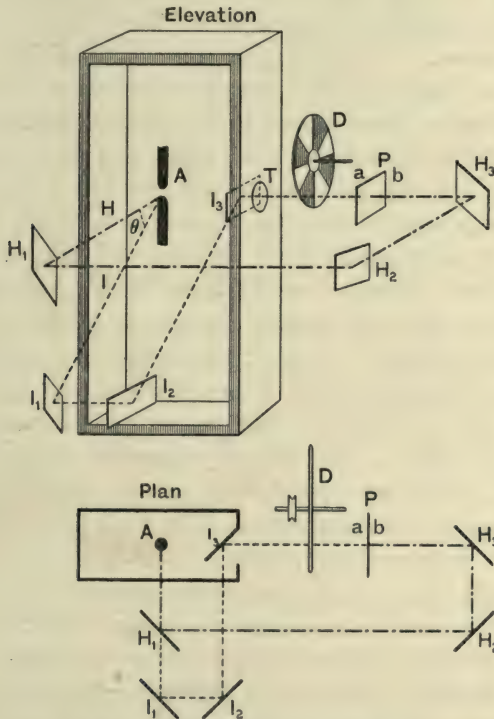


FIG. 33.—Arc Lamp Photometer (Fleming).

A Arc Lamps.  
H<sub>1</sub> H<sub>2</sub> H<sub>3</sub> I<sub>1</sub> I<sub>2</sub> I<sub>3</sub> Mirrors.

P Photometer Disc.  
D Fox-Talbot Sector Disc.

ordinary photometer, such as the Bunsen disc. Each of these rays is, of course, weakened by reflection, but, since both the rays are reflected three times at a constant incidence of 45 degrees, they are weakened in the same proportion.

In order to compare the rays conveniently together, one of them, namely, the stronger, may be weakened by the interposition of a pair of rotating sectors, or another device may be adopted. In front of the first mirror may be placed a lens of such focal length as to convert the diverging beam from the arc into a parallel beam, and a lens of this description is placed to catch both inclined and horizontal rays. The two beams, therefore, which come off the two last mirrors are parallel beams. If we interpose in the path of these parallel beams two double concave lenses of the same focal length, we again cause these beams to become divergent, and if a Bunsen disc is placed between these two last lenses and moved to and fro, a position will be found in which a photometric balance can be obtained between the light coming off horizontally from the arc and that coming off in an inclined direction, both of them having been weakened in the same ratio by passing through a similar system of lenses and mirrors. In fact, the apparatus separates out from the arc two sets of rays, one of which has an intensity equal to that of the horizontal intensity of the arc, and the other one equal to the inclined intensity, and places the sources of these rays respectively at the principal foci of the two last double concave lenses.

We can in this manner obtain the ratio between the radii of the polar curve of luminous intensity and the horizontal luminous intensity. An observation may then be taken by any independent means of the mean luminous intensity in the standard direction of the arc, employing for this purpose any suitable photometer, and we are at once able to set out the polar curve in terms of the selected unit of luminous intensity. In making this photometric comparison, the most convenient standard of reference to employ is an incandescent lamp, which is worked at high efficiency—*i.e.*, at about  $2\frac{1}{2}$  or  $2\frac{3}{4}$  watts per candle-power. A glow lamp at this temperature can be compared without much difficulty, when the horizontal light from an arc and the

incandescent lamp can itself be photometered against a standard glow lamp in the way already described.

In any case, therefore, in which the distribution of light from a source is symmetrical with respect to a vertical axis, as it is in the case of an arc lamp, we can determine the mean spherical candle-power by the process of measuring the candle-power in various directions at known angular distances above and below the horizon, and then setting off the results in the form of a Rousseau curve.

If the source of light is an incandescent lamp, the light of which is not symmetrical with respect to any axis, then we can only obtain a polar curve of candle-power by causing the lamp to rotate on its vertical axis at the rate of about 180 revolutions per minute, and at the same time taking the candle-power in different direction with respect to the vertical axis of the lamp.

The tedious nature of these observations, and the special difficulties which interfere with accurate measurements in the case of the arc lamp, owing to the incessant variation of the light, have led to many attempts to measure by one observation the mean spherical candle-power. One of the most ingenious of these is that due to Mr. C. P. Matthews\*, called an *integrating photometer*. The principle of this appliance is as follows:—

Referring again to Fig. 33, let A be an arc lamp of which the mean spherical candle-power is required. Suppose two mirrors,  $I_1 I_2$ , fixed to a rocking frame so as to reflect the ray proceeding from the lamp at an angle,  $\theta$ , below the horizon back again parallel to its original direction.

At a point, T, on the horizontal line through the arc let the ray  $I_2 I_3$  be received on a vertical white surface, such as a plaster of paris surface, which obeys Lambert's law. Suppose the intensity of the ray leaving the arc in this direction to be denoted by the symbol  $I_\theta$ . Let the distance  $AI_1 I_2 I_3$  be called D, and let K be the coefficient of reflection of the mirrors at an angle of 45deg. Then the illumination on the vertical white screen due to this ray is equal to

$$K^2 \frac{I_\theta}{D^2} \cos \theta.$$

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\* See *Transactions of the American Institute of Electrical Engineers*, Vol. XIX., p. 1465, November, 1902, "An Integrating Photometer for Glow Lamps and Sources of like Intensity."

If, then, we place a series of  $n$  pairs of such mirrors at angular intervals of  $\pi/n$  round a vertical semi-circle having the arc as its centre, the brightness  $B$  of the vertical white screen at  $T$  will be due to the sum of all the illuminations, and will be expressed by

$$B = \frac{K^2}{D^2} \sum_{\theta=0}^{\theta=\pi} (I_{\theta} \cos \theta).$$

If  $n$  is sufficiently large, then the mean spherical candle-power  $I_0$  is obtained from the  $n$  observations of the candle-power in the direction  $\theta$  as follows:—

$$I_0 = \frac{1}{2R} \int R d\theta. \quad I_{\theta} \cos \theta,$$

where  $R$  is the radius of the semi-circle, and  $ds$  is an element of its periphery. We may write  $\pi/n$  for  $d\theta$  if  $n$  is greater, say, than 12, and then we have

$$I_0 = \frac{\pi}{2n} \sum_{\theta=0}^{\theta=\pi} I_{\theta} \cos \theta.$$

Hence

$$B = \frac{K^2}{D^2} \cdot \frac{2n}{\pi} I_0 = CI_0,$$

or the brightness of the vertical screen at  $T$  is proportional to the mean spherical candle-power.

Mr. Matthews has therefore constructed an integrating photometer by arranging twelve pairs of mirrors,  $M$ , round a semi-circular support,  $A$  (see Fig. 34), and employing a Lummer-Brodhun screen photometer,  $S$ , as the white screen above mentioned. In this case we can balance the integral brightness produced by all the rays reflected from the twelve pairs of mirrors by a direct ray from a standard lamp placed on the other side of the photometer. The constant,  $C$ , of the integrating photometer may be determined by placing in the instrument a lamp the mean spherical candle-power of which has been determined by independent observations and the employment of a Rousseau diagram. Let  $i$  be the candle-power of this standard lamp and  $d$  its distance from the screen of the Lummer-Brodhun photometer, then  $i/d^2$  is the illumination produced on the screen by it when the ray from the standard falls normally on the screen. If this brightness balances that due to the known mean spherical candle-power  $I_0$  of the lamp employed in the integrating photometer, then we have

$$\frac{i}{d^2} = CI_0 \text{ or } C = \frac{i}{I_0 d^2}.$$

Hence the constant  $C$  becomes known.

There are certain corrections, however, which must be noted. The plaster of paris screen of the Lummer-Brodhun photometer obeys Lambert's law of the cosine up to an incidence of 50deg. Beyond this there is, however, a departure, and the illumination is greater than that due to the cosine law. It is possible, however, to correct for this by setting the pairs of mirrors at and beyond 50deg. above or below

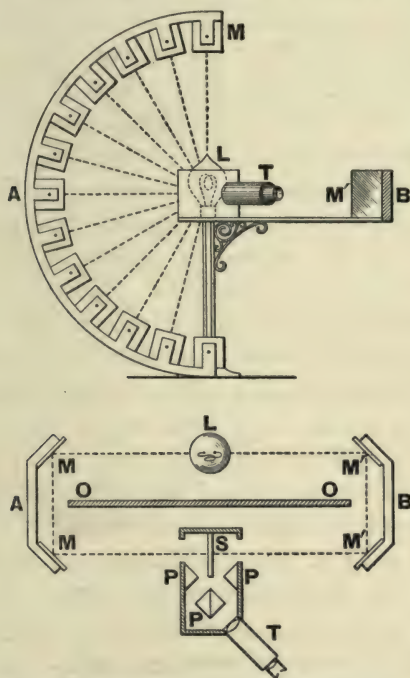


FIG. 34.—Matthews Integrating Photometer.

the horizon a little farther away from the light, and for this purpose they are carried on movable pins.

An experimental adjustment has therefore to be made, in the first place, of the instrument to make it fulfil the law of its theoretical basis. When, however, this is done, and the constant determined, one single photometer observation enables us to determine the mean spherical candle-power of

any lamp. If the lamp is an arc lamp it may be kept at rest, but if it is an incandescent lamp then it must be rotated round its vertical axis during the observation of candle-power.

For additional details of the manipulation of the instrument the reader must be referred to the original Paper.

§ 11. **Heterochromatic Photometry.**—At this point it is important to consider a little more carefully the precise nature of the comparison which we are making when we compare photometrically together two lights of different spectral characters. With this object it is necessary to particularly notice the various visual powers of the human eye. Outside of ourselves, or objectively, light consists in a wave motion in the ether or luminiferous medium. Rays of light are classified as pure and impure. The purity of a ray is measured by the degree to which it is a mixture of radiations of various wave-lengths. Radiation is called *pure* or *monochromatic* when it consists only of rays of one wave-length, and *impure* or *polychromatic* if, when analysed by a prism, it presents itself as a mixture of various wave-lengths. Assuming the ray to be a pure ray it has two particular properties, viz., (1) *wave-length*, (2) *amplitude*. Subjectively, these two qualities correspond with what we call the colour and intensity of the light. Corresponding to these external properties of a ray of light, the human eye possesses powers of sensation or sensibilities of two kinds. It appreciates variation in wave-length or variation in purity in the light falling on it as a *colour sensation*, and it appreciates the amplitude of the wave motion as a sensation of *brightness* apart altogether from the colour sensation. This last sensibility is probably connected intimately with the amount of energy conveyed by the impinging radiation. In order that we may discriminate the outlines, or form, of external bodies, it is necessary that the stimulus of the retina shall be of that kind that there are differences in sensations of colour or brightness created on the different portions of it. The image on the retina corresponding to any object which

we see consists, therefore, of a patchwork, the different portions of which produce different sensations of colour or brightness when received upon the central portion. If the image on the retina consists of two portions, respectively corresponding to two external surfaces which are unequally illuminated, or reflect light of different wave-length, we can make a judgment between the two and decide whether they are alike or unlike in what we call brightness or colour; but we have no means of estimating quantitatively, without external assistance, how much brighter one surface is than the other. It is necessary, therefore, at the outset, to distinguish carefully between sensation and stimulus. The sensation is a result of the structure of the eye, the external stimulus is a consequence of some physical condition in the body seen. Broadly speaking, in the case of all sensations, there is an increase in the degree or amount of the sensation with an increase in the stimulus up to a certain point, but beyond that, increase in stimulus is not accompanied by increase in sensation. It may even be accompanied by a decrease in sensation, as when we are blinded by an exceedingly bright light. In the case of retinal sensation, the connection between stimulus and sensation may be stated as a particular instance of what is known in psychology as Fechner's Law.

This law may be stated generally in the following terms:—*Sensation varies as the logarithm of the ratio between the stimulus and the minimum stimulus which will cause any appreciable sensation.\** Or symbolically, if  $S$  is the sensation and  $I$  the stimulus, and  $i$  the minimum stimulus, then  $S = k \log \frac{I}{i}$ , where  $k$  is some constant. Otherwise the law may be stated thus: If increase in stimuli causes increase in sensations which are in arithmetical progression, as regards magnitude or degree, then the corresponding stimuli increase

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\* See Principal C. Lloyd Morgan, F.R.S., in the Croonian Lecture, "Studies in Visual Sensation." *Proc. Roy. Soc. Lond.*, 1901, Vol. XCVIII., p. 459.

in geometrical progression. If, therefore, we could represent by a curve the ratio of luminous sensation to stimulus, the curve would be a logarithmic curve of the general form shown in Fig. 35. If, therefore, we are comparing together visually two surfaces adjacent to one another, which in effect is comparing together the sensations produced on the retina by them, the degree to which we are able to decide on their likeness or unlikeness will depend on their absolute value. The more intense the stimuli the less will be the minimum percentage difference which we are able to perceive. Thus, if two surfaces are illuminated feebly, we may probably not be able to detect a difference between them of less than 10 per cent. of either ;

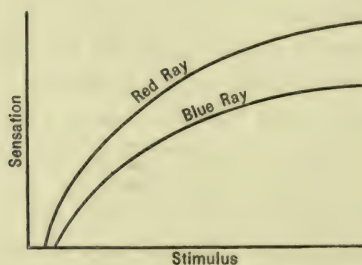


FIG. 35.—Curves Illustrating the Relation of Luminous Sensation to Stimulus for Rays of Different Colours.

but if the two surfaces are very brilliantly illuminated, we may be able to detect a difference between them of  $\frac{1}{2}$  per cent. In addition to these sensations of brightness and colour, it has been contended that the eye possesses a power of *detail discrimination* which may be described as the power of detecting a pattern consisting of black lines or dots drawn upon a white surface. In the exercise of reading a book we are concerned with an operation of this kind. The printed letters consist of fine or coarse black lines drawn upon a white surface. It may be argued, however, that this detail discriminating power essentially depends upon the brightness discriminating power of the eye. If, for instance, we are

reading a book or examining a pattern of black lines drawn upon a white surface, we really have before our eyes a surface, the various portions of which are of unequal reflecting power, and hence when illuminated there is a difference in brightness between the parts. We guide the eye along the boundary of a letter by the aid of the difference in brightness between the adjacent parts, and we cannot distinguish any pattern on a surface in which, between the adjacent portions, there is no difference either in brightness or in colour. On the other hand, it has been asserted by L  pinay and Nicati that if we illuminate two portions of a white surface, one by light of one colour and the other by light of another, and adjust these lights so that the two portions of the illuminated surface appear as a whole to have equal integral brightness, then the eye will not be able to see with equal ease an identical black pattern drawn on the two surfaces. Thus, the above investigators have stated that, if yellow and a blue light produced by any prismatic means are adjusted to produce equal apparent brightness when falling upon two parts of a uniform white surface, then, when these rays are allowed to fall upon a printed book, the type is more easily read which is illuminated by the yellow light than that illuminated by the blue light.\* If this is the case, we must include amongst the powers of the eye a definite detail discriminating power which is not identical with its power of appreciating the general illumination of a white surface. It may be noted in passing that we cannot detect the existence of a black line on a white ground unless the width of the lines subtends a certain minimum angle at the eye, which is dependent upon the degree of the illumination. A line having an angular width of one minute of an arc can certainly be seen; this corresponds with a line 1mm. in width viewed at a distance of 3,440mm., or about 11ft.; but a black line on a white ground having a width subtending an angle of one second of an arc certainly

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\* Mac   de L  pinay and Nicati, *Journal de Physique*, Vol. II., p. 75, 1883.

cannot be seen under any illumination. The same thing applies to black dots on a white ground or white dots on a black ground. If, for instance, we rule a number of black lines on white paper we shall find that the power to discriminate these lines depends on their width and on the general illumination. As the illumination is reduced it becomes more difficult to see these lines separately, and below a certain degree of illumination the lines cannot be seen.

This effect, however, is much complicated by a reflex action of the pupil of the eye. In proportion as the illumination diminishes, the pupils of our eyes automatically expand to admit more light, and beyond a certain limit we are conscious of a "strain" in the eye. The injurious effect of trying to read or write by an imperfect illumination is a consequence of the continued unconscious muscular effort to preserve the aperture of the pupil expanded to its utmost limits. If, therefore, we regard a pattern of black lines on white paper at a certain distance at which we can see these lines distinctly, aided, if necessary, by spectacles, then, if the illumination on the paper is reduced, it will be found that, owing to the automatic action of the pupil of the eye, there is no very sharply marked limit below which we cease to see the lines. We can, however, eliminate this reflex action by the following device:—If we make in a thin sheet of metal or card a small pin-hole and look through it, the amount of light which reaches the eye is determined solely by the size of the pin-hole, and no expansion or contraction of the pupil behind it will affect the retinal stimulation. Provided with such an artificial pupil, if we try the experiment of looking at a pattern of fine lines, either black on white or white on black, we shall find that, corresponding to a certain minimum illumination, we cease to see these lines and the surface appears uniformly illuminated.

We have, therefore, two methods by which we may compare together the illuminations of two surfaces, or, more correctly speaking, make a judgment as to their equality in

illumination. First, we may compare together the two surfaces as a whole, assuming them to be uniform and, technically speaking, white—*i.e.*, reflecting all the rays that fall upon them—and we may make a judgment as to their integral or general brightness apart altogether from their difference of colour. Secondly, we may make a judgment as to their equality in illumination by noticing whether a particular pattern or black lines can be seen on the two surfaces with equal distinctness. It has been stated, therefore, that a ray of light possesses two qualities: (1) its luminous intensity, and (2) its visual intensity, which we may perhaps translate somewhat freely by calling them, (1) the power of creating brightness, (2) the power of revealing detail.

We may make a comparison between two illuminants, either in respect of their luminous intensity or their visual intensity. The purposes for which we require artificial light are partly for revealing what we call colour differences between objects, and partly for revealing detail. In the one case, the chromatic equality of the light is of great importance, in the second it is not of so much importance, provided it is sufficiently intense, either as regards its luminous or visual intensity. Thus, for the purpose of reading we are far less concerned with the chromatic quality of an illuminant than we are when providing an illuminant for a picture gallery or dye-house.

Another point in connection with the comparison of lights of different spectral character of special interest is that known as Purkinje's phenomenon, which is illustrated by the following experiment:—We take a white right-angled wedge and illuminate one side by a red light and the other side by a blue light, and adjust the distances of these two illuminants until we obtain what we consider is an equal illumination on the two adjacent sides of the wedge. If we then move in both these lights to half their distance from the wedge—in other words, make the *objective brightness* of the surfaces fourfold—we find that the retinal stimulation, or the *apparent brightness* of

the two surfaces are no longer the same : the red-lighted side would appear the brighter. It is, therefore, clear that, although the retinal sensation of brightness increases with the objective or actual illumination of the surface, it does not increase according to the same law for all colours. It follows from this that, if we illuminate such a simple wedge photometer on one side by a candle and the other side by an arc lamp, the ratio of the distances at which these two illuminants must be placed in order to produce what we deem to be an equal brightness on the two surfaces will depend upon the degree of that surface brightness. Hence there is no fixed ratio between the luminous intensities or the so-called candle-powers of an arc lamp and a candle, in regard to the brightness they produce on a white surface apart altogether from their colour difference. The ratio is a function of the brightness produced which is declared to be equal.

Viewing these qualities, great authorities, such as von Helmholtz, have declared that there was no such thing as heterochromatic photometry—in other words, no possibility of comparing light of different spectral compositions. With regard to colour-distinguishing power, or colour-revealing power, it is perfectly clear that no scientific meaning can be attached to the term “candle-power.” Our standard light, as regards revealing the so-called natural colours of objects, is *daylight*—say the light from a northern sky, such as that which an artist admits to his studio. The same surfaces viewed by other illuminants create totally different sensations in the eye, and it is doubtful whether any single numerical co-efficient can be attached to these illuminants defining their colour-revealing powers in terms of normal daylight taken as a standard. On the other hand, if we separate out the sensation of brightness from that of colour, we can define the power of an arc in terms of that of a candle, as regards its power of producing brightness on a white surface, *provided we define the degree of that brightness*. If we take as our standard of brightness 1 *candle-foot*—i.e., the illumination produced

by one candle placed at a distance of 1 foot from a white surface, then we can by one single number express the ratio of the two lights in producing brightness of this kind ; but the same ratio is not applicable to other degrees of brightness, and hence, in consequence of Purkinje's phenomenon, there is no such thing as an absolute "candle-power of an arc."

It has, therefore, been proposed that we shall define the ratio of two heterochromatic lights by means of their detail-distinguishing powers. This could be done if we agree on a certain pattern which shall afford, as it were, a standard of discrimination.

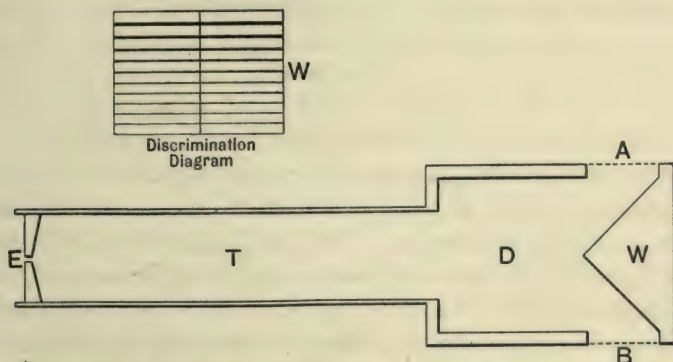


Fig. 36.—Discrimination Photometer (Fleming).

W Ritchie Wedge covered with  
Ruled Discrimination Dia-  
gram, W.

T Eye-piece Tube.  
E Pinhole Eye-piece.  
A and B Two Sources of Light.

Based on these principles we can construct what may be called a *discrimination photometer* in the following manner:—A wooden box is provided with an eye-tube (*see* Fig. 36); the end of the eye-tube nearest the eye is closed with a metal or cardboard disc, in which is made a pin-hole. In the box at the end opposite to the eye-tube is placed a wooden wedge covered over with fine white paper, on which are ruled a number of parallel black lines of gradually decreasing width, some of them zigzag and others broken up into dots. These lines decrease in width,

so that the angle subtended by the finest of them is about one second of an arc. The box has two windows at the sides, by which the two opposite sides of the wedge can be illuminated respectively by two lights. The photometer is used in the following manner:—It is placed between two heterochromatic lights, say an arc lamp and a candle, or an incandescent lamp and an arc lamp, and the observer, placing his eye at the pin-hole, sees the two sides of the wedge unequally illuminated. The wedge is then moved to and fro between the lights until, without regard to any difference in colour of the two white grounds, or difference in general brightness, the observer can see the lines on the two sides of the wedge equally distinct and equal in number. When this is the case, the observer is able to say that he can discriminate the pattern of black lines on a white ground equally well when illuminated by the two lights respectively, and under these circumstances the visual intensity of the two lights may be said to be proportional to the square of their distances from the wedge. The justification for this latter assumption—viz., that the visual intensity varies inversely as the square of the distance—is based on the experimental fact that a light of four candles placed at a distance of 2 feet from a white surface enables us to discriminate equally well with a light of one candle placed at 1 foot, or nine candles at 3 feet, using the word “candle” to imply a constant small unit which is multiplied in the respective sources.

Various forms of discrimination photometer may be devised, but they are all based upon the principle that two lights are said to be equal in visual intensity when they enable us to discriminate the details of a black pattern on a white surface equally well when placed at the same distance from it. If the experiment be tried with lights of very different spectral character, using a discrimination photometer as above and also an ordinary intensity photometer, it will be found that the number which represents the visual intensity of a source of light is not the same as that which represents

its luminous intensity. Sources of light in which the blue rays predominate are less useful for the general purposes of vision than sources of light in which the red rays and yellow rays predominate, as far, at any rate, as regards distinguishing the detail of objects, and especially their utility for purposes of reading and writing.

§ 12. **Illumination Photometry.**—The construction of an Illumination Photometer has already been described. By means of such an instrument as that devised by Mr. A. P. Trotter or Sir W. H. Preece, we can measure the illumination on the floor or at any height above the floor of any room or building or street which is lighted in any manner, natural or artificial. The unit in which the results are recorded is generally the *candle-foot*—*i.e.*, an illumination equal to that produced by one standard candle held at a uniform distance of 1 foot from a perfectly white surface. This illumination is a convenient one for reading. The unit of illumination in France has generally been the *bougie-metre* or the *carcel-metre*, the latter being a unit of illumination not very different from that of the candle-foot. The *bougie-metre* is, however, too small a unit of illumination for practical purposes. The name “lux” was proposed by Sir William Preece for the unit of illumination, and has been variously applied either to a *bougie-metre* or else, as originally by him, to an illumination produced by a standard candle at a distance of 12·7 inches. A unit proposed by the Author is the *lamp-metre*—*i.e.*, an illumination equal to that produced by 10 British standard candles, or by the Harcourt 10-candle pentane lamp at a distance of 1 metre. In general, it is more convenient to record illuminations by such a name as the candle-foot or the lamp-metre rather than by a single name such as lux, because the double name is self-explanatory and analagous to such phrases as ampere-hour or watt-hour. The illumination of a surface, or at various points along a surface, may be represented by an illumination curve or illumination surface



the horizon is represented by  $\theta$ ,  $AB=I$  and  $BD=I \sin \theta$ . The illumination at the point P is in candle-feet equal to  $AB$  divided by  $(AP)^2$  multiplied by the sine of the angle  $APX$ . Hence, if we draw a vertical line  $PM$  through P, and make  $PM$  equal to  $BD$  divided by  $(AP)^2$ , then  $PM$  will represent the illumination in candle-feet at the point P.

By carrying out this construction at different points on the horizontal line, we can delineate a curve  $XY$ , which is called the curve of illumination of the arc on the horizontal plane, and by finding the mean ordinate of this curve, we can determine the mean illumination. If there are two arc lamps in proximity instead of one, then the illumination at any point on the street is the sum of the illuminations of the two separately. In general it is not necessary to consider more than the two nearest arc lamps in predetermining the illumination at any point in a street.

On a street plan lines can be set out of equal illumination. These lines are like contour lines, and connect together the points on the surface at which the illuminations are equal.

An illumination survey of a street comprises drawing the sectional illumination, and also the plan contour curves of illumination. In the case of the sectional illumination curves we can draw a mean ordinate or line at the height of the mean ordinate, which represents the mean illumination along the street, and it is generally necessary to determine the position and amount of *maximum illumination*, the position and amount of *minimum illumination*, and the *mean illumination*, in order to have a definite conception of the nature of the street illumination.

These measurements should in general not be made on the street surface, but on a level about 4 feet above the street surface, as it is at this height that we are most concerned with the illumination. It appears from the results of Mr. Trotter's measurements in 1891-2\* that the lowest illumination on the pavement surface which should be reached in a well-lighted street is 0.03 of a candle-foot, whilst the maximum should certainly not fall below 1 candle-foot, and for good lighting should reach nearly 2 candle-feet. Broadly speaking, it may be said that the higher the sources of light are placed above the street the more uniform will be the lighting but the less will be the average illumination.

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\* See "Distribution and Measurement of Illumination," *Proc. Inst. C.Eng.*, Vol CX., part 4, 1892.

As regards gas-lighted streets, illuminations as low as 0.005 candle-foot are not uncommon, and the illumination will rarely rise as high as half a candle-foot. In streets lighted by arcs illuminations as low as 0.025 candle-foot are sometimes found, although the average is usually about 0.15 candle-foot. In streets well lighted by arc lights, maximum illuminations varying between 1 and 2 candle-feet are quite usual. In any street or building the illumination at any one point is, of course, the sum of that due to each individual light separately, but, as a matter of fact, in street lighting the illumination at any one point of the street is in general entirely determined by the illuminating power of the two nearest lights. Thus, in street lights, with continuous-current electric arcs taking 10 amperes, and placed on posts 20 feet above the road surface and 120 feet apart, the maximum illumination would be about 1.5 candle-feet, and the minimum about 0.15. If the lamps were raised on posts 40 feet high and kept at the same distance apart, the maximum illumination would fall to 0.3 candle-foot, and the minimum would rise to 0.2 candle-foot.

The following Table, composed of results obtained by Mr. Trotter, will indicate the general character of the maximum and minimum illumination produced by street arc lamps at various heights and distances :—

Arc lamp current in amperes.	Height of lamp above road in feet.	Distance apart in feet of the lamps.	Horizontal illumination of the pavement in candle-feet.	
			Maximum.	Minimum.
10.0	20	120	1.85	0.12
10.0	25	120	1.17	0.15
10.0	40	120	0.5	0.28
6.8	20	90	1.1	0.21
6.8	40	120	0.3	0.17

The above illuminations for street lighting may be compared with some figures which have been given by various authorities for the illumination within well lighted buildings. For

instance, in a report on the action of light on water-colours, published in 1888 by Dr. Russell and Sir W. Abney, measurements taken by Dr. Festing with the Preece photometer in the South Kensington Museum are given. In one water-colour gallery lighted by gas the illumination was 1·81 candle-feet, in another gallery 2·32 candle-feet, in a third, lighted by electric glow-lamps, 1·72 candle-feet, and in fourth and fifth galleries, lighted by arc lamps, the illuminations were respectively 2·26 and 3·12 candle-feet.

It is not stated, however, whether these numbers are the mean or maximum illumination, or whether they apply to the wall or floor surface.

The following Table gives some further measurements of illumination in candle-feet (C.F.) taken in various localities:—

	C.F.
In a well-lighted room on the floor or tables .....	1 to 3
On a theatre stage .....	3 to 4
On a railway platform .....	0·05 to 0·5
In a picture gallery .....	0·65 to 3·5
The mean daylight in May, in the interior of a room	30 to 40
A white surface in full sunlight .....	7,000 to 10,000
A white surface in full moonlight .....	$\frac{1}{16}$ to $\frac{1}{128}$

With regard to the illumination produced in the interior of ordinary rooms by daylight, it will vary with every passing cloud, but generally lies between 10 and 50 candle-feet. Much depends, however, upon the nature of the reflecting surfaces, such as the ceilings and walls.

**§ 14. The Measurement of Coefficients of Absorption and Reflection.**—Intense sources of light produce a paralysing effect upon the retina, and have to be surrounded or included in a semi-transparent globe of some kind for the purpose of diffusing the light. An intense point source of light, such as a naked arc lamp, is not very useful for purposes of vision for two reasons—first, because the small surface of the source causes it to cast very sharply marked shadows, and secondly because the pupil of the eye is caused to contract by looking at a very bright point of light; and the effect is not only to

reduce the light reaching the retina, but to paralyse that portion of it on which the image falls, and to render it incapable of receiving for a short time afterwards less intense impressions. Hence the image of the source of light must be spread over a larger area, and this is done by enclosing it in a semi-transparent or diffusing globe, whereby the intrinsic brilliancy of the source of light is made less, as well as the luminous intensity; but the source of light is made more useful for the purposes of ordinary vision. Electric lamps, both arc and incandescent, are therefore usually enclosed in globes or bulbs, which may be either simply sand blasted or formed of ground glass, or they may be made of opal glass or some semi-transparent material. There is, however, a great loss of light, amounting to 50 to 80 per cent. This is illustrated by the results of some experiments made on arc lamp globes by Messrs. Guthrie and Redhead in the following table:—

*Effect of Diffusing Globes on Arc Lamp Candle-power.*

—	Naked Arc.	Arc in clear globe.	Arc in rough glass globe.	Arc in opal globe.
Mean spherical C.P. ....	319	235	160	144
Mean hemispherical C.P. ....	450	326	215	138
Percentage value of transmitted light .....	100	53	23	19
Percentage apparent absorption...	0	47	77	81

In order to avoid this large loss of light by absorption, glass shades or globes have been invented by Mr. Trotter, M. Fredereau and others, which consist of clear glass grooved in such a manner as to diffuse the light uniformly with less absolute loss by absorption. In a globe of this description, called the holophane globe, an arc placed in the interior causes the globe to appear uniformly luminous, and yet not more than 20 per cent. of the light is absorbed. The *coefficient of apparent absorption* of such a globe can be determined by photometering an incandescent lamp in the ordinary manner

and then enclosing the lamp in the globe to be tested. For this purpose, it is best to employ a type of incandescent lamp known as a *Focus Lamp*, in which the filament is crinkled up into a very compact form, so as to concentrate the light into a focus. If such a lamp is photometered against a fixed standard, both with and without the enclosing globe, the ratio of the first reading to the second, expressed as a percentage, gives the coefficient of apparent transmission, and this last number, expressed as a percentage, gives us the percentage apparent transmission coefficient of the globe. The use of a double-enclosing globe, as in enclosed arc lamps, causes the resultant luminous intensity of the lamp to be still more decreased. Thus, for instance, if the lamp is enclosed in two opal globes, both of which would separately transmit 50 per cent. of the light, then, when the globes are used one inside the other, the result is to transmit only 25 per cent. of the total light. Hence, in these cases, it is advisable to use clear glass either for the outer or inner globe for the sake of avoiding the great loss by absorption which would otherwise take place.

Methods for the measurement of the true coefficients of absorption and transmission of light by different translucent surfaces have been given by Dr. W. E. Sumpner (see *The Electrician*, Vol. XXX., pp. 381, 411, 439; see also *Phil. Mag.*, S. 5, Vol. XXXV., 1892, p. 81). From these Papers a number of interesting remarks are extracted in the following paragraphs.

The following definitions must be recalled to mind:—The *illumination on a surface* means the flux of light incident upon it per unit area. The *reflecting power of a surface* is the ratio of the amount of light reflected by it to the total amount incident upon it. There are two kinds of reflection, called respectively regular and irregular reflection. In the first case, every reflected ray follows the ordinary law of reflection, that the incident and reflected rays make equal angles with the normal to the surface at the point of

incidence. In the case of irregular reflection this law is not followed, but the incident light is scattered in all directions. The illumination due to a source of light of intensity  $I$  on a surface at a uniform distance  $D$  is measured by  $I/D^2$ . It will be measured in candle-feet if the intensity is measured in candle-power, and the distance in feet. The flux of light received upon a surface having an area  $A$  square feet placed perpendicular to the rays of light, and at a distance  $D$  from a source of intensity  $I$ , is equal to  $IA/D^2$ . The ratio  $A/D^2$  is called the solid angle subtended by the area  $A$  at the source of light, so that the flux of light given out by a source of intensity  $I$  within a solid angle  $\Omega$  is  $I\Omega$ . If the source of light radiates equally in all directions, the total flux of light given out by a source of intensity  $I$  is  $4\pi I$ , or nearly  $12.7 I$ . This flux of light, as already explained, is measured in *lumens*, so that the total flux of light from a source of unit intensity, is  $12.7$  lumens. The term "quantity of light" is sometimes used instead of flux of light. The term quantity of light should, however, be reserved for the product of flux and time, and is, therefore, properly measured in lumen-hours or lumen-seconds.

Another term which is necessary to define is the word *brightness*, or *intrinsic brightness*. A luminous surface may be said to have a brightness of one candle per square foot when a area of one square foot of the surface acts like a lamp of one candle-power in the direction normal to the surface. If  $B$  is the brightness of a surface of area  $A$ , the illumination produced by it at a distance  $D$ , in the direction perpendicular to the surface, is  $BA/D^2$ , or  $B\Omega$ , where  $\Omega$  is the solid angle subtended by the area at the point at which the illumination is measured. As already shown, if the luminous intensity or candle-power of a bright surface in directions inclined to the normal varies as the cosine of the angle between the ray and the normal, then the total flux of light radiated per unit of area from a surface having a brightness  $B$  is equal to  $\pi B$ . Starting from these definitions, we can obtain expressions for

the true absorbing or reflecting power of a diffusing surface. No instrument exists by which flux of light can be directly measured, but the luminous intensity or candle-power of a surface which is throwing off light can be measured by an intensity photometer, and the brightness of the diffusing surface is then given by the quotient of that intensity measured in the direction of normal to the surface, by the area of the surface. The total light reflected from the surface is therefore measured by the quantity  $\pi AB$ , where  $A$  is the area of the surface and  $B$  the brightness. To connect this with the reflecting power of the surface, we have the equation  $\pi B = rI$ , where  $I$  is the calculated illumination of the surface, and  $r$  the coefficient of reflection.  $I$  is, by definition, the amount of light incident upon the surface per unit of area, and, consequently,  $rI$  must be the amount reflected—*i.e.*,  $\pi B$ .

The following figures taken from measurements made by Prof. L. Weber (see *The Electrician*, Vol. XXV., p. 404) will give some notion of the values of the brightness of various illuminated surfaces:—

Reflecting surface.	Brightness in candle-power per square foot.
White paper exposed to summer sky .....	4,300
White paper within an ordinary room in daylight .....	5 (about)
White paper at night in room with illumination 1 candle-foot .....	0.25
White paper exposed to rays of full moon .....	0.006
Brightness of walls in ordinary room during day .....	from 1 to 5
Walls of ordinary room during night .....	from 0.05 to 0.25
Black velvet exposed to summer sky at noon .....	6.0

From the above figures it will be seen that the brightness of white paper exposed respectively to full sunshine and full moonlight varies between 4,300 candles per square foot, and 0.006 candle per square foot, or in the ratio of 700,000 to 1; yet the eye can endure being exposed to surfaces having these extremes of brightness. The figures show also that the blackest surfaces, such as black velvet, are capable of reflecting a large amount of light.

*Measurement of Reflecting Power.*—The following is a convenient method, according to Dr. Sumpner, of measuring the irregular reflection from a diffusing surface. The surface to be examined is attached to a large screen of black velvet placed perpendicularly to a photometer bench carrying on it a Lummer-Brodhun photometer. It is illuminated by a glow-lamp of about 20 c.p. placed just above and fixed to a Lummer-Brodhun photometer, and the light received by the photometer from the reflecting surface is balanced against the light from another source placed on the opposite side of the Lummer-Brodhun photometer. This last light will be called the *balancing light*, and the first one the *illuminating light*, and their intensities denoted respectively by  $I_1$  and  $I_2$ . Then, if  $x$  is the distance from the screen of the illuminating light, and therefore of the photometer, and if  $y$  is the distance of the balancing light from the photometer, the true reflecting power of the surface ( $r$ ) is given by the following equation

$$r = \frac{I_1}{I_2} \frac{x^2}{y^2} \left\{ \frac{\pi x^2}{A} + 1.75 \right\},$$

where  $A$  is the area of the diffusing surfaces.

With some surfaces there is regular as well as irregular reflection. If  $r_1$  is the proportion of the incident light which is diffused, and  $r_2$  is the proportion which is reflected according to the regular law, then it can be shown\* that

$$\frac{I_1}{I_2} \frac{x^2}{y^2} \left\{ \frac{\pi x^2}{A} + 1.75 \right\} = r_1 + \frac{1}{4} \left\{ \frac{\pi x^2}{A} + 1.75 \right\} r_2.$$

The true reflecting power  $r$  is, therefore, given by the equation

$$r = r_1 + r_2.$$

Dr. Sumpner gives (*loc. cit.*) some results made with four surfaces—white blotting paper, white cartridge paper, tracing cloth and tracing paper—which may be taken as examples of substances which have no regular reflection and substances

\* The proof of these formulæ will be found in a Paper by Dr. W. E. Sumpner on "The Diffusion of Light" (*Phil. Mag.*, Vol. XXXV., p. 81, 1893). The proof is too long to give here in detail.

which have a little regular reflection, and the values of the coefficients in percentages are as follows:—

Reflecting Surface.	Diffused. $r_1$ .	Coefficients of Reflection.	
		Regular. $r_2$ .	Total. $r$ .
Blotting paper .....	82.0	0	82.0
Cartridge paper .....	80.0	0	80.0
Tracing cloth (shiny side) .....	30.9	4.3	35.7
"    " (rough side) .....	31.6	2.7	34.3
Tracing paper .....	19.8	2.7	22.0

There is one peculiarity of a diffusing reflecting surface, such as white blotting paper, which is important. An ordinary mirror, such as looking-glass, reflects light almost equally well at all directions of incidence, and where any difference exists it is found that rays which make a large angle with the normal to the surface suffer less absorption than those which are nearly perpendicular to it. With a diffusive reflector, on the other hand, the light is mostly sent off in the normal direction. This can be shown by placing behind a glow lamp, first, a piece of common looking-glass or a plane metallic mirror, and, secondly, a sheet of white paper. In the case of the mirror, the candle-power of the lamp is found to be nearly doubled, but in the case of the white paper it is more than doubled. In the former case, however, the candle-power is doubled in all directions on one side of the mirror, while in the latter the statement is only true for directions in the neighbourhood of the normal to the paper reflector. A piece of white paper is, therefore, a more efficient reflector if it is desired to send the rays of the lamp simply in a direction perpendicular to the paper, but a mirror is more efficient if it is desired to increase the light in all directions.

*Measurement of Absorbing Power.*—Semi-transparent substances, such as tracing paper, ground glass and opal glass, absorb a certain proportion of the rays falling upon them, reflect others, and transmit the remainder. If we denote by the letters  $r$ ,  $a$  and  $t$  the fractions of the light which are

respectively reflected, absorbed and transmitted, the relation between these quantities is  $r + a + t = 1$ . If  $Q$  is the flux of light given out per second by a source of light which is placed entirely within a semi-transparent envelope, the flux of light incident upon the surface of the envelope will, owing to internal reflection, be increased to  $Q'$ , where  $Q' (1 - r) = Q$ , and, therefore, the flux of light absorbed and transmitted will respectively be  $aQ'$  and  $tQ'$ , and the ratios that these quantities bear to  $Q$  will be respectively  $a/1 - r$  and  $t/1 - r$ . Hence, if a source of light radiates equally in all directions, and we then surround it with a semi-transparent globe which transmits a fraction of the light, the ratio of its candle-power, *after* putting on the envelope, to the original candle-power will not be  $t$  but  $t/1 - r$ .

The above statements will be more easily seen to be true if it is remembered that, when light falls on the interior of a translucent globe, some of this is reflected and falls on other parts, and is again partly transmitted. Thus, if the source of light emits a flux,  $Q$ , and of this a quantity,  $Qr$ , is reflected from the envelope and  $Qt$  transmitted, then the reflected portion again falls on the surface of the envelope, and a portion,  $Qr^2$ , is reflected and  $Qt^2$  is transmitted. Hence the total flux which is incident on the inner surface will be equal to

$$Q + Qr + Qr^2 + \&c.;$$

$$\text{or to } Q(1 + r + r^2 + \&c.) ; \text{ or to } \frac{Q}{1 - r} = Q/1 - r.$$

Hence, if this latter quantity is called  $Q'$ , we have

$$Q' = Q/1 - r \text{ or } Q' (1 - r) = Q,$$

as already stated.

Hence the proportion of the flux of light which is transmitted will be  $Q' t$  or  $Qt/(1 - r)$ , and hence the ratio of the flux after putting on the envelope to the original flux will not be  $Qt$  to  $Q$  or  $t$ , but  $Qt/1 - r$  to  $Q$ , or  $t/1 - r$ , as stated. Hence, internal reflection increases transmission.

It is important to notice that the influence of internal reflection is to increase *both* the absorption and the transmission,

and unless it is taken into account large errors may be made in estimating the coefficients. A simple way of showing the effect of internal reflection consists in surrounding a glow lamp with a white paper cylinder open at the top, and measuring the external illumination by means of a photometer. If, now, a piece of white paper is put on the top of the cylinder so as to shut in the vertical rays of the lamp, the candle-power in a horizontal direction will be found to increase considerably. In the following experiments, made by Dr. Sumpner, the lamp was surrounded with an envelope of tracing cloth or blotting paper, or some other semi-transparent material, and the luminous intensity or candle-power measured through the envelope. If  $K_0$  is its original candle-power and  $K_1$  is its candle-power after surrounding it completely with an envelope of the substance tested; the apparent absorption is given by the fraction  $\frac{K_0 - K_1}{K_0}$ , and the true absorption coefficient is given by

$$a = (1 - r) \frac{K_0 - K_1}{K_0}.$$

The following table gives the apparent absorption and the true absorption coefficient of various substances:—

Substance.	Apparent Absorption.	True Absorption Coefficient
	Per cent.	Per cent.
White blotting paper .....	77	13·8
White cartridge paper .....	61	12·2
Tracing cloth .....	23	15·0
Tracing paper .....	9	7·0

Three large glass globes made for arc lamps were also tested, and, as the reflective powers of the globes could not easily be found, only the apparent absorption was measured. One globe of opal glass, almost transparent, had an apparent absorption of 15 per cent., a second of ground glass absorbed 42 per cent., and a third of ordinary opal glass 39 per cent.

*Measurement of Transmitting Power.*—The amount of light transmitted through surfaces can be measured in a similar

manner to that by which the irregular reflective power is determined. A screen of semi-transparent material is placed on the photometer bench. A source of illumination, say a glow lamp, is placed at a distance,  $x$ , behind it. A photometer is placed at a distance,  $x$ , in front of it, and on the other side of the photometer, at a distance  $y$ , a balancing light is placed. If, as before,  $I_1$  and  $I_2$  represent the luminous intensities of the balancing and illuminating lights respectively, and  $A$  is the area of the surface, then the coefficient of transmission  $t$  is given by the following formula:—

$$t = \frac{I_1}{I_2} \frac{x^2}{y^2} \left\{ \frac{\pi x^2}{A} + 1.75 \right\},$$

assuming that all the light transmitted is diffused. It may be noticed that some substances, like thin opal glass, which are sufficiently transparent to enable the eye to clearly distinguish objects through them, have a character different from translucent surfaces, such as tracing paper, which, while transmitting a proportion of the light, permit no vision through them. These two portions may be called respectively the regular and the diffused transmission, and the values of the coefficients of the diffused ( $t$ ) regular ( $t_2$ ) transmission are connected by the equation\*

$$\frac{I_1}{I_2} \frac{x^2}{y^2} \left\{ \frac{\pi x^2}{A} + 1.75 \right\} = t_1 + \frac{t_2}{4} \left\{ \frac{\pi x^2}{A} + 1.75 \right\}.$$

Dr. Sumpner has given the following Table I. of the transmitting powers of the four substances above mentioned, and the whole of the results are also collected in the Table:—

**TABLE I.**  
*Transmitting Powers.*

Substance.	Coefficients of Transmission.		
	Diffused. $t_1$ .	Direct. $t_2$ .	Total. $t$ .
	Per cent.	Per cent.	Per cent.
Blotting paper .....	6.5	2.7	9.2
Cartridge paper .....	8.7	2.5	11.2
Tracing cloth .....	41.0	13.4	54.4
„ paper .....	46.0	29.8	76.0

\* For the proof of these formulæ the reader is referred to Dr. Sumpner's Paper in the *Phil. Mag.* for 1893, Vol. XXXV., p. 81.

TABLE II.

Substance.	Reflection. <i>r</i> .	Absorption. <i>a</i> .	Transmission. <i>t</i> .	$r + a + t$ .
	Per cent.	Per cent.	Per cent.	Per cent.
Blotting paper ...	82	13·8	9·2	105·0
Cartridge paper ...	80	12·2	11·2	103·4
Tracing cloth .....	35	15·0	54·4	104·4
„ paper .....	22	7·0	76·0	105·0

It will be noticed that the sums in the last column are all over 100 per cent. Dr. Sumpner accounts for this by a known fact that the law of the cosines (Lambert's law) is not exactly fulfilled for the above surfaces. A very slight departure from this law would account for the discrepancy.

Dr. Sumpner measured also the reflecting power of the following surfaces, with results as follows :—

Substance.	Reflecting Power in percentage.
White blotting paper .....	82
„ cartridge paper .....	80
Tracing cloth .....	35
„ paper .....	22
Ordinary foolscap .....	70
Newspapers .....	50 to 70
Tissue paper (one thickness) .....	40
„ „ (two thicknesses) .....	55
Yellow wall-paper .....	40
Blue paper .....	25
Dark brown paper .....	13
Deep chocolate-coloured paper .....	4
Planed deal (clean) .....	40 to 50
„ „ (dirty) .....	20
Yellow cardboard .....	30
Parchment (one thickness) .....	22
„ „ (two thicknesses) .....	35
Yellow painted wall (dirty) .....	20
„ „ „ (clean) .....	40
Black cloth .....	1·2
„ velvet .....	0·4

From this table it can be seen that even a dull looking wall will reflect as much as 20 per cent. of the light incident upon it, while a good white surface reflects more than 80 per cent. A room, therefore, with surfaces which are highly reflective increases in a high degree the illumination produced

upon any surface by one or more sources of light. Thus, suppose a room to be illuminated by one source of light emitting a flux of light  $Q$  per second, and let  $Q'$  be the flux falling per second upon the walls of the room, and let  $r$  be the coefficient of reflection of these walls; then the above quantities are related by the equation

$$Q' = Q + rQ'.$$

The truth of this is seen when we remember that  $Q'$  must consist of two parts—viz., that which is absorbed by the walls and that which is reflected. By definition,  $rQ'$  must be the latter quantity, and the former must be equal to  $Q$ , because the quantity falling upon the walls is made up of that which is absorbed and that which is reflected.

Hence, 
$$Q' = Q/1 - r.$$

Accordingly, it will be seen that a room with white walls needs only one-fifth the candle-power to produce any given amount of illumination on a surface in a room that it would need if the walls and ceilings were all dead black. Everyone will remember the increase in general illumination, as represented by the greater ease in reading in any position, which is produced in a room, by laying a white table-cloth or hanging up white curtains in it. Hence, the number of lamps that would be required to illuminate a room properly—i.e., to give it such an illumination that we can read easily in any part of the room—is largely dependent upon the nature of the walls and the ceiling. Suppose that  $A_1$ ,  $A_2$ ,  $A_3$ , &c., are respectively the areas of the ceiling and walls and the floor of the room, and if the reflecting powers of these surfaces are respectively  $r_1$ ,  $r_2$ ,  $r_3$ , then the average reflecting power is

$$r = \frac{A_1 r_1 + A_2 r_2 + A_3 r_3}{A_1 + A_2 + A_3}.$$

Thus, let the reflecting power of the ceiling be 40 per cent., the walls 20 per cent., and the floor 20 per cent., and, if the room is approximately cubical, the average reflecting power

would be 23·3 per cent., and the illumination in the room will be 1·3 times that which would be produced by the same lights arranged in the same way in empty space; or, to put it in another way, the illumination will be 30 per cent. greater than it would be if all the surfaces were dead black. Suppose, next, the ceiling to be whitewashed and the walls to be covered with a light-tinted paper: the average reflecting power would increase to nearly 70 per cent., and the illumination on any surface in the room would now be represented by the number 3, or more than double that which it was before the walls of the room were whitened. It will thus be seen that the nature of the surfaces immensely influences the number of lamps which are required to illuminate a room in a satisfactory manner, and no hard and fast rules, such as providing so many 16 candle-power lamps per square foot of floor surface, is a satisfactory guide to good practice.

For additional information, and for the proof of the formulæ given in this section, the reader is referred to the Paper by Dr. Sumpner quoted (*Phil. Mag.*, Vol. XXXV., 1893, p. 81; or *Proc. Phys. Soc. Lond.*, Vol. XII., p. 10, 1893).

The following list gives the titles of some of the principal works on Photometry, and original Papers, which have been published during the last thirty years, some of the references being taken from Krüss's "Electrotechnical Photometry," but the majority have been collected by the Author:—

MASCART. "Traité d'Optique," Vol. III., pp. 145-269.

1896. VON HELMHOLTZ. "Physiological Optics," p. 416 *et seq.*

JACQUES ABADY. "Gas Analyst's Manual," Chaps. I, II, and III.

W. J. DIBDIN. "Practical Photometry."

W. M. STINE. "Photometrical Measurements."

Dr. H. KRÜSS. "Die Elektro-Technische Photometrie."

Dr. A. PALAZ. "Traité de Photométrie Industrielle," translated into English by G. W. and M. R. Paterson.

## (a) General Literature.

1882. H. KRÜSS. "Die Grundlagen der Photométrie." *Abhdlg. des Naturw. Ver. Hbg.* (7) 2, 28; *Jour. f. Gasbel.*, 1883, 49; *C. Z. f. Opt. u. Mech.* 4, 124.
1882. M. DE LÉPINAY and A. NICATI. "Recherches Expérimentales sur le Phénomène de Purkinje." *Jour. de Phys.* (2), 1, 42 and 86.
1882. M. DE LÉPINAY and A. NICATI. "Relation entre la Loi de Bouguer-Masson et le Phénomène de Purkinje." *Comptes Rendus*, 94, 785.
1884. E. L. NICHOLS. "A Photometric Study of Pigments." *Am. Jour. of Science*, Vol. XXVIII., p. 342, 1884.
1887. J. T. BOTTOMLEY. "On Radiation from Dull and Bright Surfaces." *Proc. Roy. Soc. Lond.*, Vol. XLII., p. 433.
1889. E. MERRITT. "Some Determinations of the Energy of the Light from Incandescent Lamps." *Am. Jour. of Science*, Vol. XXXVII., p. 167.
1889. H. NAKANO. "The Efficiency of the Arc Lamp." *Trans. Am. Inst. E. E.*, May, 1889.
1889. E. L. NICHOLS. "The Efficacy of Methods of Artificial Illumination." *Trans. Am. Inst. E. E.*, Vol. VI., No. 5.
1892. A. P. TROTTER. "The Distribution and Measurement of Illumination." *Proc. Inst. Civil Eng.*, Vol. CX., Part 4, 1892.
1892. CH. HAUBTMANN. "Current and Candle-power Life Curves for Incandescent Lamps." *The Electrician*, Vol. XXIX., p. 590.
1892. A. P. TROTTER. "Notes on the Light of the Electric Arc." *Jour. Inst. E. E.*, Vol. XXI., p. 360.
1892. E. S. FERRY. "Persistence of Vision." *Am. Jour. of Science*, Vol. XLIV., p. 192.
1892. E. L. NICHOLS. "The Age Coating of Incandescent Lamps." *Am. Jour. of Science*, Vol. XLIV., p. 277.

1893. W. E. SUMPNER. "On the Diffusion of Light." *The Electrician*, Vol. XXX., pp. 381, 411, 439; also *Phil. Mag.*, Vol. XXXV., p. 81, February, 1893.

Gives tables of the reflecting, absorbing and transmitting powers of various surfaces.

1893. G. S. RAM. "A Wattmeter for Lamp Measurement." *The Electrician*, Vol. XXXI., p. 612.

1894. C. WIENER. "On the Diffusion of Light from Dull Surfaces." *The Electrician*, Vol. XXXIII., pp. 549 and 621.

Contains very valuable historical notes.

1894. J. M. BARR and C. E. S. PHILLIPS. "The Brightness of Light, its Nature and Measurement." *The Electrician*, Vol. XXXII., p. 525.

A very valuable discussion of photometric methods. The authors give a diagram showing the relative accuracy of various photometers, and attempts are described to construct a selenium photometer.

1894. A. BROCA. "Investigations on the Visual Sensations and on Photometry." *The Electrician*, Vol. XXXIII., p. 752; also *Jour. de Phys.*, May, 1894.

1894. Sir W. DE W. ABNEY. "Cantor Lectures on Photometry." *Jour. Soc. of Arts*, 1894.

1894. A. BLONDEL. "Photometric Magnitudes and Units." *The Electrician*, September, 1894.

1895. AYRTON and MEDLEY. "Tests of Glow Lamps." *Phil. Mag.*, Vol. XXXIX., p. 389, May, 1895.

1896. FLEMING and PETAVEL. "An Analytical Study of the Alternating Current Arc." *Phil. Mag.*, April, 1896; or *Proc. Phys. Soc. Lond.*, Vol. XIV., p. 115.

1896. A. BLONDEL. "Rapport sur les Unités Photométriques." Congrès International des Electriciens, Geneve, 1896.

1896. F. P. WHITMAN. "On a Flicker Photometer." *Phys. Rev.*, Vol. III., p. 241.

1896. P. JANET. "Temperature of Glow Lamp Filaments." *The Electrician*, Vol. XXXVIII., p. 241.

Contains reference to Violle's work on the specific heat of carbon above  $1,000^{\circ}\text{C} = S = 0.355 + 0.00006 t$ . The boiling point of carbon is  $3,600^{\circ}\text{C}$ .

1896. Sir W. H. PREECE. "Electric Glow Lamps and the General Post Office Specification for Incandescent Lamps." *The Electrician*, Vol. XXXVII., p. 733.
1897. O. LUMMER. "Temperature of Initial Incandescence." *Science Abstracts*, Vol. I., p. 8, Abstract 19; see also *Annal. Phys. Chem.*, Vol. LXII., p. 14.
1898. O. W. ROOD. "On a Flicker Photometer." *Science*, Vol. VII., p. 757; also Vol. VIII. p. 11; also *Science Abstracts*, Vol. II., p. 10.
1898. E. F. GIBBON. "The Life and Efficiency of Incandescent Lamps." *Science Abstracts*, Vol. I., p. 308, Abstract 600; see also *Electrical World*, Vol. XXXI., p. 357, 1898.
- He gives typical curves of ageing.
1899. A. A. CAMPBELL SWINTON. "Luminosity of Rare Earths Heated in Vacuo by Cathode Rays." *The Electrician*, Vol. XLIII., p. 373.

### (b) Photometers.

1834. FOX-TALBOT. "Photometry." *Phil. Mag.*, Vol. V., p. 331.  
He describes the method of revolving sectors for reducing the intensity of a ray of light.
1854. H. KRÜSS. "Zwei Sätze über das Bunsen'sche Photometer." *Verhdlgn. d. Naturw. Ver. Hbg. N.F.*, 5, 61; *Zeitschr. f. Ang. Elektr. Lehre.*, 2, 460; Carl's Rep., 18, 54.
- 1854-62. FR. ARAGO. *Oeuvres Complètes*, 10, 184. (Polarisation Photometer.)
1879. AYRTON and PERRY. "A Dispersion Photometer." *Phil. Mag.* (5), 8, 117 and 9, 45; also *Proc. Phys. Soc.*, Vol. III., p. 184.
1879. F. GUTHRIE. "A New Photometer." *Chem. News*, 40, 262.
1880. A. CORNU. "Etudes Photométriques." *Jour. de Phys.*, 10, 189, and *La Lum. Electr.*, 3, 221.
1880. D. NAPOLI. "Un Nouveau Photomètre." *Séances de la Soc. de Phys. Franc.*, 1880, 53.

1881. H. KRÜSS. "Photometer und Helligkeitsmessungen." *C. Z. f. Opt. u. Mech.*, 2, 2.
1881. R. SABINE. "On a Wedge and Diaphragm Photometer." *Phil. Mag.* (5), 15, 22.
1882. E. C. PICKERING. "Wedge Photometer." *Nature*, July 13; *Zeitschr. f. Instrk.*, 2, 340.
1882. AYRTON and PERRY. "A Simplified Dispersion Photometer." *Proc. Phys. Soc. Lond.*, Vol. V., p. 109.
1883. L. SIMONOFF. "Sur un Photomètre Optique." *Comptes Rendus*, 97, 1,053.
1883. L. WEBER. "Zur Photometrie." *C. Z. f. Opt. u. Mech.*, 5, 181.
1883. M. HARTLEY. "Photométrie." *La Lum. Elec.*, 10, 58.
1883. Sir JOHN CONROY. *Proc. Roy. Soc.*, Vol. XXXV., p. 28; or *Phil. Mag.*, Vol. XV., p. 425.
1884. H. KRÜSS. "Eine Neue Form des Bunsen Photometers." *Abhdlgn. d. Naturw. Ver. Hbg.*, 8, 55; *C. Z. f. Opt. u. Mech.*, 5, 181; *Jour. f. Gasbel.*, 1884, 587; *Rep. d. Phys.*, 20, 729; *C. B. f. Elektrot.*, 6, 781.
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1885. W. J. DIBDIN. "Further Notes on the Radial Photometer." *Jour. Soc. of Chem. Ind.*, April 29.
1885. H. KRÜSS. "Das Compensations Photometer." *Jour. f. Gasbel.*, 1885, 685; *C. Z. f. Opt. u. Mech.*, 6, 219; *D. Gastechn.*, 5, 49; *C. B. f. Elektrot.*, 7, 716; *La Lum. Electr.*, 19, 118.
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1890. E. L. NICHOLS. "Note on a New Photometer." *Trans. Am. Inst. E. E.*, Vol. VII., May, 1890.
1891. LUMMER and BRODHUN. "Photometers." *The Electrician*, Vol. XXVI., p. 665; also Vol. XLIX., p. 324.

Good diagrams of an early form of Lummer-Brodhun photometer.

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1893. A. P. TROTTER. "A New Photometer." *Proc. Phys. Soc. Lond.*, June, 1893 ; or *Phil. Mag.*, July, 1893.

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An abstract of a Paper read before the International Congress of Electricians at Chicago, 1893. This Paper contains some excellent remarks on heterochromatic photometry ; also curves are given representing Purkinje's phenomenon and the distribution of visual and luminous intensity in the spectrum.

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1894. LUMMER and KURLBAUM. "On the Construction of a Bolometer." *The Electrician*, Vol. XXXIV., pp. 168 and 192.

1894. A. CROVA. "On the Degree of Incandescence of Glow Lamps." *The Electrician*, Vol. XXXIII., p. 754.

1895. PREECE and TROTTER. "A Portable Photometer." *The Electrician*, Vol. XXXV., p. 671.

Gives diagrams of an illumination photometer.

1898. GERMAN ELECTROTECHNICAL ASSOCIATION. "Rules for Photometry of Incandescent Lamps." *The Electrician*, Vol. XII., p. 426.

1900. J. VIOLLE. "Photometry." *The Electrician*, Vol. XLV., p. 858.

Abstract of a Report presented to the International Congress of Electricians at Paris.

1900. C. G. KNOTT. "On Swan's Prism Photometer." *Phil. Mag.*, January, 1900, Vol. XLIX. ; see also *Phil. Mag.*, June, 1900, Vol. XLIX.

## (c) Standards of Light.

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1879. E. L. NICHOLS. "Ueber das von Glühendem Platin Ausgestrahlte Licht." *Inaug. Diss. Göttingen*.
1879. J. VIOLLE. "Sur la Radiation du Platine Incandescent." *Comptes Rendus*, 88, 171.
1879. J. W. DRAPER. "A New Unit of Light." *Phil. Mag.*, 9, 76.
1881. H. GIRAUD. "Expériences sur les Bougies-Etalons." *Jour. d. Usines à Gaz.*, Déc., Paris.
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1883. V. HEFNER-ALTENECK. "Ueber Elektrische Lichtmessungen und Lichteinheiten." *Elek. Zeits.*, 4; *Jour. f. Gasbel.*, 1883, 830; *C. B. f. Elek.*, 5, 65.
1883. J. VIOLLE. "Intensités Lumineuses des Radiations Émises par le Platine Incandescent." *Comptes Rendus*, 92, 866.
1883. H. KRÜSS. "Optisches Flammenmass." *Jour. f. Gasbel.*, 1883, 717; *C. Z. f. Opt. u. Mech.*, 4, 277; *C. B. f. Elektrot.*, 5, 57.
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## CHAPTER IV.

### MAGNETIC AND IRON TESTING.

§ 1. **Magnetic Definitions.**—In practical magnetic measurements we are concerned with three quantities which have direction as well as magnitude in the mass of the material tested, and are therefore called the magnetic vectors. They are the *Magnetic Force*, the *Magnetisation*, and the *Magnetic Flux*. As a preliminary definition, we may say that the magnetic force is the name for the cause of magnetisation and magnetic flux considered as effects. As regards the distinction between these two last quantities, it may be said that, whilst magnetisation is the name for the state into which certain metals can be thrown, the term magnetic flux is more general, and describes a physical condition which can be produced in all substances, or even in empty space.

The magnetic flux and magnetisation produced by any given magnetic force are determined by the specific nature and geometrical form of *Magnetic Circuit*. This latter may be described as the path or region in which these effects take place. The magnetic circuit may be either empty space or air, or it may consist of various material substances. We may arrange all substances of which a magnetic circuit can be formed in three classes:—(1) Ferromagnetic, (2) Feebly-magnetic, (3) Diamagnetic. The magnetisation and magnetic

flux are thus names used to describe the physical condition produced in the various portions of a magnetic circuit by magnetic force.

Mathematically speaking, quantities are called *circuital* when they can only exist in any place, if they exist at the same time, all along a certain closed line called the circuit. The magnetic flux is always a circuital quantity, but the magnetisation may or may not be circuital. Since these quantities are vectors, they may be delineated by lines, the direction of which at any point indicates the direction of the vector. In the case of circuital quantities, these lines will be self-closed lines.

In ferromagnetic bodies, if the state called magnetisation exists all along certain closed lines, or is circuital, the magnetic condition is called *apolar*. If, on the other hand, the lines of magnetisation are terminated, then the terminations are termed *magnetic poles*, and the magnetisation is said to be *polar*. In this case the ends of the lines of magnetisation are fresh sources of magnetic force.

The magnetisation as a physical state, although always produced by magnetic force, may remain in ferromagnetic bodies after the force is withdrawn, and in this case the body is called a *permanent magnet*. The quality of the substance in virtue of which magnetisation remains when the magnetic force is withdrawn is called its *retentivity*. In the case of the feebly magnetic bodies and diamagnetic bodies there is no retentivity, and hence no permanent magnetisation, as in the ferromagnetic bodies, can exist. Corresponding to any given force, the magnetisation may be more or less, according to the magnetic *susceptibility* of the body. The susceptibility in the case of magnetic bodies is considered to be positive, and, in the case of diamagnetic, negative.

As examples of ferromagnetic bodies characterised by the possibility of becoming permanently magnetised, we have iron, nickel, and cobalt, and certain oxides, carbides, and sulphides of these metals. As examples of feebly magnetic bodies, we may

instance oxygen, palladium and manganese, and as examples of diamagnetic bodies, antimony, bismuth, and phosphorus.

A magnetic circuit which consists wholly of ferromagnetic bodies is called an iron circuit, or ferromagnetic circuit, or sometimes a closed magnetic circuit. A circuit which consists partly of ferromagnetic and partly of feebly magnetic or diamagnetic bodies is called a compound magnetic circuit or an open magnetic circuit. In this last form of circuit magnetisation can exist in the parts which are ferromagnetic, but magnetic flux can exist in the whole of the circuit. When the magnetic circuit is wholly or partly through the air, this part is generally spoken of as a *magnetic field*, but the term may also be applied to denote any space in which magnetic flux exists.

There are two sources of magnetic force—polar magnets, and conductors traversed by electric currents. In the neighbourhood of these bodies magnetic flux exists distributed in the space round them, and, if ferromagnetic bodies are placed in this space, these masses will exhibit magnetisation either polar or apolar, according to their shape and disposition.

If we consider a very long thin bar of ferromagnetic material having a cross-section of  $S$  square centimetres possessing a uniform magnetisation,  $I$ , in the direction of its length, then the product  $IS$  is called the *magnetic strength* of the bar. The magnetic force at a short distance,  $r$  centimetres from one of the ends, is numerically equal to  $IS/r^2$  C.G.S. units. At any other point the magnetic force is the resultant of two forces of magnitude respectively equal to  $IS/r_1^2$  and  $IS/r_2^2$ , where  $r_1$  and  $r_2$  are the distances from the point in question to the two ends or poles of the bar.

The magnetic force in the neighbourhood of a long straight conductor conveying an electric current  $C$  (reckoned in electromagnetic units) is numerically equal to the product  $2C/r$ , or to  $A/5r$  if the current  $A$  is measured in amperes, where  $r$  is the shortest distance from the point to the conductor. The magnetic force at any point,  $P$ , due to any element of length,  $\delta x$ , of a circuit conveying a current,  $C$ , in E.M. units is equal to  $\frac{C \cdot \delta x \cdot \sin \theta}{r^2}$ , where  $r$  is the distance from  $P$  to the element and  $\theta$  is the angle between this radius vector  $r$  and the direction of the element  $\delta x$ .

For the proper expressions for the magnetic force due to conductors of various forms traversed by electric currents,

the reader is referred to the Author's treatise on "The Alternating Current Transformer," 3rd edition, Vol. I, p. 15, *et seq.*\*

By the term *line-integral of magnetic force* along any line is meant the sum of all the products obtained by multiplying the length of each element of that line by the magnetic force at its centre reckoned in the direction of the element. Thus, if at any point in the field the magnetic force is  $H$ , and if an element of length of any line drawn in that field makes an angle,  $\theta$ , with the direction of the force at that point, the product  $H \cos \theta \delta x$ , summed up all along the line, is the line-integral of magnetic force.

We have two cases to consider :—(i.) When the line chosen is a terminated line. In that case the line-integral of magnetic force is called the *difference of magnetic potential* between the ends of the line or the *fall of magnetic potential* along the line. (ii.) If the line is a self-closed line, then, if it is drawn in a magnetic field produced by polar magnets, the line-integral is always zero. If, however, the closed line is drawn in a field which is produced by electric currents, so that the loop embraces a conductor and is perforated by a current, then it can be shown that the line-integral of magnetic force taken once round this closed line is equal to  $4\pi$  times the total current passing through it; and the line-integral in this case is called the *magnetomotive force* along this line, and is denoted by M.M.F.

If  $C$  stands for the total current in electromagnetic units, then the above law is expressed as follows :

$$\text{M.M.F.} = 4\pi C. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the current is measured in amperes,  $A$ , then, since an

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\* For a method of deducing the expressions for the magnetic force due to a straight, circular or solenoidal current see Messrs. C. T. Hutchinson and E. N. Carichoff, *The Electrician*, Vol. XXXIV., p. 606. Also, the reader may consult articles on the subject by Prof. Minchin, *The Electrician*, Vol. XXXIV., p. 48, and Mr. A. Russell, *The Electrician*, Vol. XXXVI., p. 50, and by Prof. Everett, in *The Electrician*, Vol. XXXVI., p. 83.

ampere is one-tenth of an absolute electromagnetic unit of current, we have

$$\text{M.M.F.} = \frac{4\pi A}{10} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the electric current is conveyed  $N$  times through the circuit along which the magnetomotive force is reckoned, by means of  $N$  turns of insulated wire, and if each wire carries a current of  $A$  amperes, the relation is expressed as follows:

$$\text{M.M.F.} = \frac{4\pi}{10} NA \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The quantity  $\frac{4\pi}{10}$ , or  $0.4\pi$ , is so nearly equal to  $1.25$ , and its reciprocal is so nearly equal to  $0.8$ , that the above equation may be written in the following manner:

$$NA = 0.8 \text{ M.M.F.}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

that is,

The Magnetomotive Force =  $1\frac{1}{4} \times$  The Ampere-turns,  
or, reciprocally,

The Ampere-turns =  $0.8$  times the Magnetomotive Force.

Another important relation is that which connects magnetomotive force and magnetic flux. In any simple magnetic circuit the ratio of the magnetomotive force reckoned along the circuit to the magnetic flux in the circuit is called the *reluctance* of that circuit. If the magnetic circuit is a compound circuit, and if the flux in the circuit is identical at all points of the cross section, then the law may be generalised by saying that the magnetomotive force along the circuit is numerically equal to the product of the magnetic flux and the sum of the reluctances of each part of the circuit. This may be symbolically expressed as follows:

$$\text{M.M.F.} = \text{flux} \times \Sigma (\text{reluctances}). \quad . \quad . \quad . \quad (5)$$

In order to fix our ideas, we may consider a simple magnetic circuit consisting of an iron ring, the section of the ring being  $S$  and the mean perimeter of the ring being  $L$ , the radius of cross-section of the ring being small compared

with the mean radius of the ring itself. Imagine the ring to be wound uniformly with  $N$  turns of insulated wire so as to form an endless solenoid of  $N$  turns. We have then two interlinked circuits: First, an electric circuit composed of the insulated wire, which we will suppose traversed by a current of  $A$  amperes, the total current therefore flowing round the ring is  $NA$  amperes. Secondly, we have a magnetic circuit formed of the iron ring of length  $L$  and section  $S$ . By the first circuital law the magnetomotive force round the magnetic circuit is equal to  $4\pi/10$  times the total current embracing it. Hence in this case we have

$$\text{MMF} = 0.4\pi NA.$$

Let  $Z$  stand for the *total flux* in the iron, which is assumed to be the same at all sections, and let  $R$  stand for the reluctance of the magnetic circuit, then, in virtue of the above stated connection between magnetomotive force and magnetic flux, we have the following equation:

$$\text{M.M.F.} = ZR.$$

Hence

$$NA = 0.8ZR.$$

The reluctance of a simple magnetic circuit of the above form is proportional to its length and inversely as its cross-section, and proportional to a specific quality of the substance called its *reluctivity*, denoted generally by the symbol  $\rho$ ; hence  $R = \frac{\rho L}{S}$ . Substituting this value in the above equation, we have finally

$$NA = 0.8Z \frac{\rho L}{S}.$$

The above is the fundamental magnetic equation for a simple magnetic circuit. This equation tells us the ampere-turns which must be put upon the ring to produce a given flux in the ring. The reciprocal of the *reluctivity*  $\rho$  is called the *permeability* of the material, and is denoted by the letter  $\mu$ . Hence the above equation may also be written

$$NA = 0.8Z \frac{L}{\mu S} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The reciprocal of the *reluctance* is called the *permeance* of the magnetic circuit. Hence the permeance is measured by the quantity  $\frac{\mu S}{L}$ .

We can only make use of the above equations to determine the ampere-turns corresponding to a given flux when we know the value of the permeability of the material. Although in mathematical form these equations for the magnetic circuit resemble the various modes of stating Ohm's law for the electric circuit, there is yet an important difference. The value of the electrical resistance or resistivity of the electric circuit is independent of the current, whereas the magnetic reluctance and reluctivity depend upon the flux density in the circuit considered. The ratio of the flux density (**B**) to the magnetic force (**H**) at any time and place in the magnetic circuit is a measure of the permeability ( $\mu$ ) of the material at that time and place, and this last is therefore defined by the equation

$$\mathbf{B} = \mu \mathbf{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

An important magnetic measurement is, therefore, the determination of the permeability of a magnetic material. The measurement of the flux density **B** may be made by taking advantage of Faraday's fundamental discovery that, if a magnetic circuit is embraced by an electric circuit, and if any variation is made in the total magnetic flux in the former, an E.M.F. is created in the latter the magnitude of which, at any instant, is measured by the rate at which the flux is changing. Symbolically this may be expressed as follows:—If  $e$  is the E.M.F. created in an electric circuit, which embraces a magnetic circuit, and if the last circuit has a section **S** and the flux density in it at any moment is **B**, then

$$e = - \frac{d(\mathbf{SB})}{dt},$$

or

$$e = -S \frac{d\mathbf{B}}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This is sometimes written

$$e = -S\dot{\mathbf{B}} = \dot{Z},$$

where  $\dot{\mathbf{B}}$  stands for the time-rate of change of  $\mathbf{B}$  and  $\dot{Z}$  for the time-rate of change of  $Z$ .

The above equation is the symbolical expression for the fact that when an electric circuit and a magnetic circuit are linked together, then any variation of the total magnetic flux in the one sets up an E.M.F. in the other which at any instant is measured by the time-rate of change of the flux.

We may, therefore, give a more exact definition of the meaning of the term Magnetic Flux ( $Z$ ) by saying that it is that quality or state of a magnetic circuit the time variation of which produces an E.M.F. in a conducting circuit linked with that magnetic circuit. If the flux is distributed uniformly over the cross-section of the circuit, then the flux per square centimetre is the flux density ( $\mathbf{B}$ ). The magnetic flux is measured in units such that the change of one unit per second through a conducting circuit of one turn produces an E.M.F. of one unit in that circuit. If our unit of E.M.F. is one C.G.S. electromagnetic unit, then the corresponding unit of flux is called *one line*. If the unit of E.M.F. is one volt ( $=10^8$  C.G.S. units), then the corresponding unit of magnetic flux is called *one weber* ( $=10^8$  lines). One thousand "lines" is usually called one *kiloline*.

The magnetic permeability of air or empty space is taken as equal to unity in the C.G.S. system. Hence, in the case of air circuits the same number denotes both the magnetic force and the magnetic flux density. Thus, if an electric current of strength  $A$  amperes flows in an infinitely long straight circuit, both the magnetic force and the magnetic flux density in the air at any place distant by  $d$  centimetres are numerically equal to  $\frac{A}{5d}$ . Hence the magnetomotive force or line integral along a line drawn round such a straight current is equal to  $\frac{4\pi}{10}A$ , and is independent of the path.

If a long straight bar of steel is permanently magnetised with a magnetisation  $I$ , and if the section of the bar is  $S$ , then  $IS$  is called the magnetic strength of the bar. The magnetic force and magnetic flux density at a place distant  $d$  centimetres from one end, provided  $d$  is small compared with the length of the bar, is  $IS/d^2$ . If, therefore, we describe round one end of the bar a small sphere of radius  $d$ , the whole flux through this sphere must be

$$\frac{IS}{d^2} \times 4\pi d^2 = 4\pi IS.$$

Hence the whole magnetic flux up and along the bar must be  $4\pi I S$ , and the flux density in the bar  $\mathbf{B}$  must be such that  $\mathbf{B}=4\pi\mathbf{l}$ . We have, therefore, the two important relations between fundamental quantities in the case of an electric current and a permanent magnet,

$$\text{M.M.F.} = 4\pi C.$$

$$B = 4\pi I,$$

where  $C$  is the total current in an electric current and  $M.M.F.$  is the magnetomotive force along any closed line drawn round it, and  $I$  the magnetisation in any magnetic circuit and  $B$  the flux density in it.

Suppose, then, that we have an iron ring of section  $S$  and mean perimeter  $L$  wound over with  $N$  turns of insulated wire, and this wire is traversed by a unit current. The current creates in the interior of the closed solenoid a magnetic force  $H$  equal to  $4\pi \cdot N/L$ . If the ring on which the wire is wound is only feebly magnetic or diamagnetic, the flux density in the interior is represented by the same number as the magnetic force. If, however, the core is iron, then it becomes magnetised, and has created in it an additional flux equal to  $4\pi IS$ . Hence the total flux  $Z$  in the iron ring is given by the equation

$$Z = HS + 4\pi IS,$$

and the flux density  $Z/S = \mathbf{B}$  is given by

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The above is the general equation connecting together magnetic flux density, magnetisation and magnetic force in a complete ferromagnetic circuit. If the flux and force are known or measured, the equation enables us to calculate the magnetisation. In a ferromagnetic circuit, when the magnetic force changes its value the flux density and the magnetisation do not change immediately to the corresponding new values, but there is a delay, which has been named *hysteresis*. If the magnetic force is removed, then, generally speaking, magnetisation will remain in a sample of ferromagnetic material, and this quality is called its *retentivity*. The retentivity, therefore, is merely a particular instance of hysteresis.

We may, then, summarise the magnetic equations for a simple closed ferromagnetic circuit, such as an iron toroid or ring above described, as follows:—

1. Equation connecting magnetic force  $H$ , magnetic flux density  $B$ , and defining permeability  $\mu$ :

$$B = \mu H, \quad \text{or} \quad \mu = B/H.$$

2. Equation connecting magnetic force  $H$ , magnetisation  $I$ , and defining susceptibility  $k$ :

$$I = kH, \quad \text{or} \quad k = I/H.$$

3. Equation connecting magnetic force  $H$ , magnetisation  $I$ , and magnetic flux density  $B$ :

$$B = H + 4\pi I.$$

4. Equation connecting permeability  $\mu$  and susceptibility  $k$ :

$$\mu = 1 + 4\pi k.$$

5. Equation connecting magnetomotive force M.M.F., magnetic force  $H$ , and length of magnetic circuit  $L$ , in the case of a uniform and uniformly magnetised simple circuit:

$$\text{M.M.F.} = HL.$$

6. Equation connecting magnetomotive force M.M.F. and exciting ampere-turns AN for the above circuit :

$$\text{M.M.F.} = \frac{4\pi}{10} \text{AN} = 1\frac{1}{4} \text{AN},$$

or, reciprocally,  $\text{AN} = 0.8 \text{ M.M.F.}$

7. Equation connecting total magnetic flux Z, magnetomotive force M.M.F., and magnetic reluctance R, or relativity  $\rho$  and permeability  $\mu$  :—

$$Z = \frac{\text{M.M.F.}}{R},$$

or 
$$\text{M.M.F.} = ZR = Z \frac{\rho L}{S} = Z \frac{L}{\mu S}.$$

8. Equation connecting the exciting ampere-turns AN with the permeability  $\mu$  and dimensions of the circuit and the total flux in it :

$$\text{AN} = 0.8Z \frac{L}{\mu S} \text{ (for a simple circuit),}$$

$$\text{and AN} = 0.8Z \left( \frac{L_1}{\mu_1 S_1} + \frac{L_2}{\mu_2 S_2} + \&c. \right) \text{ (for a compound circuit).}$$

For additional information on the Laws of the Magnetic Current the reader is referred to the following books :—

J. A. FLEMING. "Magnets and Electric Currents." Chap. III., p. 4, 2nd ed. (Spon.)

The reader may be referred in particular to § 11 of Chapter III., in which Mr. Oliver Heaviside's system of rational magnetic units is explained.

J. A. FLEMING. "The Alternate Current Transformer." Vol. I., Chap. II., 3rd ed. ("The Electrician" Printing and Publishing Co., London.)

H. DU BOIS. "The Magnetic Circuit in Theory and Practice." (English Translation by Atkinson.) (Longmans.)

J. A. EWING. "Magnetic Induction in Iron and other Metals." ("The Electrician" Company.)

S. P. THOMPSON. "The Electromagnet." (Spon.)

SHELFORD BIDWELL. Article on "Magnetism." *Encyclopædia Britannica Supplement*, Vol. XXX., 1902.

J. A. FLEMING. Article on "Electromagnet." *Encyclopædia Britannica Supplement*, Vol. XXVIII., 1902.

§ 2. **Magnetisation Curves.**—The relation between the magnetic force, the magnetic flux, and magnetisation in certain standard-shaped masses of ferromagnetic materials can be represented graphically by means of curves called magnetisation curves. Magnetisation curves are of two classes: (1) Single-valued, and (2) cyclical. In these curves the abscissæ are generally taken to represent magnetic force and the ordinates magnetic flux or magnetisation, and the curve may be marked with an arrow to show whether it corresponds to increasing or decreasing magnetic forces. A cyclical magnetisation curve shows the change in magnetisation or magnetic flux which takes place in a magnetised body when the magnetic force is carried round through a complete cycle of changes, beginning with a zero value, rising up to a certain maximum value, then falling down again to zero, then becoming reversed in direction and reaching an equal negative maximum, and, lastly, returning again to zero.

It can easily be shown that the area enclosed by such a cyclical magnetisation curve represents the amount of work done in carrying the ferromagnetic material through the magnetic cycle.

Thus, suppose the above operations are conducted upon an iron ring, the cross-section of which is  $S$  and the mean perimeter  $L$ , the diameter of the section  $S$  being small compared with the mean diameter of the ring. Let the ring be wound over with  $N$  turns of insulated wire, by means of which a current of  $A$  amperes is applied to magnetise it. Let  $N$  be the terminal potential difference in volts at the ends, and  $R$  the electrical resistance in ohms of this circuit. Then we have the following equation connecting the current and voltage:—

$$A = \frac{V - SN \frac{db}{dt} \frac{1}{10^8}}{R},$$

where  $b$  represents the flux density in C.G.S. units at any instant existing in the iron. Hence it follows that

$$AVdt = A^2Rdt + \frac{1}{10^8}SNAdb. \quad (10)$$

The term on the left-hand side represents the energy in joules given in the small time  $dt$  to the system, and the first term on the right-hand side represents the energy dissipated in heating the electric circuit in the same time. Hence the term  $\frac{1}{10^8}SNAdb$  must represent the energy in joules expended in changing the flux by an amount  $db$ . Since 1 joule =  $10^7$  ergs, we may write the expression for this magnetising energy expressed in ergs in the form  $0.1(NA)d(Sb)$ , and we see that the energy in ergs required to make the total flux change  $d(Sb)$  is equal to the product of one-tenth part of the ampere-turns and the total flux change in C.G.S. units. Again, since the volume of the iron is  $SL = v$ , and since the magnetic force  $H$  is equal to  $\frac{4\pi}{10} \frac{AN}{L}$  in C.G.S. units, we have

$$0.1(NA)d(Sb) = \frac{4\pi}{10} \frac{NA}{L} \frac{v}{4\pi} db = \frac{v}{4\pi} Hdb.$$

Hence the energy required per cubic centimetre to make a change in the flux density equal to  $db$  is equal to

$$\frac{1}{4\pi} Hdb.$$

Integrating this throughout any range of flux density change, we see that the energy reckoned in ergs per cubic centimetre required to change the flux density from  $B_1$  to  $B_2$  in a field  $H$  is equal to

$$\frac{1}{4\pi} \int_{B_1}^{B_2} Hdb. \quad (11)$$

Again, since  $B = H + 4\pi I$ , and since the integral  $\oint HdH$  is zero throughout any complete cycle, we find that the energy required to carry any magnetic substance through a complete cycle of magnetisation reckoned in ergs per cubic centimetre is given by the integral  $\oint HdI$ . If, therefore, we draw any cyclical magnetisation curve in terms of  $H$  and  $B$ , and take the area of this curve, this area, divided by  $4\pi$ , represents the work done in ergs per cubic centimetre in making one complete magnetic cycle of flux.

The area must be measured in terms of that of a rectangle one side of which is the length taken to represent one C.G.S. unit of magnetic force, and the other side the unit of magnetic flux density, or *one line*.

Magnetisation curves can be plotted in terms of *Magnetic Force* reckoned on the C.G.S. system ( $H$ ) and *Magnetic Flux Density* reckoned in "lines" ( $B$ ), or in terms of *Ampere-turns per Centimetre* ( $AT$ ) and of "kilolines." Each is equal to

1,000 C.G.S. units of magnetic flux density. This last method is the usual one in the case of curves drawn for use in electrotechnical work.

In Figs. 1, 2 and 3 are shown the typical forms of magnetisation curve for iron forgings, cast steel and armature stampings delineated in terms of the flux density  $B$  in C.G.S. units or lines, and the magnetic force  $H$  in C.G.S. units.

The curves on pp. 379, 380 and 381 are taken from a Paper on magnetic materials for dynamos published in *The Electrician* for Sept. 17, 1897, Vol. XXXIX., p. 670. They are not merely laboratory experiments, but are taken from experiments made in the testing room on forgings, castings and stampings actually used in the best practice in dynamo building. The group of curves in Fig. 1 are iron or steel forgings technically known as ingot-iron or in German as flusseisen. They are practically soft steels with a very small percentage of carbon in them—in fact, very nearly pure iron. The two dotted curves are from experiments on wrought iron made by Dr. J. Hopkinson in 1885, whilst the firm line curves are 1897 materials. The curves in Fig. 2 are steel castings, and show for very high flux densities equal or greater permeability than the iron forgings. The curves are drawn in two sets, the group B being simply the lower parts of the curves in group A. In Fig. 3 are given the magnetisation curves of armature stampings. The curves 2 and 4 are for best Swedish iron. Curve 1 is a very special Swedish iron of high permeability at low flux densities, and therefore suitable for transformer cores.

### § 3. Standard Forms of Magnetic Test Pieces and Coils.—

The practical magnetic measurements which have to be made in the electrical laboratory to decide the value of an iron or steel for electrotechnical purposes involve tests for the purpose of determining the magnetic permeability and hysteresis corresponding to certain magnetic flux densities. These experiments have to be made on “test pieces” of the

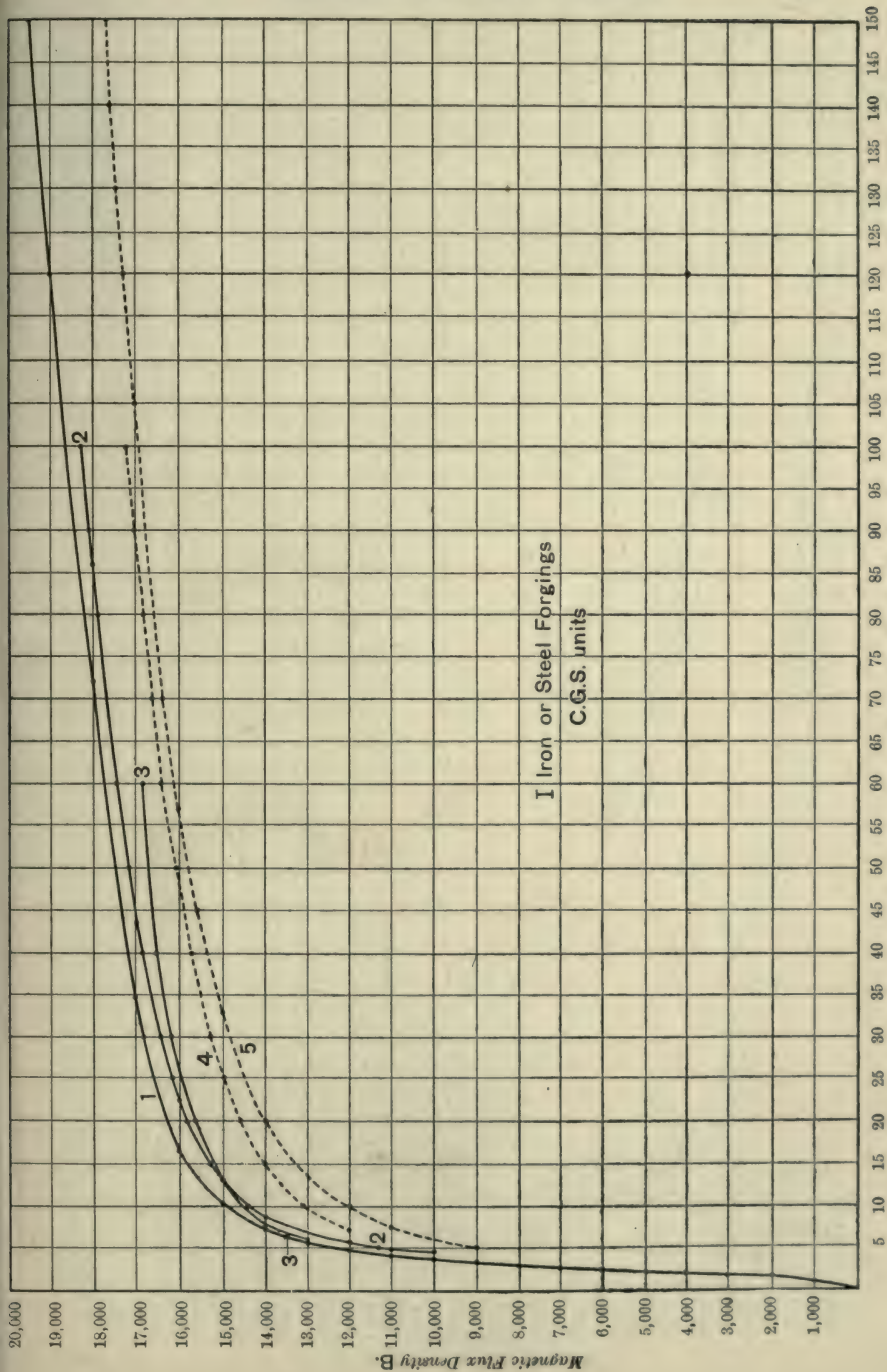


FIG. 1.

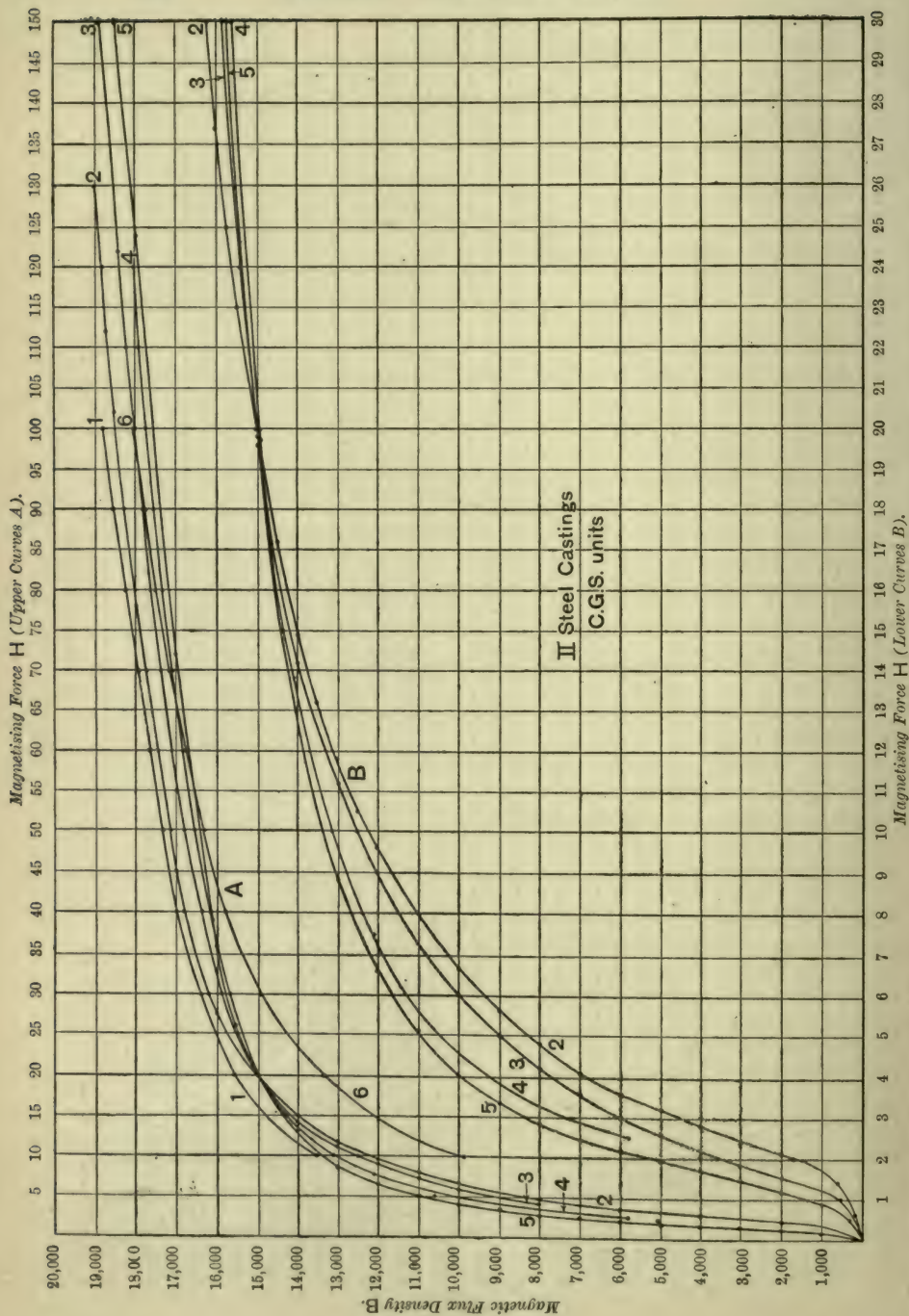


Fig. 2.

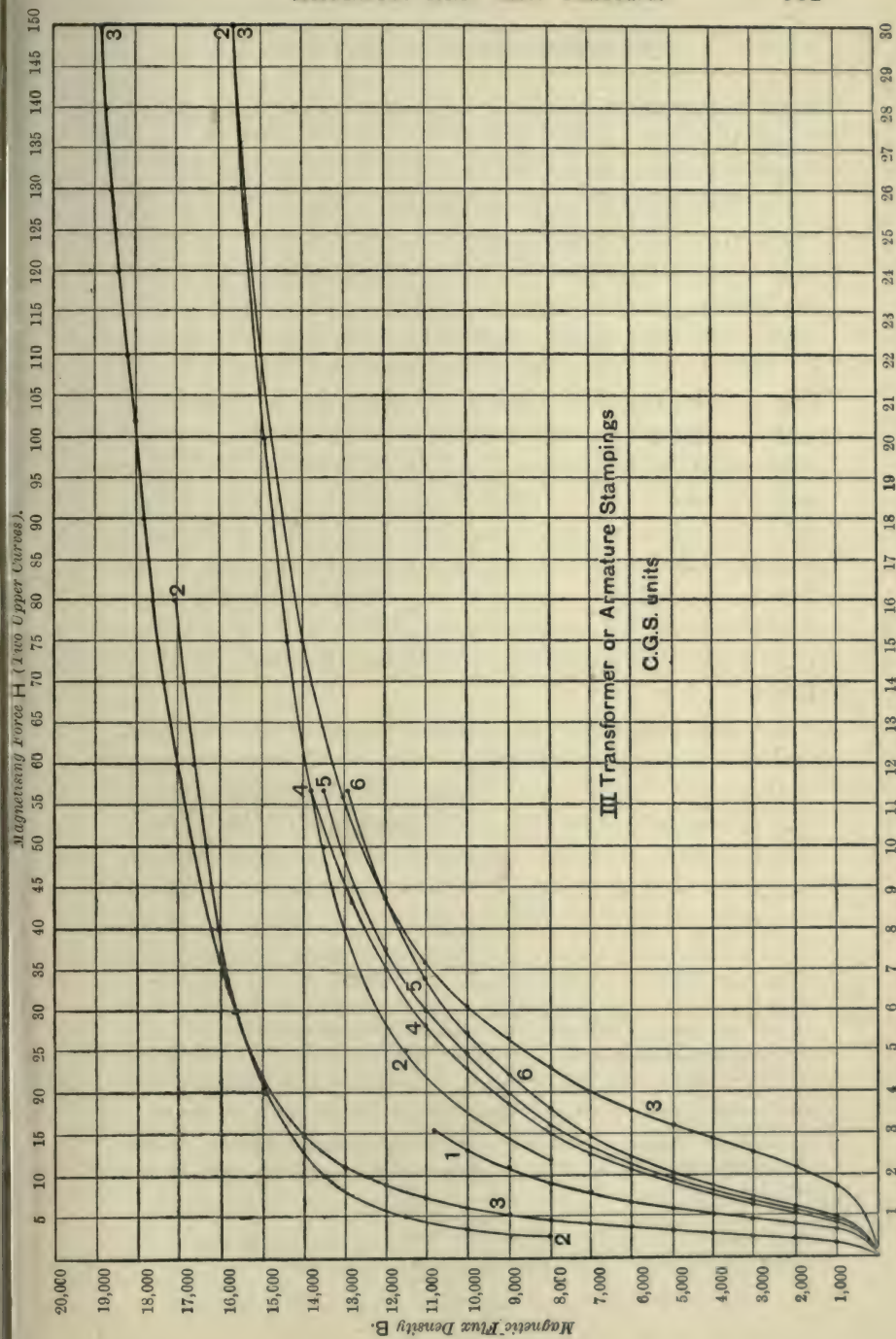


FIG. 3.

metal in question, which must be provided in certain standard shapes. We also require magnetising coils of certain standard forms to create known or predeterminable magnetic forces.

Since our object is to determine the magnetic state in the interior of the iron when subjected to a known magnetic force, it is clear that the form of the specimen must be such that the effective magnetic force on it can be calculated from the form of the magnetising coil and the current passing through it. There are only a very limited number of cases in which this can be done. Generally speaking, the magnetisation which a sample of ferromagnetic substance takes when subjected to magnetic force is polar and irregular. This polar magnetisation creates a counter-magnetic force which reacts upon the impressed or external force, so that at any time it is difficult to determine the resultant or effective magnetic force. In other words, the magnetic force operative when the ferromagnetic substance is placed in a field of force is by no means the same as that at the same point when the specimen is not there.

The most simple and, in fact, unique case of magnetisation is that of a ring magnetised by the magnetic force due to an endless solenoid. We may consider, in the first place, the field or magnetic force produced in the interior of an endless solenoid when a current passes through it. (*See* Note B, Appendix to Vol. I., 3rd edition, "The Alternate Current Transformer," by J. A. Fleming.)

Let the solenoid have a circular section and a circular perimeter, let  $a$  be the mean radius of the circular cross-section of the solenoid, and let  $R$  be the radius of the circular axis of the solenoid. The whole cross-section of the solenoid may be considered to be resolved into elementary solenoids. Let  $dS$  be the cross-section of one of these, and let  $x$  be the radius of the circular axis of the circular elementary solenoid. Let  $N$  be the total number of turns of wire on the ring. Then, for the elementary solenoid there are  $\frac{N}{2\pi x}$  turns per unit of length. Hence the magnetic force in the interior of the elementary solenoid is  $H$  along the line of radius  $x$ , and is given by the equation

$$H = \frac{4\pi}{10} \frac{NA}{2\pi x} = \frac{NA}{5x},$$

where  $A$  is the current in the solenoid reckoned in amperes.

If the core is non-magnetic, the magnetic force is the same as the flux density, as far as mere value is concerned. Hence the magnetic flux in the interior of an elementary solenoid of section  $dS$  is  $\frac{NA}{5x} dS$ . Hence the total flux  $Z$  through the whole solenoid is obtained by integrating this last expression over the area of the solenoid, viz.,  $\pi a^2$ . Therefore, between proper limits,

$$Z = \frac{NA}{5} \int \frac{dS}{x} \quad \dots \dots \dots (12)$$

Now, the integral  $\int \frac{dS}{x}$  taken over the circular cross-section of radius  $a$  can be shown to be equal to  $2\pi \{R - \sqrt{R^2 - a^2}\}$ , and hence,

$$Z = \frac{2\pi NA}{5} \{R - \sqrt{R^2 - a^2}\} \quad \dots \dots \dots (13)$$

This, then, is the expression which should be employed for the total flux through the circular cross-section of the ring if the mean radius of cross-section  $a$  is not very small compared with the radius of the circular axis  $R$ .

If  $\frac{a}{R}$  is a small quantity, then, since

$$\{R - \sqrt{R^2 - a^2}\} = \frac{1}{2}R \left( \frac{a^2}{R^2} + \frac{1}{4}\frac{a^4}{R^4} + \frac{1}{8}\frac{a^6}{R^6} + \&c. \right),$$

we have  $R - \sqrt{R^2 - a^2} = \frac{a^2}{2R}$  nearly, when  $\frac{a}{R}$  is small, and hence

$$Z = \frac{4\pi NA}{10} \frac{a^2}{2R} = \frac{4\pi}{10} \frac{NA}{2\pi R} \pi a^2,$$

and

$$B = Z/\pi a^2 = 0.4\pi NA/2\pi R.$$

Accordingly, the total flux is  $4\pi$  times the current turns per unit of length of the solenoid multiplied by the area of cross-section of the solenoid, and for a very large thin circular solenoid the magnetic force in the centre is  $4\pi$  times the current turns per unit of length. The magnetic force in the interior of the solenoid is everywhere in the direction of the circular axis. Hence, when the solenoid surrounds an iron ring, the magnetisation and magnetic flux in the iron are of the same type. The magnetisation is *apolar*—there are no *ends* and no self-demagnetising force. Accordingly, the effective magnetic force is the same as the impressed magnetic force. The flux can be experimentally determined as shown subsequently, and we have, therefore, all the information necessary to determine the magnetisation curve.

In order, however, that all parts of the metal may be subjected to the same magnetising force, the solenoid must have

a form such that its radial width is small compared with the mean radius of the solenoid. Hence, it is best to give it a rectangular section, and in cases in which the diameter of the solenoid over all cannot be very large, the form should be that produced by winding wire round a thin cylinder, as shown in Fig. 4. There is then no doubt that the magnetic force acting on the metal is everywhere the same, and is equal to the value of  $1.25 AN/L$ .

In the construction of a ferromagnetic ring for experimental purposes, it is essential that the ring should be cut out of the solid material, and that no joint should be made. It is, for instance, not permissible to construct a ring of iron

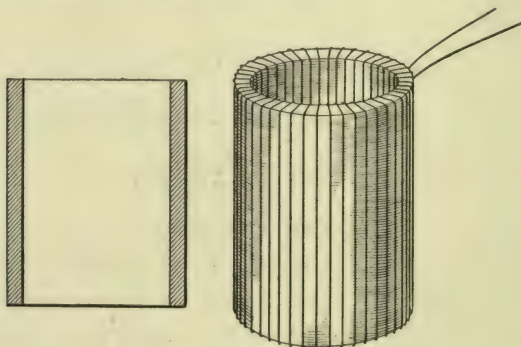


FIG. 4.

by welding the ends of a bar together, as the weld has always different magnetic qualities to the rest of the material.

After the ring has been cut to shape it should be annealed, and its dimensions carefully taken in order to determine the length of the mean perimeter and the mean cross-sectional area. For this purpose it is generally sufficient to take eight measurements, at angular distances of  $45^\circ$ , of the inside and outside diameters of the ring, and the depth of the ring. From these dimensions we determine the mean diameter of the ring, and therefore the mean perimeter. A ring of this kind may also be formed by piling, one on the top of the

other, flat circular rings of sheet metal, and for some purposes, such as hysteresis measurements, it is essential that these rings should be very thin, and should be insulated from one another by varnish or thin paper. In this case it is necessary to deduct from the total cross-section of the ring the amount of space occupied by the insulating material in order to arrive at the true cross-section of the metal.

In the case of the measurement of permeability where a large specimen of the metal is not available, the best form to give the specimen is that of a thin-walled circular cylinder. Such a sample may, for instance, take the form of a cylinder 4 or 5 centimetres in height, 3 or 4 centimetres internal diameter, and 3 or 4 millimetres in thickness. A cylinder of this description is easily turned out of the solid metal, and then can be annealed without losing its regularity of form. In all cases where the ballistic galvanometer is employed it is essential to bear in mind that the magnetic flux does not rise up to its full value instantly in a mass of ferromagnetic material on the application of a certain magnetic force. In a large mass of solid iron which is subjected to sudden magnetic force, or sudden reversal of the magnetic force, the changes in magnetic flux proceed very slowly from the surface to the centre of the mass. Thus, in a large bar of iron 12in. in diameter, magnetised by an electric current flowing round its surface through a magnetising coil, about 60 seconds would elapse before the magnetic flux was established uniformly over the cross-section of the iron, assuming the magnetic force to be such as would produce the state popularly known as saturation. In bars or rods of different sizes the time taken to establish the steady magnetic state varies as the square of their diameter, and even in a comparatively small rod quite a sensible time elapses after the application of a magnetic force before the corresponding magnetic flux is fully established in all parts of the iron. Hence, since the ballistic galvanometer can only be employed on the assumption that the flow of electricity through it which

has to be measured is completed before the needle or coil is sensibly displaced from its zero position, it follows that the ballistic method is only valuable in those cases in which the change of flux in the iron corresponding to a change of magnetic force is practically instantaneous. This is only the case when we are employing rings, cylinders or rods the thickness of which does not exceed, say, a centimetre.

On this subject the reader may, with advantage, consult a Paper by Dr. J. Hopkinson and Prof. E. Wilson, on the "Propagation of Magnetisation into Iron," *Trans. Roy. Soc.*, or *The Electrician*, Vol. XXXIV., pp. 510, 535 and 792; or *Journal Inst. Elec. Eng.*, Vol. XXIV., p. 194, 1895. It is there shown that the time required to establish any given state of magnetisation varies as the square of the linear dimensions. In the case of a cylinder of iron 12in. in diameter, magnetised by an encircling magnetisation coil, the time required to establish the steady magnetic state approaching uniform saturation is about 60 seconds. Hence, in a bar only 1in. in diameter the time required is still quite perceptible. The magnetic flux, so to speak, soaks in from the surface to the centre of the mass. The law of governing this inward propagation of magnetic flux is the same as that controlling the similar surface-to-centre soaking of electric current in an electric conductor. It is a case of diffusion. The magnetic flux and the electric current both begin at the surface of the circuit and diffuse inwards. In addition to this there is a time effect which depends on a quality which has been called magnetic viscosity.\*

On this subject the reader may consult O. Heaviside's "Electromagnetic Theory," Vol. I., § 191, Chapter IV.; also J. A. Ewing's "Magnetic Induction in Iron and other Metals," Chapter VI., § 88 ("The Electrician" Printing and Publishing Co., London).

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\* See also Dr. J. Hopkinson and Mr. B. Hopkinson on "Magnetic Viscosity," *The Electrician*, Vol. XXIX., p. 510; also Hopkinson, Wilson and Lydall on "Magnetic Viscosity," *The Electrician*, Vol. XXXI., p. 69. For other references to magnetic after-effects see I. Klemencic, *The Electrician*, Vol. XL., p. 411, and C. Fromme, *The Electrician*, Vol. XLI., p. 243.

The moment, however, that we deal with a specimen of ferro-magnetic material which is not ring-shaped we are confronted by the complication introduced by the self-demagnetising force of a non-endless magnetised bar. This may be illustrated most simply as follows:—Imagine an infinitely long straight solenoid in the interior of which there is a uniform magnetic force  $H_0$ . Let a thin iron wire of section  $s$  be placed in this field, with its length parallel to the field. Then it takes a magnetisation  $I$ , and from each pole a magnetic force radiates equal to  $\frac{Is}{r^2}$  at a distance  $r$  from the pole. The magnetic force due to these free poles acts on the mass of the wire in a direction which is opposed to that of the field  $H_0$ . Hence it is called a counter-magnetic force or *demagnetising force*. This latter has a different value at all points in the wire, but it is otherwise proportional to the magnetisation of the wire. We may say, then, that the true or effective magnetic force  $H$  acting at any point in the wire is the difference between the impressed magnetic force  $H_0$  and the demagnetising or counter-magnetic force  $H_1$  due to the action of the ends of the bar, or  $H = H_0 - H_1$ .

This last term  $H_1$  is proportional to  $I$  and to a factor  $N$  called the *demagnetisation factor*, which is a function of the dimensions of the wire. Hence,

$$H = H_0 - NI. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For instance, it can be shown that the demagnetisation factor for a sphere is equal to  $4\pi/3$ .\* Hence, if we place a sphere in a uniform magnetic field  $H_0$  the actual magnetic force inside the sphere is given by  $H_0 - \frac{4\pi I}{3}$ , where  $I$  is the magnetisation. But  $I = kH$ , where  $k$  is the susceptibility, and it is, therefore, easily shown that

$$\frac{H}{H_0} = \frac{1}{\frac{4\pi}{3}k + 1}.$$

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\* See J. A. Ewing's "Magnetic Induction in Iron and other Metals."

Hence, if  $k$  has any such value as 100, which it may easily have for soft iron in a sufficient field, then

$$\frac{H}{H_0} = \frac{1}{420}.$$

Accordingly, the actual magnetising force under these circumstances inside the iron would not be  $\frac{1}{420}$ th part of the force which would exist at that point if the sphere were removed.

The only forms for which the demagnetisation factor can be calculated exactly are the varieties of the ellipsoid. The ellipsoid is thus the typical form of open magnetic circuit, just as the toroid or ring is the typical closed magnetic circuit. We have two cases to consider in the case of the ellipsoid: (i.) The prolate ellipsoid of revolution, or the *Ovoid*. This is generated by the revolution of an ellipse round its major axis. Let such an ovoid be placed in a uniform magnetic field with its long axis parallel to the field; then it is everywhere magnetised uniformly along lines parallel to the field. If  $2a$  is the long diameter of the generating ellipse and  $2b$  its shorter diameter, its eccentricity is

$$e = \sqrt{1 - \frac{b^2}{a^2}},$$

and its demagnetisation factor  $N$  is

$$N = 4\pi \left( \frac{1}{e^2} - 1 \right) \left( \frac{1}{2e} \log \frac{1+e}{1-e} - 1 \right). \quad \dots \quad (15)$$

When the ellipse is very elongated the above expression can be written

$$N = 4\pi \frac{b^2}{a^2} \left( \log \frac{2a}{b} - 1 \right).^* \quad \dots \quad (16)$$

The above expressions can be thrown into another useful form. Let  $m$  stand for the *dimension-ratio*—that is, the ratio of length to diameter of the ellipsoid or rod or cylinder

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\* For the proof of these formulæ the reader is referred to Maxwell's "Treatise on Electricity and Magnetism," Vol. II., 2nd edition, §§ 437, 438.

considered. Then, in the case of the ovoid above considered, we have  $m = \frac{a}{b}$ , and hence  $e^2 = 1 - \frac{1}{m^2}$ . Making this substitution, we can write the expression for  $N$  as follows :

$$N = \frac{4\pi}{m^2 - 1} \left\{ \frac{m}{\sqrt{m^2 - 1}} \log (m + \sqrt{m^2 - 1}) - 1 \right\}, \quad (17)$$

and if  $m$  is large—that is, the ellipsoid very prolate—then

$$N = \frac{4\pi}{m^2} (\log 2m - 1).*$$

If  $m$  has a value exceeding 50, the above simple expression will give the value of the demagnetisation factor very closely. In all the above expressions the logarithms are Napierian.

In the next case we have to consider the oblate spheroid. Let an ellipse whose major axis is  $2a$  and minor axis  $2b$  revolve round its minor axis. It generates an oblate spheroid. Let this be placed with its shorter axis  $2b$  in the direction of the force of a uniform field. Then it is magnetised uniformly. In this case the eccentricity  $e = \sqrt{1 - \frac{b^2}{a^2}}$  as before, and the demagnetisation factor  $N$  is given by

$$N = 4\pi \left\{ \frac{1}{e^2} - \frac{\sqrt{1 - e^2}}{e^3} \sin^{-1} e \right\}.$$

This last expression can be transformed into a function of  $m$  as follows :

$$N = \frac{4\pi}{1 - m^2} \left( 1 - \frac{m}{\sqrt{1 - m^2}} \cos^{-1} m \right). \quad (18)$$

In a material as permeable as iron we can neglect the value of  $H$  in comparison with  $4\pi I$ , and we can consider that the flux density  $B$  is numerically equal to  $4\pi I$ . Accordingly, the expression for the difference between the impressed and effective magnetic forces is therefore given by

$$H = H_0 - NI,$$

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\* The above expressions are taken from a work by Dr. H. du Bois entitled "The Magnetic Circuit," translated by Dr. Atkinson. See Chap. II., § 29.

or by 
$$H_0 - H = \frac{NB}{4\pi}, \quad . . . . . (19)$$

and the true permeability of the material is measured by the ratio of  $B/H$ .

It will be seen that the quantity  $\frac{N}{4\pi}$  for this open magnetic circuit resembles, in a sense, a coefficient of self-induction, or an inductance in the case of an electric circuit. We can, therefore, make use of an ellipsoid of ferro-magnetic material placed in a uniform impressed magnetic field generated in the interior of a long solenoid to measure the permeability of the material. For we can measure, as shown in a subsequent section, the total magnetic flux through its equatorial section by winding the ellipsoid over with an equatorial secondary coil, and we can calculate the demagnetisation factor from the form of the ellipsoid, and hence the actual magnetic force operative in the interior of the metal. We can, therefore, delineate a magnetisation curve for the material and measure the permeability corresponding to any flux density.

We have thus two standard forms in which we can fashion a mass of ferro-magnetic material for magnetic measurements:

(i.) A toroid or ring of ferro-magnetic material wound over with an endless solenoid, which provides a magnetic force of predeterminable value.

(ii.) A prolate ellipsoidal (preferably very prolate) shaped core of ferro-magnetic material placed in the interior of a very long straight magnetising solenoid with the long axis of the core parallel to that of the solenoid.

If in both cases the exciting current in the solenoid is  $A$  amperes and the turns per centimetre  $n$ , then in the case (i.) the effective magnetic force  $H$  is equal to  $1.25 An$ . But in the case (ii.) the effective magnetic force  $H$  is equal to  $1.25 An - \frac{NB}{4\pi}$ , where  $B$  is the flux density in the equatorial section and  $N$  is the demagnetisation factor of the ellipsoid as given above.

In experiments made with ellipsoids, however, it is necessary that the geometrical form shall be exceedingly accurate, as a very little variation makes a considerable difference in the value of  $N$ .

The only other practical form of ferro-magnetic material adapted for quantitative research is the long circular-sectioned wire of which the dimension-ratio  $m$  has a value of 100 and upwards. In the case of such a long wire the demagnetisation factor has a different value at various distances from the ends. It can be shown, however, that its mean value is equal to  $45/m^2$ .

In the case of a rod for which  $m=400$  or upwards, the demagnetisation factor is so small that the wire is, in a magnetic sense, *endless*, and we may take the effective magnetic force as equal to the impressed force. The other extreme case is that of a flat circular disc magnetised perpendicularly to its own plane, for which case  $N=4\pi$ .

The reader can with advantage consult on this part of the subject the works of the following authors:—

H. DU BOIS. "The Magnetic Circuit," Chap. II., p. 23 *et seq.*

J. A. EWING. "Magnetic Induction in Iron and other Metals," Chap. I.

CLERK MAXWELL. "Treatise on Electricity and Magnetism," Vol. II., 2nd edition, Chap. V.

**§ 4. Magnetic Field or Flux Density Measurement in Air-Spaces.**—The measurement of magnetic flux density in an air-space, or as it is sometimes called the strength of a magnetic field, may be made by availing ourselves of the fundamental property of magnetic flux—viz., that when removed from a conducting circuit it produces in that circuit electromotive force. If a small loop of wire is placed in a magnetic field with the plane of the loop perpendicular to the direction of the magnetic flux, and if the loop is then suddenly removed from the field or turned with its plane parallel to the magnetic flux, or if the magnetic flux is suppressed or reversed in direction, all these actions remove the flux from

the circuit, and therefore give rise in the circuit to an electromotive force. If the conducting loop is connected to a ballistic galvanometer, and if the resistance of the whole circuit—viz., the coil of the galvanometer, the connecting wires, and the loop—is  $R$  ohms, then the removal of the flux from the loop sets up in the loop circuit a transient E.M.F. which results in the production of a transient electric current, and therefore in the movement of a certain quantity of electricity through the galvanometer circuit. If  $\epsilon$  is the E.M.F. at any instant in the loop, and  $i$  is the current at the corresponding instant, then, by Ohm's law,  $iR = \epsilon$ . If the area of the loop is  $S$ , and if the loop has on it  $N$  turns of wire, and if the flux is uniform over the cross-section of the loop and has a density at any moment equal to  $B$ , then the total flux which is linked with the loop is equal to  $SNB$  at any moment, and the rate at which this flux is changed is represented by  $SN \frac{dB}{dt}$ . Hence, at any moment we have  $Ri dt = SN dB$ ; or, if  $dq$  denotes the quantity of electricity set flowing in the circuit by the change of flux  $dB$ , we have  $R dq = SN dB$ . If we integrate the last equation throughout the complete period of time during which the flux change lasts, then the term on the left-hand side becomes equal to  $RQ$ , where  $Q$  is the whole quantity of electricity that has passed through the galvanometer; and the term on the right-hand side becomes equal to  $SNB$ , where  $B$  is the value of the flux density, or of the field in which the coil is placed. Hence we have

$$B = \frac{RQ}{SN} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The needle or coil of the galvanometer therefore makes a movement or "throw," and assuming the galvanometer to be calibrated, as shown in Chap. I. of this volume (*see* p. 18), we can determine from the throw of the galvanometer the quantity of electricity which is passed through it reckoned in coulombs or microcoulombs. If, then, the resistance of the

circuit is measured in ohms, and the quantity of electricity indicated by the throw of the galvanometer reckoned in coulombs, the area  $S$  of the coil being in square centimetres, the flux density  $B$  is calculated out in webers, one weber being equal to  $10^8$  C.G.S. units of magnetic flux. One unit of magnetic flux in the C.G.S. system is generally called one "line"; hence, if the quantity of electricity  $Q$  sent through the galvanometer is measured in microcoulombs and the resistance  $R$  of the whole circuit is measured in ohms, the flux density  $B$  (in lines per square centimetre) will be given by the equation

$$B = \frac{100 R \text{ (in ohms)} \times Q \text{ (in microcoulombs)}}{N \times S \text{ (in square centimetres)}}. \quad (21)$$

By means of a calibrated ballistic galvanometer and the employment of an exploring loop, we can, therefore, measure the flux density in any magnetic field by placing the loop in the field and suddenly snatching the loop away.

The measurement of the earth's magnetic field or flux density can in this way be made by holding a circular coil of a known number of windings and known area in the terrestrial field, with the plane of the coil perpendicular to the direction of the field, and then turning the coil suddenly in a direction such that its plane is parallel to the field. In calculating out the results of such a measurement, the product  $SN$ , which appears in the above equation (*viz.*, the number of turns on the exploring coil multiplied by the area of each turn), is in reality the total area included by all the windings of the coil. If the coil is wound uniformly on a circular bobbin in a certain number of layers, it will generally be sufficient to take as the area of the coil the area of a circle whose diameter is equal to the mean of the inner and outer diameters of the wire windings, and the product  $SN$  is then obtained by multiplying this area by the number of windings.

In making an experiment of this kind with a movable coil galvanometer it is essential to notice that, since the circuit of the galvanometer remains almost closed, we cannot assume

that the logarithmic decrement or the ballistic constant of the galvanometer, under the circumstances of the experiment (viz., when the galvanometer circuit is closed), is the same as when the galvanometer circuit is open; hence an uncertainty exists as to the magnitude of the correcting factor  $\left(1 + \frac{\lambda}{2}\right)$ , by which the galvanometer "throw" must be corrected to obtain the real quantity of electricity sent through the galvanometer.\* This difficulty is overcome by the employment of a standard magnetic field and of a secondary standardising coil placed in the circuit of the galvanometer. A standard magnetic field having a flux density of known value may be obtained in the following manner:—On a long wooden or pasteboard tube insulated wire is coiled in a known number of turns, so as to construct a long solenoid. The length of this solenoid should be at least 10 or 12 times its internal diameter. If the length of this bobbin is  $L$ , if it contains  $N$  windings of insulated wire, and if a current of  $A$  amperes is sent through the wire, then the strength of the magnetic field, or the magnetic flux density near the middle in the interior of the above coil, is given by the expression

$$B = \frac{4\pi}{10} \frac{SN}{L} = 1.25 \frac{SN}{L} \dots \dots \dots (22)$$

That is to say, the field in the interior of such a coil is given numerically by  $1\frac{1}{4}$  ampere-turns per centimetre of length. Since one inch is nearly equal to  $2\frac{1}{2}$  centimetres, it follows that the above rule may be reduced to the more simple form: *Magnetic flux density or field strength equals ampere-turns per half-inch for the interior of any long solenoid or bobbin for regions near the centre.*† If, therefore, in the interior of such a long bobbin we insert another coil of wire, consisting of one or more layers of insulated wire wound smoothly on a

\* For this reason the discharge from a condenser of known capacity cannot be used to calibrate the galvanometer, as is sometimes stated.

† This rule is accurate within 1 per cent. As a matter of fact, ampere-turns per half-inch are 1 per cent. greater than the magnetic force in C.G.S. units,

cylinder or rod of wood, and if this secondary coil has a number of turns  $n$ , each turn of wire having an area  $a$  (*i.e.*, if the total area included by all the windings is  $na$ ), then, when this secondary coil is placed in the interior of the above-described primary coil, the secondary circuit will have linked with it a magnetic flux  $Z$  represented by the expression

$$Z = \frac{4\pi}{10} \frac{N}{L} A n a = 1.25 (NA) (na)/L \quad . \quad . \quad . \quad (23)$$

Hence, if the length of the primary coil is reckoned in half-inches, the total flux included by the secondary or inner coil is numerically equal to the product of the ampere-turns per half-inch on the outer coil and the total area included by the whole of the windings on the inner coil.

Supposing, therefore, we join this secondary circuit in series with the galvanometer and the exploring coil which is to be used to measure the strength of some unknown magnetic field, and if the whole circuit of galvanometer secondary coil and exploring coil has a resistance  $R$  ohms, we can at once determine the total flux removed from the exploring coil by any movement by a comparison reading made by the aid of the secondary coil in the standard field.

The practical arrangements for determining magnetic field strength or magnetic flux density by the ballistic galvanometer are as follows :—A coil should be provided consisting of a long straight solenoid which forms a standard field coil. This may consist of a pasteboard tube with wooden cheeks at the ends. The coil may be preferably 125 centimetres long and 6 to 8 centimetres internal diameter, and may have on it 1,000 turns of double cotton-covered copper wire No. 18 S.W.G. A secondary coil is then provided, consisting of a cylinder of boxwood or vulcanised fibre, turned true, the diameter of which may be 5 centimetres. On this last cylinder is wound in one layer No. 32 S.W.G. silk-covered copper wire, say also 1,000 turns. In the case of this last coil the diameter of the cylinder should be carefully measured

at different places, and also the diameter over the outside of the wire when the winding is complete. The mean of these

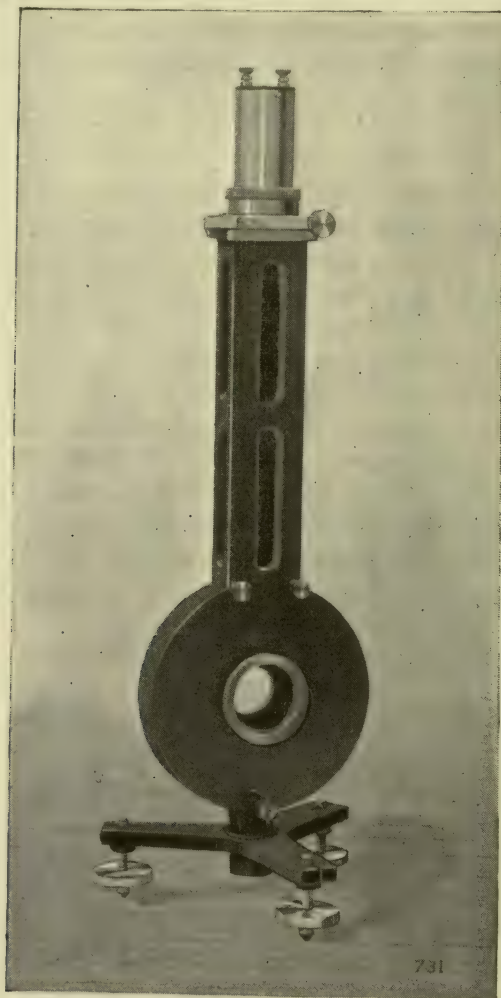


FIG. 5.—Crompton's Suspended Coil Galvanometer.

two measurements will give the diameter of the circular axis of each turn of the wire ; and if the area of this circle is

calculated and multiplied by the total number of turns we have a total area enclosed by all the windings of this secondary bobbin. The number may approximate to 10,000.

The ballistic galvanometer employed in these tests should be of a movable coil type having a long time period of oscilla-

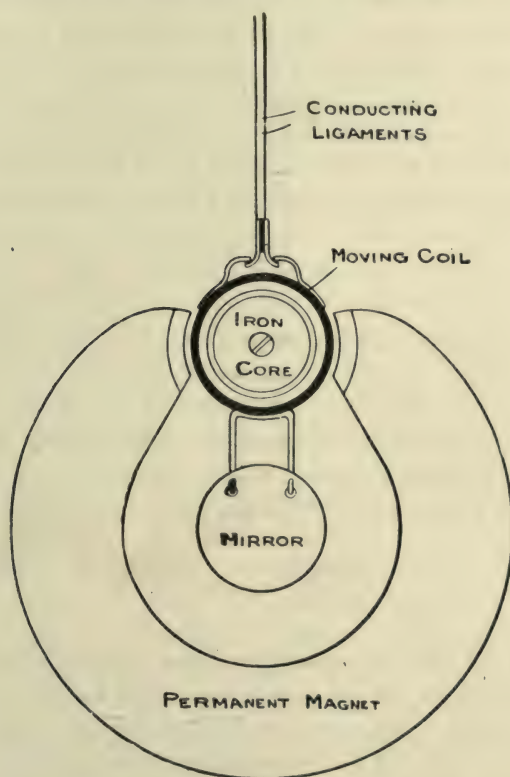


FIG. 6.—Arrangement of Magnet and Coil in Crompton Galvanometer.

tion. An excellent form is one of Crompton's latest pattern with bifilar suspension of coil (*see* Figs. 5 and 6). In series with the standard primary field coil above described is placed an ammeter, preferably a Weston ammeter, and a double-pole

quick reversing switch by means of which the current through the coil can be instantly reversed in direction. The secondary coil above described should be suspended in the centre of the primary coil, and connections should be made between this secondary coil and the galvanometer circuit by means of two twisted insulated wires. The secondary coil is joined in series with the galvanometer and with an exploring coil, which is placed in the magnetic field to be measured.

If we denote by the symbols  $L_1$ ,  $A_1$ ,  $N_1$  respectively the length, number of turns, and the current flowing through the primary bobbin, then the magnetic force  $H$  in the interior of the primary bobbin or flux density is measured by the value

$$H = \frac{4\pi}{10} \frac{N_1 A_1}{L_1},$$

and this expression also gives us the value of flux density in the same place. If we denote by  $n_1$  the number of turns on the secondary coil, and by  $a_1$  the area of each turn, then the product  $n_1 a_1$  is the total area of all the secondary windings, and the flux change through this secondary coil on *reversing* the primary current through the field coil is given by the expression

$$\frac{8\pi}{10} \frac{N_1 A_1}{L_1} n_1 a_1 = \Delta F_1.$$

Let  $B$  be the flux density of the magnetic field to be measured, let an exploring coil be placed in this field with its plane perpendicular to the lines of the field, and let  $n$  be the number of turns on this exploring coil and  $a$  the area of each turn. Then, if this field is reversed, the flux change through the exploring coil will be given by the expression  $2Bna = \Delta F$ . Suppose this last reversal produces a throw  $\theta$  on the galvanometer, and that the current  $A_1$  in the primary standard field coil is adjusted so as to create when reversed a throw  $\theta_1$  on the galvanometer, which is nearly the same as the throw  $\theta$  obtained when the field to be measured is reversed,

then, since these "throws" are proportional to the flux changes, we have

$$\frac{\theta}{\theta_1} = \frac{2Bna}{8\pi \frac{N_1 A_1}{10} \frac{n_1 a_1}{L_1}} = \frac{5}{2\pi} \frac{BnaL_1}{N_1 A_1 n_1 a_1}.$$

Hence 
$$B = \frac{2\pi}{5} \frac{N_1 A_1}{L_1} \frac{n_1 a_1}{na} \frac{\theta}{\theta_1}. \quad \dots \dots \dots (24)$$

If we make  $\theta_1$  nearly equal to  $\theta$ , we eliminate all the error which is due to non-proportionality of "throw" to quantity in the case of the galvanometer, and by preserving the secondary circuit unchanged in the two experiments we eliminate all error due to change in logarithmic decrement. The conditions for success are, however, that the galvanometer coil must have a long periodic time, so that the coil does not move sensibly from its zero position until the whole of the flux change is completed. The galvanometer merely serves to test the equality of two flux changes which are respectively reversed through its circuit. If we use the exploring coil by snatching it away out of an air field, then the flux removed or unlinked from it is measured by  $Bna$  and not by  $2Bna$ , and the formula given above needs correction accordingly.

An exploring coil may be used as above described to measure the flux density or field at any point in the neighbourhood of a polar magnet. In this case the exploring coil should be very small—should consist, say, of 100 turns of No. 40 silk-covered copper wire wound on a little flat boxwood bobbin, and should have a length of twisted flexible cord attached to its ends. The boxwood bobbin can be affixed to a handle, which enables it to be placed in the field in the right position, and then snatched away, and by the use of a calibrated galvanometer, as above described, we can obtain the absolute measure of the flux density or field at that point. The flux density may, in fact, be defined as measured by the time integral of the E.M.F. produced in a coil of one turn having an area of 1 sq. cm. when placed in the field, with its

plane perpendicular to the direction of the field, and suddenly snatched away. The importance of the quantity called the magnetic flux is, therefore, that it is the measure of the quality of the magnetic field or state in space, or in any material in virtue of which an E.M.F. is created in a circuit placed in that field when the flux is reversed or suppressed.

The ballistic method of field measurement has a large range of application. It can be employed equally well to measure very strong and very weak fields, since we have great control over the galvanometer throw by the size of the coil and number of turns on it which we employ to place in the field. Hence, if the field is weak, a coil with a large number of turns may be employed.

**§ 5. Measurement of Magnetic Flux Density in Air Spaces by Means of the Variation in Resistance of Bismuth.—**

Another method of magnetic field measurement, which, however, is only applicable to rather strong fields, is one dependent upon the variation produced in the resistance of a bismuth wire. It was discovered by Mr. Tomlinson\* in 1881 that the resistance of a rod of bismuth was increased when placed with its length parallel to a magnetic field, and a year or two later Prof. Righi discovered that a more marked increase in resistance was produced when the magnetic field was transverse to the bismuth rod. Hence Leduc proposed to employ this increase to measure magnetic fields. Lenard† also investigated the effect, and a very complete investigation was made at different temperatures by Henderson‡ and by Dewar and Fleming.§

If a flat spiral of pure bismuth wire drawn from electrolytically-prepared bismuth is placed in a magnetic field with the plane of the spiral perpendicular to the direction of the

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\* *Phil. Trans.*, Part I, 1883, p. 153.

† *Wied. Ann.*, Vol. XXXVI., 1890, p. 619.

‡ *Phil. Mag.*, Vol. XXXVIII., p. 488.

§ *Proc. Roy. Soc.*, Vol. LX., p. 425.

flux of a strong magnetic field, then a considerable increase in the electrical resistance is found at constant temperature. Thus, taking the resistance of the wire to be unity in a zero magnetic field, Henderson found the electrical resistance of a pure bismuth wire kept at 18°C. to be increased in the following ratios, when the field had the value in C.G.S. units given in the first column of Table I. below :—

**TABLE I.**

*Increase in the Resistivity of Pure Electrolytic Bismuth Wire when placed transversely in Magnetic Fields of various strengths.*

(Henderson.)

Magnetic Field $H$ in C.G.S. units.	Ratio of Resistivity in the field to the Resistivity in zero field taken as unity at 18°C. ( $R/R_0$ ).	Percentage Increase in Resistivity.
0	1.000	0
5,830	1.227	22.7
6,310	1.253	25.3
6,830	1.290	29.0
7,790	1.341	34.1
8,880	1.407	40.7
10,410	1.496	49.6
12,500	1.630	63.0
15,710	1.830	83.0
20,450	2.160	116.0
23,480	2.333	133.0
26,820	2.508	150.8
27,450	2.540	154.0
27,820	2.568	156.8
28,370	2.609	160.9
29,270	2.643	164.3
30,090	2.704	170.4
31,270	2.772	177.2
32,730	2.846	184.6
33,300	2.893	189.3
35,800	3.070	207.0
36,600	3.160	216.0
38,900	3.334	233.4

Accordingly, the measurement of the increase in resistance of a pure bismuth wire enables us to determine the strength of the magnetic field in which it is placed. It was discovered by Dewar and Fleming that this increase in resistance is itself immensely increased when the bismuth is cooled below atmospheric temperatures; and by cooling the wire in liquid

air they were able to increase its resistance 150 times by placing it in the magnetic field of strength 22,000 C.G.S. units.

For practical laboratory purposes a very convenient instrument is prepared by Messrs. Hartmann and Braun, of Frankfurt, on the basis of investigations by MM. Lenard and Howard.\*

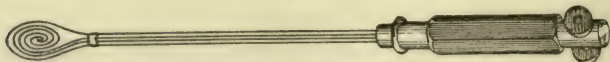


FIG. 7.—Hartmann and Braun's Bismuth Spiral for Measuring Magnetic Fields.

These experimentalists succeeded in preparing a pressed wire of electrolytic bismuth about 0.5mm. in diameter. This is wound into a flat spiral in a noninductive manner, and the ends attached to flat copper rods held in a suitable insulating handle, so arranged that the flat spiral can be placed in any magnetic field with its plane perpendicular to the lines of flux (*see* Fig. 7). A table or curve is provided with the

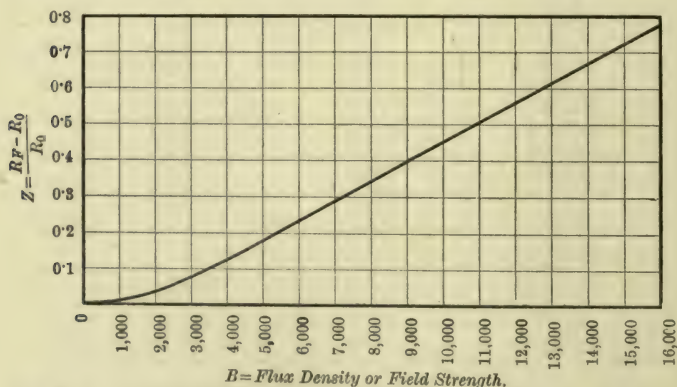


FIG. 8.—Sample of a Chart furnished with the Hartmann and Braun Bismuth Spiral.

instrument, showing the electrical resistance of the flat bismuth coil when thus placed transversely in magnetic fields

\* See *The Electrician*, Vol. XXI., p. 471.

of various strengths. Accordingly, to determine the interpolar field, say, of an electromagnet all that is necessary is to place the bismuth spiral between the poles of the magnet, connect it to a Wheatstone bridge, and measure its resistance both in the field and out of the field. From the difference in these two resistances the strength of the field can be at once obtained by consulting the chart furnished with the instrument (*see Fig. 8*).<sup>\*</sup> For, in the case of the curve drawn on this chart, one of which is furnished with every spiral, the

TABLE II.

*Resistivity of Pure Bismuth in various Magnetic Fields.*

Strength of Magnetic Field. C.G.S. units.	Resistivity of Bismuth. C.G.S. units.
Temperature 19°C.	
0	116,200
1,375	118,200
2,750	123,000
3,800	149,200
14,150	186,200
21,800	257,000
Temperature - 79°C.	
0	78,300
650	83,300
2,300	103,500
3,350	114,800
4,100	134,000
5,500	158,000
7,900	201,000
14,200	284,000
Temperature - 185°C.	
0	41,000
1,375	103,300
2,750	191,500
3,800	738,000
14,150	1,730,000
21,800	6,190,000
Temperature - 203°C.	
0	34,300
2,450	283,500

<sup>\*</sup> This instrument can be obtained in England from Messrs. O. Berend & Co., London. For particulars as to its use see *The Electrician*, Vol. XXVII, p. 128.

horizontal abscissæ are field strengths in C.G.S. units, and the vertical ordinates of the curve are the ratios of the increase in the resistance of the spiral when placed in that field to its original resistance when not in the field.

The figures in Table II. give the absolute resistivity of electrolytic bismuth at various temperatures and in various

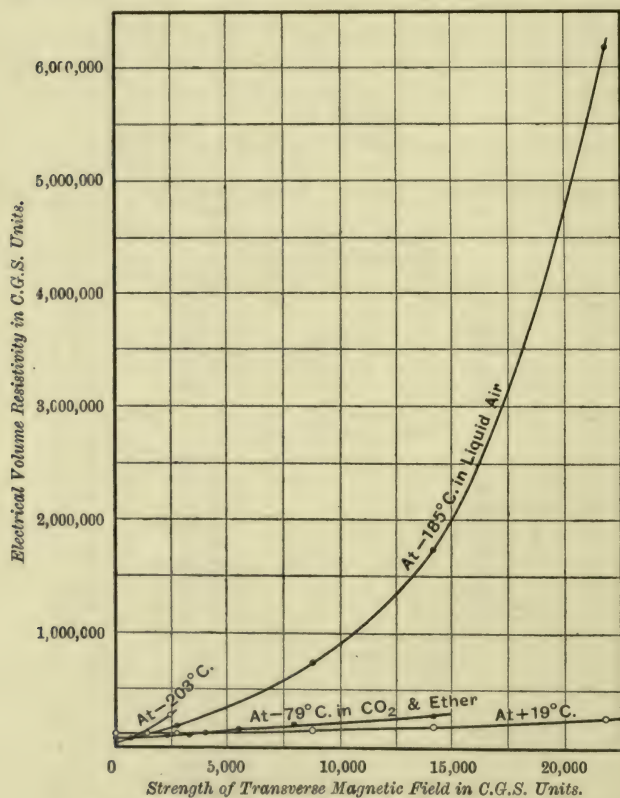


FIG. 9.—Change of Resistivity of Electrolytic Bismuth when Transversely Magnetised.

fields (*see* Fleming and Dewar "On the Electrical Resistivity of Electrolytic Bismuth," *Proc. Roy. Soc.*, Vol. LX., 1897, p. 425)

These results are graphically depicted in the curves shown in Figs. 9, 10 and 11.

The bismuth spiral is an exceedingly convenient method of measuring with fair accuracy very strong fields, such as the interpolar air-gap fields of dynamos or that of any electro-magnet. It is not, however, suitable for measuring fields of a

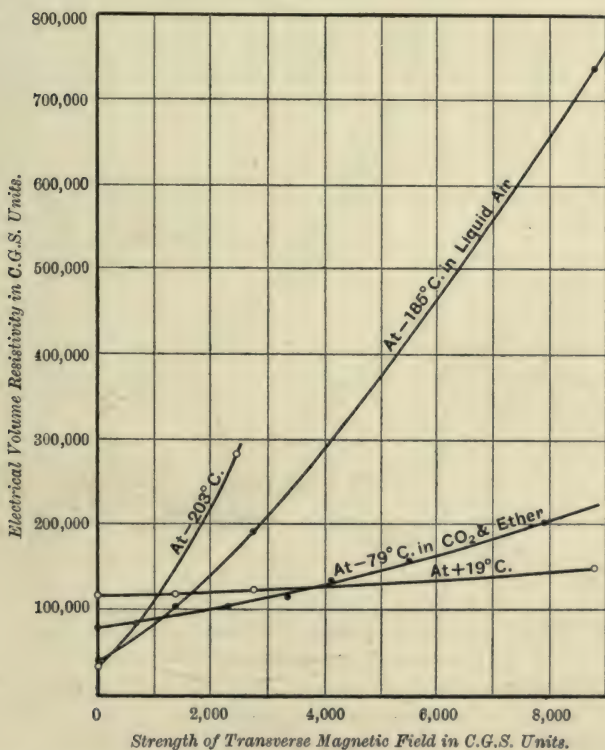


FIG. 10.—Change of Resistivity of Electrolytic Bismuth when Transversely Magnetised. (Lower part of diagram in Fig. 9 shown on an enlarged scale.)

strength much less than 1,000 C.G.S. units or lines per square centimetre. Messrs. Hartmann and Braun have devised an appliance by which the permeability of test samples of iron is

measured by determining the resistance of a bismuth spiral in a narrow gap in a magnetic circuit partly formed of the bar in question. (See *The Electrician*, Vol. XXXIII., p. 719, for details.)

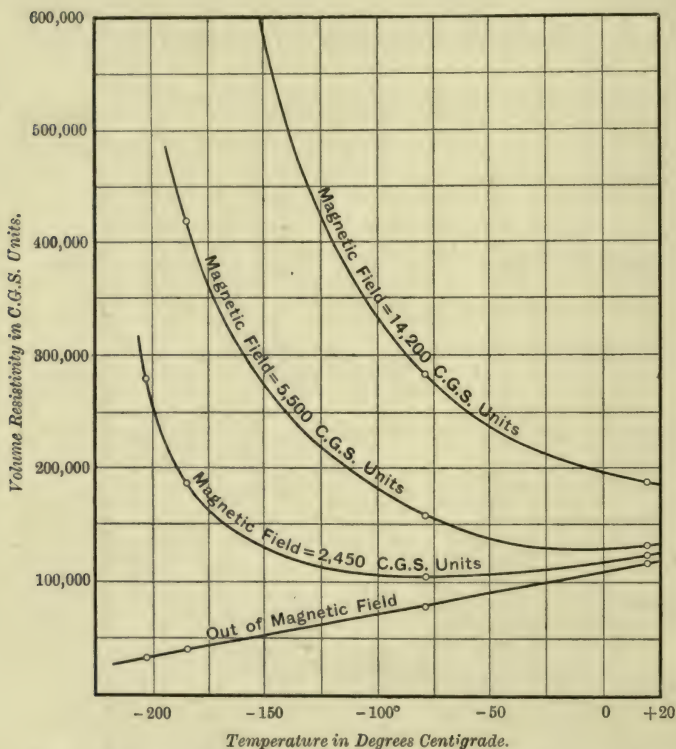


FIG. 11.—Change of Resistivity with Temperature of Electrolytic Bismuth when Transversely Magnetised.

**§ 6. Measurement of Weak Magnetic Fields by Gauss's Method.**—The measurement of the strength of magnetic fields or flux densities of the order of 1 C.G.S. unit or less, such as that of the earth's magnetic field, when sufficiently extensive and uniform, is best effected by the method devised by Gauss. This consists in determining the time of oscillation of a

suspended magnetic bar of known moment of inertia in the field to be measured and then using it to deflect another suspended magnet placed in that field. These two experiments enable us to determine both the strength of the magnetic field and the *magnetic moment* of the suspended bar.

If a cylindrical circular-sectioned bar magnet is suspended at its centre and placed in a uniform magnetic field it will place its magnetic axis in the direction of the field. If it is displaced through an angle  $\theta$  and left to itself it will execute isochronous oscillations in diminishing arcs. At any moment the torque, urging it back to the position in which its magnetic axis coincides in direction with that of the field, is proportional to  $\sin \theta$ . Hence, if we neglect frictional resistance to the motion, the equation of motion is

$$-I \frac{d^2\theta}{dt^2} = MH \sin \theta, \quad \dots \dots \dots (25)$$

where  $I$  is the moment of inertia of the bar and  $M$  its magnetic moment and  $H$  the strength of the field.

This equation can be written in the form

$$\frac{d^2\theta}{dt^2} + h^2 \sin \theta = 0, \quad \dots \dots \dots (26)$$

where  $h^2 = MH/I$ .

The above equation cannot be integrated completely in a finite form. If it is multiplied by  $\frac{d\theta}{dt} dt$  and integrated from  $\theta = 0$  to  $\theta = a$  we obtain

$$\left(\frac{d\theta}{dt}\right)^2 = 2h^2 (\cos \theta - \cos a). \quad \dots \dots \dots (27)$$

Hence we have

$$dt = \frac{d\theta}{\sqrt{2} \cdot h \sqrt{\cos \theta - \cos a}};$$

and, therefore, if  $T$  is the time of a complete oscillation,

$$T = \frac{2}{h} \int_0^a \frac{d\theta}{\sqrt{\cos \theta - \cos a}}. \quad \dots \dots \dots (28)$$

If we expand the expression under the radicle and integrate the separate terms of the series we have

$$T = \frac{2\pi}{h} \left\{ 1 + \left(\frac{1}{2}\right)^2 \sin^2 \frac{a}{2} + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 \sin^4 \frac{a}{2} + \&c. \right\},$$

or, representing by  $1 + \beta$  the value of the series within the bracket, we have for the complete periodic time

$$T = 2\pi \sqrt{\frac{I}{MH}} (1 + \beta). \quad \dots \dots \dots (29)$$

The periodic time for infinitely small oscillations is  $2\pi \sqrt{\frac{I}{MH}} = T_0$ . Hence

$$T = T_0 (1 + \beta),$$

or

$$T_0 = \frac{T}{1 + \beta}. \quad \dots \dots \dots (30)$$

The values of  $1 + \beta$  for angles from 0 to 30 degrees are as tabulated below :—

TABLE III.

$\alpha$	$1 + \beta$	$\alpha$	$1 + \beta$	$\alpha$	$1 + \beta$
deg.		deg.		deg.	
1	1·0000	11	1·0023	21	1·0085
2	1·0001	12	1·0027	22	1·0093
3	1·0002	13	1·0032	23	1·0102
4	1·0003	14	1·0037	24	1·0111
5	1·0005	15	1·0043	25	1·0120
6	1·0007	16	1·0049	26	1·0130
7	1·0009	17	1·0056	27	1·0141
8	1·0011	18	1·0062	28	1·0151
9	1·0015	19	1·0070	29	1·0162
10	1·0019	20	1·0077	30	1·0174

The observed time of oscillation  $T$  can, by means of these factors, be reduced to the equivalent value  $T_0$  for an infinitely small arc. In most cases, however, in the measurement of a time of oscillation, the swing can be kept sufficiently small to make the above correction negligible. For an angle of 7·5 degrees it is only one-tenth of 1 per cent. Suppose, then, that we have a hardened steel bar of regular shape, such as a rectangular or circular-sectioned straight bar. Let it be magnetised permanently and suspended freely from a fixed point by cocoon fibres, so as to hang horizontally in the field. It will then execute isochronous oscillations if slightly disturbed. To observe the time period of these it is best to place the magnet in a paper cradle suspended by as few silk fibres as possible from a support provided with a torsion head. The magnet should be enclosed under a glass shade to prevent disturbance by air currents. It is convenient to attach to the cradle a light mirror, so that by a lamp and scale the angular deflections may be noted. The magnet can then be removed from the cradle without disturbing the mirror, and can be set in oscillation by approaching another magnet held in the hand. In the first place the magnet must be weighed to determine its mass,  $W$ , in grammes. Its linear dimensions are next carefully taken. If rectangular, let its width be  $w$ , its depth be  $d$ , and its length  $l$ , all reckoned in centimetres. If circular-sectioned let its diameter be  $d$  and its length  $l$ .

It is shown in treatises on rigid dynamics that its moment of inertia  $I$  is then given by the formulæ

$$I = W \left( \frac{l^2 + w^2}{12} \right) \text{ for rectangular section, } \dots (31)$$

$$I = W \left( \frac{l^2}{12} + \frac{d^2}{16} \right) \text{ for circular section. } \dots (32)$$

$W$  being measured in grammes and  $l, d$  or  $w$  in centimetres, the moment of inertia  $I$  is in gramme-(cms.)<sup>2</sup>. Accordingly, from equation (29) above, we have

$$MH = \frac{4\pi^2 I (1 + \beta)^2}{T^2}, \dots (33)$$

where  $I$  has the value proper for the form of the magnet used,  $1 + \beta$  is the factor taken from the Table III., corresponding to the observed arc of oscillation, and  $T$  is the observed time of a complete oscillation.

It is best to proceed as follows:—Give the magnet a small displacement and let it oscillate. Let  $D$  be the distance from the mirror or the cradle to the scale, and let the spot of light be displaced through a distance  $S$  on the scale. Suppose  $D$  and  $S$  to be both reckoned in millimetres; then  $S/2D$  is, to a sufficient approximation, the tangent of the angle  $\alpha$ , through which the needle is displaced, and from the tables of natural tangents we can find the angle  $\alpha$ , and, therefore, the proper value of  $1 + \beta$  from Table III. We then count the time of 10 or 20 complete swings in seconds and deduce the time  $T$  of one complete oscillation. The equation (33) gives us the value of the product of the moment of the magnet and the magnetic force or field in which it is swinging.

We have next to employ this magnet to create a deflection of a very small magnet placed in the same field. A magnetometer is constructed as follows:—It is possible to buy clear mica argand chimneys as used for certain gas lamps. One of these has a pair of large corks fixed in it at the top and bottom. A very small but good concave mirror must then be obtained,

about 0.5in. in diameter, such as is used for mirror galvanometers. A piece of very thin sheet aluminium is procured and cut into the shape shown in Fig. 12. The circular part should be a very little larger than the mirror, and this latter is placed upon it and the four little projecting lugs are turned over so that the mirror is held firmly, but without distortion of form. Previously to attaching the mirror three or four small pieces of hardened steel watch spring should be attached to the back of the aluminium mirror holder with shellac and be magnetised. The whole system is then to be suspended by a single cocoon fibre from the upper cork by attaching it to a



FIG. 12

wire passing through the cork. The length of the fibre must be adjusted so that the lower pointed portion of the aluminium carrier just dips into a little paraffin oil placed in a small glass cup or beaker. The beaker should be as large as will fit easily into the mica chimney. The whole arrangement is supported upon a board with levelling screws, and forms a reflecting magnetometer.

In order to determine the magnetic force in any uniform field, say, the horizontal magnetic force of the earth, the magnetometer above described is placed in the field, and should, of course, be at a distance from all masses of iron and

magnets. A lamp and scale are then fixed so as to reflect the ray of light from the lamp on to the scale. The scale should be placed at a distance of 1 metre from the mirror on the magnetometer needle and should preferably be divided into millimetres. The source of light, or lamp, should either be an electric glow lamp with a straight filament, so that on the screen we have formed a sharp line of light which is the image of a part of the filament, or it should be a paraffin lamp with a flat wick, so that the image on the scale is a sharp barb-like patch of light which is the image of the edgewise view of the flame. In either case the image should be sufficiently sharp to be able to read a displacement of a fraction of a millimetre of the edge of the image. If the

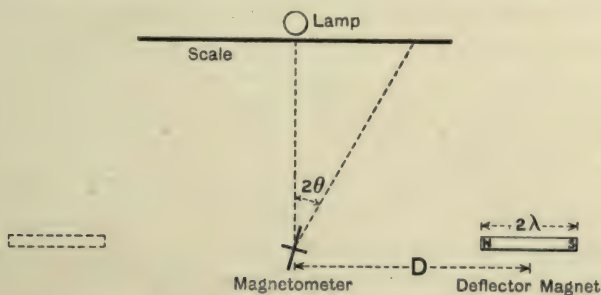


FIG. 13.

scale is at a distance of 1,000mm. from the mirror, then the tangent of the angle of deflection of the needle of the magnetometer is very nearly equal to the displacement of the scale image in millimetres from its zero position divided by 2,000.

Having arranged the magnetometer mirror and scale, the cylindrical magnet which has been employed previously to oscillate in the magnetic field in which the magnetometer is now situated is placed with its axis pointing towards the magnetometer needle, in a direction at right angles to the magnetic meridian, and at a distance great compared with its own length. The arrangement of the apparatus will be as shown in plan in Fig. 13. Under these circumstances the cylindrical

magnet, hereafter called the *deflector*, will cause a deflection in the magnetometer needle, and the tangent of the angle of deflection must be observed as above described. Having obtained one reading and one deflection  $\theta$  of the magnetometer needle when the centre of the deflector magnet is at a distance  $D$  from the centre of the magnetometer needle, the deflector magnet is moved to another distance  $D_1$  and the deflection of the magnetometer needle  $\theta_1$  is again observed.

To eliminate want of symmetry in the arrangement, it is best to proceed as follows:—Four observations are taken corresponding to one the same distance of the deflector magnet from the magnetometer needle—one pair with the deflector magnet first on the east side and second on the west side of the magnetometer needle, and one pair in each of the above positions, with the deflector magnet first with its north pole towards the magnetometer and second with its south pole towards the magnetometer. Corresponding, therefore, to one distance  $D$ , we have four values of the angle  $\theta$ , or, which comes to the same thing, of  $\tan \theta$ . The mean of these four values is then taken as the value of  $\tan \theta$  to be substituted in the equation below.

A similar experiment is performed with a deflector magnet at another distance,  $D_1$ , and it can be shown that the best results are obtained when the ratio between  $D_1$  and  $D$  is equal to 1.32. In the same manner we take four observations at the second distance, and take the mean of them as the value of  $\tan \theta_1$ . These observations will give us the means of calculating the ratio between the moment  $M$  of the deflector magnet and the strength of the field  $H$  in which the magnetometer needle rests. Assuming that that field is the same field in which the deflector was vibrated, we have the means of calculating both the moment of the deflector magnet and the strength of the field, or the magnetic force acting upon the magnetometer magnet.

Let  $M$  be, as before, the magnetic moment of the bar magnet, whose time period of oscillation in the earth's field we have determined. Then  $M$  may be

considered to be resolvable into two factors, one called the magnetic length  $2\lambda$  of the magnet, and the other called the pole strength  $m$ , so that  $2m\lambda = M$ . In the same manner the small magnetometer needle has a moment  $M'$  which may be resolved into the product of its magnetic length  $2\lambda'$  and pole strength  $m'$ , so that  $2m'\lambda' = M'$ .

When the bar magnet (now called the deflector) is placed with its centre at a distance  $D$  from that of the magnetometer needle in the first position it creates a deflection of the magnetometer  $\theta$ . If  $D$  is large compared with  $\lambda'$ , then the magnetic force due to the deflector which acts on the magnetometer needle consists solely of a torque equal to

$$\begin{aligned} & \left\{ \frac{2\lambda'm'm}{(D-\lambda')^2} - \frac{2\lambda'm'm}{(D+\lambda')^2} \right\} \cos \theta \\ &= M'm \left\{ \frac{1}{(D-\lambda')^2} - \frac{1}{(D+\lambda')^2} \right\} \cos \theta \\ &= MM' \left\{ \frac{2D}{(D^2-\lambda'^2)^2} \right\} \cos \theta. \quad \dots \dots \dots (34) \end{aligned}$$

If the magnetometer needle is in a uniform magnetic controlling field of strength  $H$ , then the field exerts on it a counter torque equal to  $M'H \sin \theta$ . Hence the equation of equilibrium is

$$M'H \sin \theta = M'M \frac{2D}{(D^2-\lambda'^2)^2} \cos \theta,$$

$$\text{or} \quad \frac{M}{H} = \frac{(D^2-\lambda'^2)^2}{2D} \tan \theta. \quad \dots \dots \dots (35)$$

This can be written  $\tan \theta = \frac{2M}{D^3H} \left( \frac{1}{1 - \frac{\lambda'^2}{D^2}} \right)^2$ ,

and, if  $\lambda'^2/D^2$  is a small quantity such that powers of it may be neglected, we can write the above equation

$$\tan \theta = \frac{2M}{D^3H} \left( 1 + \frac{\lambda'^2}{D^2} \right).$$

Multiplying by  $D^2$  we have

$$D^5 \tan \theta = 2 \frac{M}{H} D^2 + 2 \frac{M}{H} \lambda'^2. \quad \dots \dots \dots (36)$$

If, then, the deflecting magnet is moved to another distance,  $D_1$ , and creates a deflection  $\theta_1$  in the magnetometer, we have for the second observation the equation

$$D_1^5 \tan \theta_1 = 2 \frac{M}{H} D_1^2 + 2 \frac{M}{H} \lambda'^2. \quad \dots \dots \dots (37)$$

Subtracting equation (37) from equation (36) we arrive at the equation

$$\frac{M}{H} = \frac{D^5 \tan \theta - D_1^5 \tan \theta_1}{2(D^2 - D_1^2)}. \quad \dots \dots \dots (38)$$

We have, therefore, obtained values for the product  $MH$  and the ratio  $M/H$  of the moment of the deflector and the field-strength, and hence we have at once the means of obtaining the numerical value of either quantity. For if

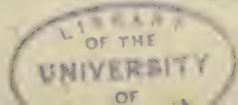
$$MH = \alpha \text{ and } M/H = \beta$$

then

$$M^2 = \alpha\beta \text{ and } H^2 = \alpha/\beta,$$

and therefore

$$H = \sqrt{\alpha/\beta}. \quad \dots \dots \dots (39)$$



It should be remarked, however, that, if the value of  $H$  is required with great exactness, there are several corrections which must be applied. In the first place the deflector magnet is under different conditions when it is being vibrated and when it is used to create deflection in the magnetometer needle. In the first case the magnetisation of the deflector needle is in the same direction as the field in which it is placed, but in the second the magnetisation of the deflector magnet is at right angles to the field in which it is placed. There is, therefore, a source of error due to the additional magnetisation induced in the deflector magnet by the field. It can, however, be shown that if the deflector magnet is glass-hard and magnetised as nearly as possible to saturation the error due to this cause will be negligibly small.

Again, there is an error due to the torsion of the suspending fibre in the case of the vibration experiments with the magnetometer. The magnetometer needle is urged back to its position not merely by the magnetic force of the terrestrial field but by the torsional elasticity of the cocoon fibre. This source of error can be practically eliminated by suspending the magnetometer needle by a single and very long cocoon fibre or by a very fine quartz fibre, or a correction can be applied for it. The moment of torsion of a twisted thread is proportional to the angle of torsion; hence, if a horizontal magnet is suspended by a cocoon fibre and is deflected through a small angle from the magnetic meridian, the restoring torque or couple due to the magnetic force is proportional to the angle of deflection, and also the moment of torsion due to the suspending fibre is proportional to the angle of its twist. Hence, relatively to the magnet, if the thread was perfectly torsionless the deflection of the needle from the meridian under the action of the opposing torques upon it would be greater than it actually is by an amount equal to the actual deflection multiplied by a factor  $1+t$ , where  $t$  is a small fraction which expresses the ratio of the torque due to torsion to the torque due to the magnetic force. To

discover the value of  $t$  we must ascertain the angle through which the suspending fibre has to be twisted in order to deflect the suspended horizontal magnet through a known angle from the meridian. If, therefore, the top end of the thread can be twisted through an angle which can be read off on a divided circle called the torsion head, and if a torsion is thus applied to the upper end sufficient to deflect the magnet by an angle  $\phi$  from the meridian, and if it is necessary to turn the torsion head, *i.e.*, to give the fibre a twist equal to an angle  $\alpha$  at the top end of the suspending fibre, the value of  $t$  is given by the expression

$$t = \frac{\phi}{\alpha - \phi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

because the above fraction expresses the ratio between the angle by which the magnet is deflected from the meridian and the angle through which the thread is twisted relatively to the magnet, and, these angles being proportional to the couples or torques, the value of the above fraction expresses the ratio of the torque due to torsion to the torque due to the magnetic forces. The correcting factor  $1+t$  has then to be applied to the expressions for the ratio of  $M$  to  $H$  and produce of  $M$  and  $H$ , so that we have finally

$$\frac{M}{H} = \frac{D^5 \tan \theta - D^5 \tan \theta_1}{2(D^2 - D_1^2)} (1+t), \quad . \quad . \quad . \quad (41)$$

$$\frac{M}{H} = \frac{4\pi I(1+\beta)^2}{T^2(1+t')}, \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where  $t'$  is the ratio of torsion for the suspended deflector magnet and  $t$  that for the magnetometer.

The correction which must be applied on account of the torsional elasticity of the suspending fibre is, in general, greater than that which has to be applied either for the reduction of the oscillations to infinitely small arcs or for the want of entire rigidity in the magnetism of the bar. Accidental sources of error are not included in the above, but in taking the observations the observer should be careful to remove

from the neighbourhood of the magnetometer all iron or steel objects, particularly those about the observer himself. Thus keys, steel spectacles, eyeglasses, or steel watch-chains must be removed before the observer approaches the magnetometer to take a reading.

Having in this manner determined the magnetic moment of a given bar, and, at the same time, the value of the earth's horizontal magnetic force in any room, the bar itself becomes a means for determining the magnetic force of any field not exceeding 1 or 2 C.G.S. units. Supposing, for instance, it is desired to determine the field due to any coil in a region in which that field is uniform over distances large with respect to the size of the magnetometer needle. The above-described magnetometer may be placed in that field, and the magnetic bar for which the moment has already been determined can be placed at some distance with its axis on a line passing through the centre of the magnetometer needle and the direction of that axis at right angles to the direction of the magnetic field. If, then, we take two observations of the deflections  $\theta$  and  $\theta_1$ , produced on the magnetometer needle by the magnetic deflector placed at different distances,  $D$  and  $D_1$ , we have, as the expression for the magnetic force, the following equation :—

$$H = \frac{2M(D^2 - D_1^2)}{(D^5 \tan \theta - D_1^5 \tan \theta_1)(1 + t)}, \quad \dots \quad (43)$$

where  $M$  is the magnetic moment of the deflector magnet, which has already been determined.

Another method by which we can determine the absolute value of a uniform magnetic field of considerable extent is by placing in it a long coil of insulated wire, having at its centre in the interior a magnetometer needle made and suspended as above described. The coil should be so placed that the axis of the coil is in the direction of the lines of the field the strength of which is to be determined. Under the action, therefore, of the field to be measured the

magnetometer needle will stand in one direction, and under the action of the magnetic forces of the coil taken alone it will stand in the same direction. A current can then be sent through the coil of such a magnitude that no field exists at all inside it. This will be the case when the current has such a direction and strength that its field, numerically expressed by  $0.4\pi$  times the ampere-turns per centimetre on the coil, is equal in magnitude and opposite in direction to the other field. The absence of any resultant field is shown by the magnetometer needle setting differently in any direction, and when this is the case the strength of the original field is equal to  $0.4\pi$  times the ampere-turns per centimetre on the long coil.

A third method, due to Messrs. Edser and Stansfield, consists in determining the torque acting on a coil traversed by a known current when the coil is held with its plane parallel to the direction of the field. (See *Proc. Phys. Soc. Lond.*, May 13, 1892, Vol. XI., p. 338; or *The Electrician*, Vol. XXIX., pp. 71 and 118.)

**§ 7. Determination of a Magnetisation Curve for Iron by the Ballistic Galvanometer.**—If a sample of iron is furnished for which it is required to determine the form of the curve connecting magnetising forces and flux density, commonly called the ( $B/H$ ) curve, it is in every way preferable to give the metal the form of a ring, which may be either square, rectangular or round in section. Generally speaking, the most convenient size is a ring the mean diameter of which is 10cm., the width  $\frac{1}{2}$ cm., and the depth 2cm. The cross-section of the ring is, therefore, 1 sq. cm., and the mean length of the perimeter is  $10\pi$  cm. This ring should be carefully annealed after being cut to shape, the sharp edges having been taken off and the dimensions of the ring carefully measured. The ring should be wound over with silk tape and shellaced. Upon the ring is then placed at one or more places a secondary coil of, say, 100 turns of

silk-covered wire. This wire may conveniently be No. 36 single silk-covered copper wire. After putting on these coils, the ring is wound over uniformly with one or more layers of a primary coil of No. 18 double cotton-covered copper wire, forming a magnetising coil. The ends of the fine wire inner secondary coil are brought out through the turns of the outer primary coil, which is put on over the top of it. In some cases it is desirable that the iron ring should be laminated—*i.e.*, not formed of solid metal, but of thin rings stamped out of sheet metal which are piled

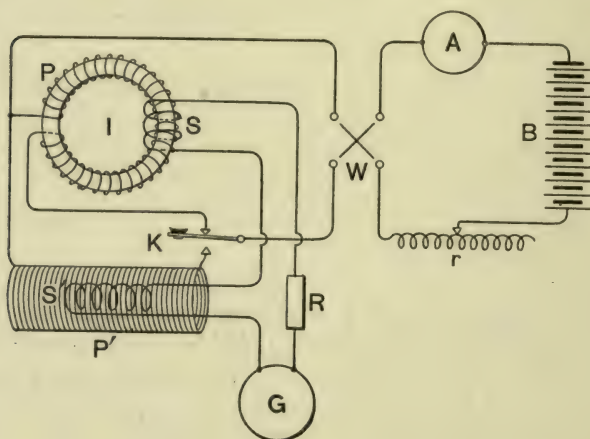


FIG. 14.

I Iron Ring.	S' Secondary Search Coil.	R Rheostats.
P Primary Coil.	W Current Reverser.	A Ammeter.
P' Standardising Field Coil.	B Battery.	K Switch Key.
S Secondary Coil.	G Galvanometer.	

one on the other. In this case each ring should be varnished so as to prevent conducting contact with its neighbour, and the dimensions of the rings measured and the number of them noted. We can then proceed to take observations on the magnetising force and flux density as follows:—An ammeter is put in series with the primary coil, and also a double-pole reversing switch, by means of which a current

can be reversed through the coil. If the number of turns on the primary coil is  $N$ , if the mean length of the perimeter is  $L$  centimetres, and if a current of  $A$  amperes is passed through the primary coil, then the uniform magnetising force  $H$ , to which the iron is subjected, is equal to  $\frac{4\pi}{10} \frac{AN}{L}$

C.G.S. units. The secondary coil  $S$  upon the iron ring is then connected in series with a ballistic galvanometer and with another secondary calibrating coil,  $S'$ , which can be placed in the field of a standardising coil,  $P'$ , as described in § 4, the arrangements being as in Fig. 14. If, then, the primary current employed to magnetise the ring is reversed by the reverser  $W$ , we obtain a throw,  $\theta$ , on the galvanometer  $G$ . We then switch the current off the coil  $P$  into the coil  $P'$  by the switch  $K$ , and we can then adjust the current through the standardising field-coil until, when this last current is reversed, it creates a throw on the galvanometer  $\theta_1$  as nearly as possible equal to  $\theta$ . We have then all the means of calculating the flux density  $B$  in the iron coil by the formula

$$B = \frac{2\pi}{5} \frac{N_1 A_1}{L_1} \frac{n_1 a_1}{na} \frac{\theta}{\theta_1} \dots \dots \dots (44)$$

where  $n$  and  $a$  are the number of turns and area of each turn of the secondary coil on the iron ring,  $N_1 L_1$  the turns and length of the standard field-coil,  $A_1$  its current, and  $n_1$  and  $a_1$  the turns and area of each turn of the secondary calibrating coil enclosed by the standard field-coil.

These operations should be repeated, increasing step by step the current in the primary coil magnetising the iron ring, and we then obtain a series of values for  $B$  and the corresponding values of  $H$ , increasing step by step. The primary current magnetising the iron ring should be so adjusted that the value of  $H$  increases by nearly equal steps from  $H=0$  to  $H=100$ , and in some cases it is desirable to plot out the values of  $B$  and  $H$  for small values of  $H$  on a larger scale. The value of  $H$  is obtained at once, because it

is equal to  $\frac{4\pi}{10} \frac{AN}{L}$ , where  $L$  is the mean perimeter of the ring,  $A$  the magnetising current and  $N$  the turns of the primary coil.

When these values are set out graphically, we get a curve as shown in Figs. 1, 2 and 3, which is called a  $(B/H)$  curve, for the iron in question. It will be found that the curve generally rises up quickly to a "knee," and then becomes more flat; and for good magnetic qualities of iron or cast steel the value of  $B$ , in C.G.S. units or lines per square centimetre, corresponding to  $H=100$ , should approximate to 17,000 or 18,000. On performing this experiment for various samples of iron and steel, as used in dynamo work, it will generally be found that, although the iron has a lower value of the flux density for low magnetising forces than the cast steel, yet for high magnetising forces the cast steel generally exceeds the iron in flux density for a given high magnetising force.

The extreme tediousness of tests made with the ballistic galvanometer renders it desirable to proceed in a systematic manner, and at the same time to adopt certain precautions to secure accurate results. In the first place, the galvanometer employed should be a movable coil galvanometer, having a time period of not less than 5 seconds, and preferably of 10 or 15 seconds, for one complete swing when the coil is on open circuit. The galvanometer should be provided with a short-circuiting key, by means of which the coil circuit can be closed and the coil, therefore, quickly brought to rest after each experiment. The source of electric current should be a secondary battery, and a very convenient form is the lithanode battery (*see* Vol. I., Chap. I., p. 12). If the results of the tests are to be considered as valuable, very great care should be taken in the preparation of the ring that it represents a fair sample of the material being tested. The ring should be cut out of the solid, and not forged or cast; in any case it should be carefully examined for flaws and cracks, which will greatly reduce the average permeability.

The ring having been wound with the two circuits, primary and secondary, as already described, should, in the first place, be carefully demagnetised, if by any chance a current has already been sent through the primary coil. This demagnetisation may be accomplished by sending through the primary coil a current the direction of which is rapidly reversed, whilst at the same time the current is continually reduced in strength by means of a resistance. One way to demagnetise such a ring is to put it into connection with an alternator, and send through it an alternating current. The driving power of the alternator should then be removed, and the machine allowed to come to rest under the action of the frictional resistance. As it slows down, the alternating E.M.F. will diminish without limit to zero, and therefore also the magnetising current in the solenoid. The arrangement which is most convenient to use for these ballistic tests of iron, whether the object be to determine a single-valued or a cyclical magnetisation curve, is that which is described by Prof. Ewing in a Paper on the "Magnetic Testing of Iron and Steel."\* The disposition of apparatus is as shown in Fig. 15. The iron ring, with its two circuits P and S, is represented at I. G is the ballistic galvanometer with its short circuiting key F, and  $S_2$  is the secondary standardising coil within or around the primary standard field coil  $S_1$ . A resistance, R, is joined up in the series with the secondary coil on the iron ring, and the secondary standardising coil and galvanometer, so as to make one complete circuit. In the same manner the primary magnetising coil on the ring and the primary standardising coil  $S_1$  are joined up to a two-way switch, C, and these, together with an ammeter, A, are joined up with the battery B and ammeter A and a rheostat,  $R_1$ . These different circuits are connected through a throw-over switch, K, mounted on a board, the handle of which, when thrown over to one side, connects a split contact or a mercury cup,  $a$ , with  $c$  or  $e$ , and a similar contact,  $b$ , with  $d$

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\* *Proc. Inst. Civ. Eng.*, Vol. CXXVI., 1896.

or *f*. Another variable resistance,  $R_2$ , and circuit-closing key, *k*, are provided. It will be seen that the operation of the apparatus is as follows:—The magnetising current from the storage battery *B* is regulated by the adjustable resistance  $R_1$  and measured by the ammeter *A*. It then passes through the key *K*, which, when thrown over to one side, places *a* in contact with *c* and *b* with *d*, and when thrown over to the other side, places *a* in contact with *e* and *b* with *f*. As long

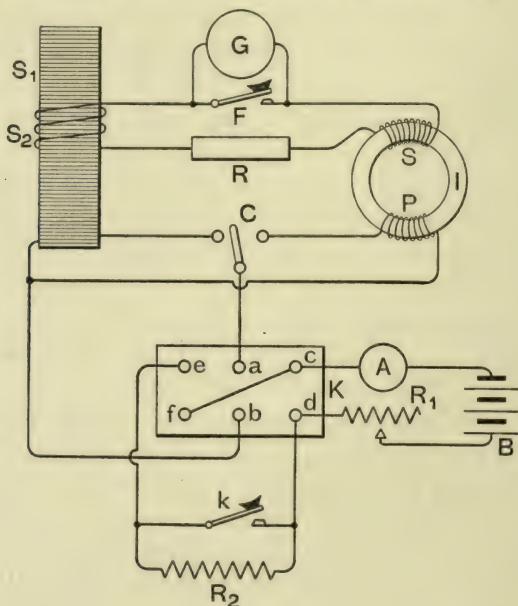


FIG. 15.

as the short-circuit key *k* is closed this movement of the switch *K* simply reverses the current through the primary coil on the iron ring, or through the primary standardising coil, according to the direction of the two-way switch *C*. Hence, by adjusting the resistance  $R_1$ , the circuit reverses a suitable current through either primary coil, and produces therefore a throw on the galvanometer due to the change in flux through the corresponding secondary coil, and, as already

explained, the standardising coil enables us to interpret the meaning of the throw obtained when experimenting with the iron ring.

If, however, the key *k* is opened, the effect of moving over the handle of the reversing switch *K* is not only to change the sign of the current but to alter its amount by throwing in the supplementary adjustable resistance *R*<sub>2</sub>. This adjustable resistance may conveniently be a liquid rheostat, consisting of a pair of amalgamated zinc plates, which are lowered more or less into a vessel containing a dilute solution of sulphate of zinc. In taking a single-valued magnetising curve to obtain the permeability of the material the best way to proceed is as follows:—After having demagnetised the ring, we begin by applying a series of small magnetising forces by passing small measured currents through the primary coil on the iron ring. After measurement this current is reversed in direction, and the result is to produce a throw on the galvanometer which, as already explained, is proportional to the total quantity of electricity sent through it, and dependent upon the corresponding flux change. Hence, as the magnetising current is increased step by step, each time being reversed, we mount up higher and higher upon the magnetisation curve. The galvanometer, in fact, measures for us the double value of the total flux in the core corresponding to each measured magnetising current. The magnetising currents are increased up to a point which corresponds with a magnetic force of 100 C.G.S. units. It will generally be found that the limit to this magnetising force is the heating of the copper coils and of the iron ring. The current cannot be increased beyond the value at which it begins to burn the insulation on the wire; hence there is generally a limit to the magnetising force that can be applied by this method. But, by the selection of a suitable battery and primary wire, it is not difficult to reach magnetising forces of 100 or 120 C.G.S. units. By a little care it is possible to make the magnetising forces increase by equal

increments to about 5 units, and also to take some observations at the lower point of the curve for much smaller forces than five. In taking the observation corresponding to each magnetising force it is well to reverse the key K several times before taking the final galvanometer reading, as this operation wipes out the effect left in the iron core from the previous stage of magnetisation.

As the result of these observations, it is possible to set out a series of numbers denoting the flux density in the ring corresponding to the known magnetising forces.

A number of observations of this kind on various samples of iron and steel are given in the Paper by Prof. Ewing above mentioned (*loc. cit.*). Some of these are collected in the following Table IV. :—

TABLE IV.

*Magnetic Flux Density corresponding to various Magnetising Forces in the case of certain Samples of Iron and Steel.*

(Ewing.)\*

Magnetising Force (H) in C.G.S. units.	Magnetic Flux Density B. (C.G.S. units.)					
	I.	II.	III.	IV.	V.	VI.
5	12,700	10,900	12,300	4,700	9,600	10,900
10	14,980	13,120	14,920	12,250	13,050	13,320
15	15,800	14,010	15,800	14,000	14,600	14,350
20	16,300	14,580	16,280	15,050	15,310	14,950
30	16,950	15,280	16,810	16,200	16,000	15,660
40	17,350	15,760	17,190	16,800	16,510	16,150
50	...	16,060	17,500	17,140	16,900	16,480
60	...	16,340	17,750	17,450	17,180	16,780
70	...	16,580	17,970	17,750	17,400	17,000
80	...	16,800	18,180	18,040	17,620	17,200
90	...	17,000	18,390	18,230	17,830	17,400
100	...	17,200	18,600	18,420	18,030	17,600

The figures under heading I. are values given in a Paper by Messrs. Lydall and Pocklington† as the results of a magnetic test of an exceptionally pure iron supplied for the

\* "Magnetic Testing of Iron and Steel" by J. A. Ewing, F.R.S., *Proc. Inst. Civ. Eng.*, Vol. CXXVI., part 4, 1896.

† "Magnetic Properties of Pure Iron," *Proc. Roy. Soc.*, Vol. LII., p. 228.

purpose of experiment by Colonel Dyer of the Elswick Works. The substances other than iron in this sample were stated to be carbon, trace; silicon, trace; phosphorus, none; sulphur, 0.01 per cent.; manganese, 0.1 per cent. The other five specimens, II. to VI., are samples of commercial iron or steel. No. II. is a sample of Low Moor bar iron forged into a ring, annealed and turned. No. III. is a steel forging furnished by Mr. R. Jenkins as a sample of forged ingot metal for dynamo magnets. No. IV. is a steel casting for dynamo magnets, unforged, made by Messrs. Edgar Allen & Co. by a special pneumatic process, under the patents of Mr. A. Tropenas. No. V. is an unforged steel casting for dynamo magnets, made by Messrs. Samuel Osborne & Co. by the Siemens process. No. VI. is an unforged steel casting for dynamo magnets, made by Messrs. Krupp of Essen.

It will be seen from the figures and the description of the materials that the steel forgings and castings have a remarkably high permeability under small magnetising force.

Table V. shows the magnetic qualities of some of these materials when tested with small magnetising forces.

TABLE V.

*Magnetic Permeabilities of Samples of Iron and Steel under Small Magnetising Forces.*

Magnetic Flux Density (B) in C.G.S. units.	Pure Iron. I.		Steel Forging. III.		Steel Casting. VI.	
	H	$\mu$	H	$\mu$	H	$\mu$
2,000	0.90	2,220	1.38	1,450	1.18	1,690
4,000	1.40	2,850	1.91	2,090	1.66	2,410
6,000	1.85	3,240	2.38	2,520	2.15	2,790
8,000	2.30	3,480	2.92	2,740	2.83	2,830
10,000	3.10	3,220	3.62	2,760	4.05	2,470
12,000	4.40	2,760	4.80	2,500	6.65	1,810

The numbers I., III. and VI. in the above Table refer to the samples similarly numbered in connection with Table IV.

When a cyclical magnetisation curve has to be taken the procedure is somewhat different. It is then necessary

always to start from a given stage of magnetisation corresponding to a certain maximum selected magnetising force, and to change this magnetising force suddenly either to a lesser value in the same direction or to another value equal or less in an opposite direction, and to observe the corresponding change in the flux which takes place in the iron core. For this purpose the variable resistance  $R_2$  (see Fig. 13) is brought into operation. The short-circuiting key  $k$  is opened, and after a magnetising current has been passed through the primary coil on the iron ring corresponding to any given magnetising force, say, 5 C.G.S. units, the force is suddenly reduced, say, to 4, 3, 2 or 1 C.G.S. units, and the corresponding change in the flux observed by means of the ballistic galvanometer.

We come, then, in due course to an observation in which we observe the flux change corresponding to a removal of the magnetising force, *i.e.*, a drop from  $H=5$  to  $H=0$ . The subsequent course of the experiment consists in suddenly changing the magnetising force from a value, say  $+5$ , to a negative value, either  $-5$  or something between  $-5$  and zero. This further stage in the process is achieved by leaving the key  $k$  open and throwing over the handle of the reversing switch  $K$ . The current is thereby reduced from its full value to a smaller value, and at the same time reversed in sign, the reduction in value being determined by the adjustment of the variable rheostat  $R_2$ .

In plotting out these results we proceed as follows:—Two axes are drawn at right angles (see Fig. 16). On the horizontal one are marked off magnetic forces and on the vertical one flux densities. The crossing point of the two lines is taken as the origin of magnetic force. Through the abscissæ points, which represent the maximum positive and negative values of the magnetic force, are drawn two other vertical lines, and on the right-hand one of these is set off a distance representing the corresponding flux density in the core, which is represented by the symbol  $B_{\max}$ . Through the

top of this ordinate a horizontal line is drawn, and the rectangle is symmetrically completed as in Fig. 15. If, then, we set off, corresponding to the various magnetising forces, ordinates measured from the top line, which to scale represent the reduction in the flux density corresponding to the magnetic change from the maximum force to the one applied, we obtain the points on a curve, which is one branch of the cyclical magnetisation curve. The other branch of the curve, as shown in Fig. 16 by the dotted line *m*, is obtained by simply drawing a symmetrical completion of the closed loop, and we obtain as the result a *hysteresis*

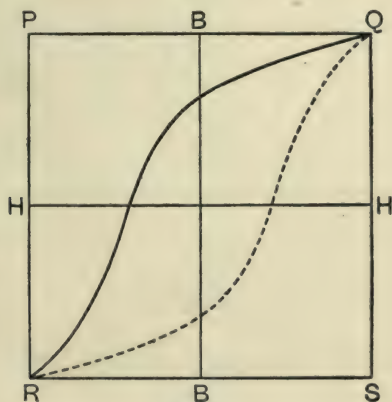


FIG. 16.

loop or cyclical magnetisation curve set off in terms of *B* and *H*. When this is done, the area of this curve has to be taken with a planimeter and reckoned out in terms of the area of a rectangle, one side of which is the length taken to correspond with 1 C.G.S. unit of magnetic force and the other the length taken to correspond with 1 unit of magnetic flux density called "1 line." Thus, for example, supposing we take, as the scale for graphical representation, 1cm. measured in a horizontal direction to represent 1 C.G.S. unit of magnetic force, and 1cm. to stand for

1,000 "lines." If we then measure the area of the hysteresis loop in square centimetres, this number, multiplied by 1,000 and divided by  $4\pi$ , is the work in ergs per cubic centimetre expended in making one complete magnetic cycle of operations in the material.

Having drawn in this manner one complete hysteresis loop, it is necessary to take another, corresponding to a different maximum magnetic force, and so to complete the series of

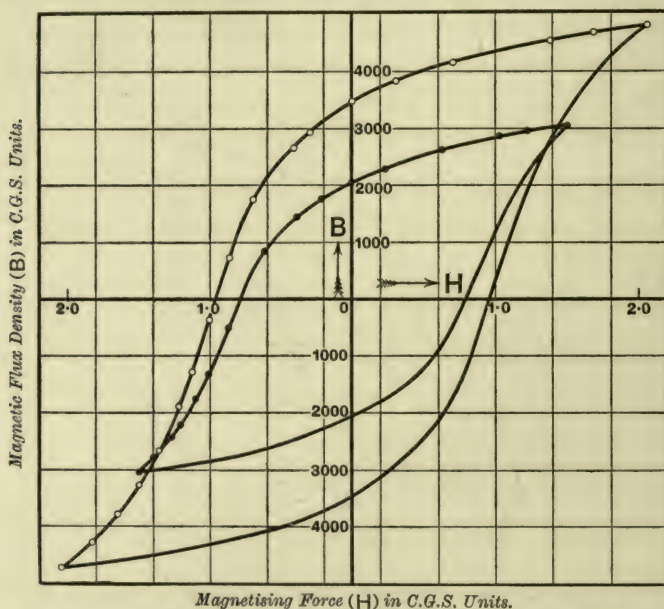


FIG. 17.—Samples of Cyclical Magnetisation Curves for good Soft Iron.

loops, as shown in Fig. 17, each one of which corresponds to a different value of  $B$ . This process is at the best somewhat laborious, not only in taking the observations, but in reading them and drawing the curve; but the result, when carefully carried out, is to give us a very accurate insight into the behaviour of the metal when subjected to simple or cyclical magnetising forces.

A series of such curves for a transformer iron ring are given in Fig. 18.

For a comprehensive series of tests on iron by the ballistic method the reader may consult a Paper by Prof. Ewing and Messrs. Klaasen, "On the Magnetic Qualities of Iron." (See *Trans. Roy. Soc.*, Vol. CLXXXIV.A, 1893, p. 985; also *The Electrician*, Vol. XXXII, pp. 637, 668, 713, and Vol. XXXIII, pp. 8, 38.)

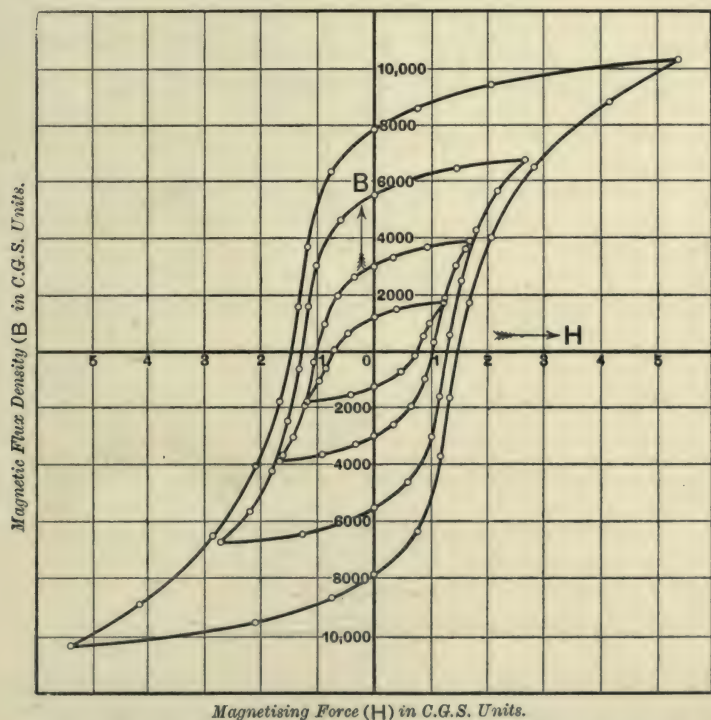


FIG. 18.—A Series of Cyclical Magnetisation Curves for a Sample of Transformer Iron.

Loss per 100  $\sim$  at the Flux Density  $B=2,500=0.659$  watts per kilogramme.

Other useful references are :—

B. S. SUMMERS. "The Relation of Magnetic and Chemical Properties of Iron." *Science Abstracts*, Vol. I., p. 150.

Prof. E. WILSON. "Magnetic Properties of Almost Pure Iron."  
*Science Abstracts*, Vol. I., p. 267; or *Electrical Review*, Vol.  
 XLII., p 313.

W. LIECK. "Some Magnetic Properties of Iron, Nickel and  
 Cobalt." *The Electrician*, Vol. XXXVII., p. 667.

§ 8. **Permeability Curves.**—Having taken a series of observations of the magnetic force  $H$  and magnetic flux density  $B$  with a ring of iron or steel as above described, we can proceed to set out a *Permeability Curve*. In this case the horizontal

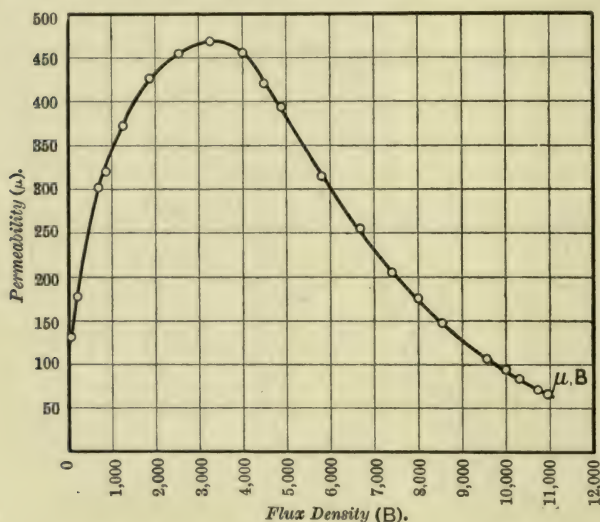


FIG. 19.—Permeability Curve for a Sample of Cast Iron.

distances are generally taken to represent flux density  $B$ , and the vertical distances or ordinates to represent the ratio of  $B$  to  $H$  or the permeability ( $=\mu$ ). If a number of values of  $\mu$  are plotted out in terms of  $B$  from the observations taken to determine a single-valued ascending magnetisation curve, it is found that the permeability curve begins at a low value and rises up to a maximum value corresponding to some flux density lying between 5,000 and 7,000 C.G.S. units or lines

per square centimetre. If the flux density is pressed up still higher the permeability falls down to a very small value,

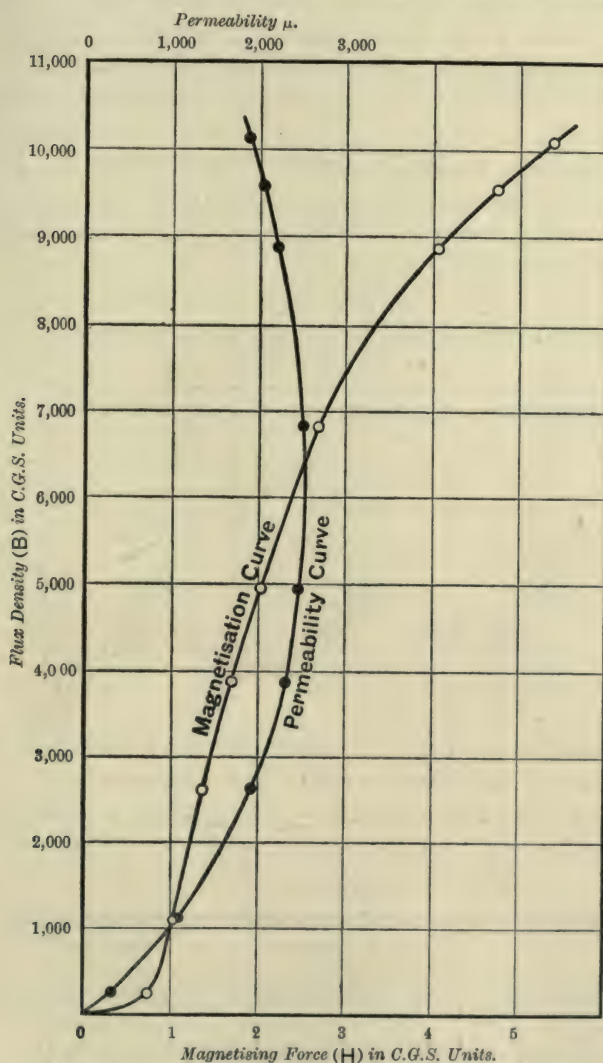


FIG. 20.—Magnetisation and Permeability Curve for a Sample of Transformer Iron.

ultimately tending towards unity (*see* Fig. 19). The permeability, however, for zero magnetic force is not zero, but (according to Lord Rayleigh) for very small forces tends towards a value  $\mu = 80$  approximately.

The curves shown in Fig. 20 are the magnetisation curve and permeability curve of a sample of good transformer sheet iron.

The following Table VI., taken from a Paper by Dr. Shelford Bidwell, shows the general nature of the variation of the permeability throughout the range of magnetising forces which have been attained :—

TABLE VI.

*Variation of Permeability of Iron with Magnetising Force.*  
(Bidwell.)

Magnetic Force = $H$ .	Flux Density = $B$ .	Permeability = $\mu$ .
0.2	80	400
0.5	300	600
1	1,400	1,400
2	4,800	2,400
4	8,800	2,200
7	11,200	1,600
11	13,200	1,200
16	14,400	900
65 (Rowland)	16,500	255
200 (Bidwell)	18,000	90
585 "	20,000	34
24,500 (Ewing)	45,300	1.9

The magnetic properties of nearly pure iron have formed the subject of investigation by several observers. Dr. J. Hopkinson\* gave the following figures (*see* Table VII.) for  $B$  and  $H$  for a ring of nearly pure iron :—

TABLE VII.

*Flux Density and Corresponding Magnetising Force for Nearly Pure Iron.*  
(J. Hopkinson.)

$B = 34$	118	467	2,700	7,060	10,980	14,160	15,590	16,570	17,120	17,400
$H = 0.15$	0.38	0.60	1.06	2.11	3.77	7.48	13.36	23.25	33.65	44.66

\* *Proc. Roy. Soc.*, Vol. LII., p. 228.

Prof. E. Wilson has also given the results of similar investigations on a ring of pure iron\* which contained no carbon, no silicon, no phosphorus—only 0·1 per cent. of manganese and 0·013 per cent. of sulphur.

The values of the permeability ( $\mu$ ) obtained by him for this iron before and after annealing are given in Table VIII. below :—

TABLE VIII.

*Permeability of Nearly Pure Iron Before and After Annealing.*

(E. Wilson.)

Flux Density B.	Permeability = $\mu$ .	
	Before Annealing.	After Annealing.
2,000	2,000	2,500
3,000	2,730	3,190
4,000	3,330	3,810
5,000	3,700	4,350
6,000	4,138	4,800
7,000	4,375	5,380
8,000	4,445	5,440
9,000	4,615	5,490
10,000	4,545	5,460
12,000	4,000	4,900
14,000	2,641	3,260
15,000	1,415	2,050

The above figures show well the manner in which permeability rises up to a maximum value and then decreases again as the flux density increases.

Variations in the chemical composition of iron and steel have an immense influence upon the permeability. Broadly speaking, an approximation to perfect chemical purity with good annealing increases the permeability to a maximum which may approach a value of 4,000 or 5,000 at ordinary temperatures.†

\* See *Proc. Roy. Soc., Lond.*, Vol. LXII., p. 369, on "The Magnetic Properties of Nearly Pure Iron."

† For valuable information on this subject see a Paper by Mr. H. F. Parshall "On the Magnetic Data of Iron and Steel," *Proc. Inst. Civ. Eng.*, Vol. CXXVI., 1896.

The following table (IX) of permeabilities for various descriptions of transformer plate iron and steel are taken from the Paper by Prof. Ewing (*loc. cit.*) :—

TABLE IX.

*Permeability Tests of Transformer Plate and Wire.*

Magnetic Flux Density $B$ in C.G.S. units.	Transformer Plate of Swedish Iron. VII.	Transformer Plate of Scrap Iron. VIII.	Transformer Plate of Steel. IX.	Transformer Wire. X.
$B$	$H$ $\mu$	$H$ $\mu$	$H$ $\mu$	$H$ $\mu$
1,000	0.81 1,230	1.08 920	0.60 1,470	1.71 590
2,000	1.05 1,900	1.46 1,370	0.90 2,230	2.10 950
3,000	1.26 2,320	1.77 1,690	1.04 2,880	2.30 1,300
4,000	1.54 2,600	2.10 1,900	1.19 3,360	2.50 1,600
5,000	1.82 2,750	2.53 1,980	1.38 3,620	2.70 1,850
6,000	2.14 2,800	3.04 1,970	1.59 3,770	2.92 2,070
7,000	2.54 2,760	3.62 1,930	1.89 3,700	3.16 2,210
8,000	3.09 2,590	4.37 1,830	2.25 3,600	3.43 2,330
9,000	3.77 2,390	5.3 1,700	2.72 3,310	3.77 2,390
10,000	4.6 2,170	6.5 1,540	3.33 3,000	4.17 2,400
11,000	5.7 1,930	7.9 1,390	4.15 2,650	4.70 2,340
12,000	7.0 1,710	9.8 1,220	5.40 2,220	5.45 2,200
13,000	8.5 1,530	11.9 1,190	7.1 1,830	6.5 2,000
14,000	11.0 1,270	15.0 930	10.0 1,400	8.4 1,670
15,000	15.1 990	19.5 770	...	11.9 1,260
16,000	21.4 750	27.5 580	...	21.0 760

No. VII. is a specimen of good transformer plate 0.301mm. thick, rolled from Swedish iron by Messrs. Sankey of Bilston. No. VIII. is a specimen of specially thin transformer plate rolled from scrap iron. No. IX. is a specimen of transformer plate rolled from ingot steel. No. X. is a specimen of the wire which was used by Mr. Swinburne some years ago to form the core of his "Hedgehog" transformer. Its diameter was 0.602mm. All these samples were tested in the form of rings by the ballistic method, the rings of sheet metal being stamped or turned in the flat. The wire ring No. X was coiled and annealed after coiling.

The permeability of cast iron and the various varieties of hard steel falls very much below that of well annealed pure iron and mild steel. The general forms of the magnetisation curves for various ferromagnetic metals are shown in Fig. 19,

taken from a Paper by Messrs. N. E. Thompson, P. H. Knight and G. W. Bacon, "On the Magnetic Permeability of Special Irons for Electrical Purposes" (*Trans. Am. Inst. E.E.*, Vol. IX., 1892).

Great improvements have been made of late years in the quality of cast iron and steel in regard to magnetic permeability, and dynamo builders are now able to obtain steel castings for dynamo field magnets of a very high permeability. A metal called "mitis" metal has been prepared from wrought iron by melting down scrap wrought iron in crucibles and rendering it fluid by the addition of a small quantity of aluminium. This mixture can then be cast in moulds, and when proper precautions are taken to ensure sound castings, a material is obtained which has a much higher permeability than ordinary cast iron, although the castings are somewhat difficult to work on account of their roughness.

In making tests of permeability, the influence of temperature has to be taken into account. A rise of temperature affects the permeability of iron in a different manner according as the magnetic force to which the iron is subjected is large or small. If small, a rise in temperature increases the permeability somewhat irregularly until a temperature of 500°C. or 600°C. is reached. At this point the permeability begins to rise very rapidly, and at a temperature rather below 800°C. it may attain a value 8,000 to 10,000 or more; but at a slightly higher temperature the permeability falls very quickly to an exceedingly small value. On the other hand, with very large magnetising forces the permeability appears to decrease with a rise of temperature, and at a temperature about 700°C. begins to drop rapidly to a small value, which is attained at a critical temperature of about 795°C. A valuable investigation on the effect of temperature on the magnetic permeability of iron was made by Dr. D. K. Morris entitled "On the Magnetic Properties and Electrical Resistance of Iron as Dependent upon Temperature."\*

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\**Proc. Phys. Soc. Lond.*, Vol. XV, p. 134, 1897; also *Phil. Mag.* Sept., 1897, Vol. XLIV., series 5, p. 213.

The diagrams given in this Paper show in a clear manner the variation of permeability with temperature throughout the above-mentioned ranges.

A very curious variation in the permeability occurs a few degrees below the critical temperature of  $800^{\circ}\text{C}$ . The permeability, after becoming reduced to a value of about 50, appears to rise again to a value of nearly 100 at  $800^{\circ}\text{C}$ ., and then falls again at about  $900^{\circ}\text{C}$ . to an extremely small value; again it rises slightly, and finally, at  $1,200^{\circ}\text{C}$ ., becomes reduced to a value which is probably not far from unity.

According to Dr. D. K. Morris, the permeability under small magnetising forces (less than 0.5 C.G.S. unit) rises at first slowly at the rate of 0.2 to 0.3 per cent. per degree Centigrade, when the temperature is first raised. Then, in the neighbourhood of  $300^{\circ}\text{C}$ ., it begins to rise rapidly. It remains nearly constant at  $400^{\circ}\text{C}$ . and  $550^{\circ}\text{C}$ ., after which it rises with increasing rapidity to some very high value, the maximum observed having been  $\mu=12,660$ , corresponding to  $H=0.078$ . After which, as above mentioned, it sinks rapidly, and its value at the critical temperature is quite insignificant. Another interesting point with regard to the specimen of iron tested in the above experiments is the diminution of permeability which it experiences in the neighbourhood of  $550^{\circ}\text{C}$ . All tests, therefore, of permeability of iron or steel, to be of any considerable value, should be accompanied by a chemical analysis of the material, and also by a statement of the temperature at which the experiments were made. In carrying out experiments with the ring form of specimen the iron is generally heated to a fairly high temperature by the current in the magnetising coils, and, as there is little or no free radiating surface, the temperature of the iron is uncertain, and probably continues to rise during the whole course of the experiments. The temperature can best be ascertained by laying next to the iron a silk-covered platinum wire the resistance of which has been ascertained previously for various temperatures. This wire is laid round

the periphery of the iron ring, and the magnetising coil put over it. The two ends of this platinum wire are brought to the outside, and by measuring its resistance we can determine very approximately the temperature of the iron ring.

**§9. Bar and Yoke Methods for Determining Magnetisation Curves.**—The labour of winding a closed ring with primary and secondary coils, and the fact that a considerable time and expenditure are also necessary to prepare the ring-shaped sample of iron or steel for permeability tests, has caused investigators to direct attention to methods by which straight bars of metal could be used instead of rings for magnetic

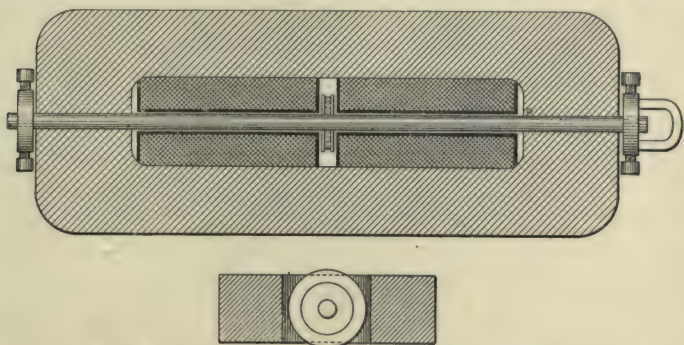


FIG. 21.—Hopkinson's Bar and Yoke Apparatus.

tests. The preparation of a short, straight, circular-sectioned rod of iron or steel is a comparatively simple matter, and samples of metal in this form can be easily slipped into bobbins of wire which are kept ready wound on formers. Hence great advantages result if the metal for magnetic tests can be employed in the above-mentioned form. The first attempt in this direction was made by Dr. J. Hopkinson, who employed a compound closed magnetic circuit which was formed partly of a short straight bar fashioned from the material to be tested, the remainder of the circuit being a yoke of soft iron of high magnetic quality. The arrangement is shown in Fig. 21, in which the shaded portion represents

the section of a massive soft iron ring or yoke. Through the two ends of this are bored clean holes, in which fit two identical samples in the form of rods of the iron or steel to be tested. One end of each rod is faced so that when the two are brought together they make a good contact surface. One rod is permanently clamped into the yoke, and the other can be pulled, with little difficulty, by means of an eye, through a hole, which it fits well, on the other side of the yoke. The two test rods are surrounded by magnetising coils, which are shown cross-hatched in the diagram. Between these coils there is another secondary coil or small bobbin which is slipped on the rod, and when one rod is pulled back by  $\frac{1}{4}$  in. this bobbin is made to spring out by an elastic band attached to it from between the two rods. The magnetising current flows through the fixed bobbins of wire, and by a measured current a magnetising force can be applied. The secondary coil is then put in place, and the two test rods made to touch one another. The secondary coil is then connected to the ballistic galvanometer, and when one of the rods is pulled back a little way the secondary coil flies out, cutting in its passage through all the flux lines which pass out of one bar into the other. This enables the total magnetic flux up the bars to be measured. The method has the advantage that it is not interfered with by any sluggishness in the magnetic circuit in either taking up or losing its magnetic flux when the magnetic force is applied or withdrawn. It has, however, the great disadvantage that there are joints in the magnetic circuit, and however well the two rods may be made to fit together, these joints introduce the necessity for a correction to obtain the true effective value of the magnetic force. The effective magnetic force cannot, therefore, be taken as quite equal to  $0.4\pi$  times the ampere-turns per unit of length of the rods. Apart also from the influence of the joints themselves, the magnetising turns are not distributed uniformly along the magnetic circuit, and, moreover the correction to be applied is

not strictly the same for all bars, even with the same value of the flux density, since part of it is due to the fact that the virtual length of the bar is greater than the clear length, and consequently greater at any given flux density the less permeable the bar. Of the whole number of ampere-turns on the magnetising coil, some are employed in overcoming the magnetic resistance of the yoke, some in overcoming the magnetic reluctance which the flux meets with in getting from the yoke into the bar, some in overcoming the magnetic reluctance of the contact-joint between the bars, and only the remainder are effected in overcoming the magnetic reluctance of the bars themselves, reckoning on the clear length of the bars free from the yoke. However permeable may be

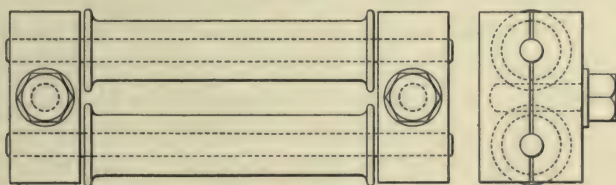


FIG. 22.—Ewing's Bar and Yoke Permeameter.

the yoke, however large its section, and however well faced the bars, the above terms cannot be neglected in the calculation of the true effective magnetic force. If the correction is omitted, considerable errors may be introduced.

In another and better arrangement, due to Prof. J. A. Ewing, the two straight test-bars are clamped between two massive end-yokes, magnetising coils having been previously slipped on to the bars. The bars are cut longer than the distance between the yokes, and this last can be altered. Each of the two yokes is a small block of soft iron with holes bored in it to fit the bars, and split across the holes to allow the bars to be introduced and tightly clamped by means of a screwbolt (*see* Fig. 22). By loosening this screw the yokes may be slipped along the bars so as to enclose between them

any portion of the whole length. The magnetic circuit is also embraced at some point by a secondary coil, which can be connected to a ballistic galvanometer. The reversal of the magnetising current then induces a secondary current, by means of which, as already explained, the flux density in the core is measured. To find the true effective magnetic force it is only necessary to take two observations, one with the yokes extended as far as the bars allow and another with their clear length reduced to half the first value.

If  $H_1 = 0.4 \cdot AN/L_1$  is the magnetising force, where  $A$  is the current through the magnetising coil,  $N$  the number of turns, and  $L_1$  the clear length of the bars. In the first case—*i.e.*, when the yokes are extended as far as possible, and if  $H_2$  and  $L_2$  are the values of the force and length when the bars are only half the length and when the flux density is the same—it is easy to show that the correction applicable to  $H_2$  is just twice as great as that applicable to  $H_1$ , and hence  $H_2 - H_1$  is the value of the correction  $K$  to be subtracted from  $H$  in order to find the true magnetising force  $H$ .\*

The proof of this proposition has been given by Prof. Ewing as follows:—We shall denote as usual by  $N_1$  and  $N_2$  the turns of the coils used in the two experiments, by  $L_1$  and  $L_2$  the clear lengths of the bars between the yokes, and by  $A_1$  and  $A_2$  the magnetising currents. Then, in all cases,  $0.4\pi$  times the ampere-turns is equal to the magnetomotive force round the circuit. The magnetomotive force in the bars in the two cases considered is  $HL_1$  and  $HL_2$  respectively, where  $H$  is the true magnetic force operative in the bar. A part of the whole magnetomotive force, which we will denote by  $m$ , is taken up in driving the flux through the yoke and the joints. Hence we have in the two cases

$$0.4\pi A_1 N_1 = HL_1 + m,$$

$$0.4\pi A_2 N_2 = HL_2 + m.$$

If we write  $H_1$  for  $0.4\pi A_1 N_1/L_1$ , and  $H_2$  for  $0.4\pi A_2 N_2/L_2$ , then we have from the above equation

$$H_1 = H + \frac{m}{L_1}$$

and

$$H_2 = H + \frac{m}{L_2}.$$

Hence

$$H_2 - H_1 = m \left( \frac{1}{L_2} - \frac{1}{L_1} \right).$$

---

\* The magnetic flux, by assumption, remains the same, and therefore also the reluctance of the joints and yokes.

But if we make  $L_1 = 2L_2$ , we have

$$H_2 - H_1 = m/L_1.$$

Accordingly

$$m = L_1(H_2 - H_1).$$

In order to determine  $H$  we have to subtract from the value of the quantity  $0.4\pi A_1 N_1/L_1$  a correction equal to the value of  $H_2 - H_1$ .

The true magnetic force in the rods is less than  $0.4\pi$  times the ampere-turns per centimetre of their length by an amount equal to the difference of the ampere-turns per centimetre in the two cases multiplied by  $0.4\pi$ . The practical procedure is, therefore, as follows:—Take a magnetisation curve for the rods when out to the full length and plot the values of  $B$  against the impressed magnetic force  $H_1$ . Again, take the same observations with a different value of the magnetic force  $H_2$ . Set back each point on the magnetisation curve ( $BH_1$ ) horizontally by a distance equal to the difference of  $H_2 - H_1$  for the same value of  $B$ .

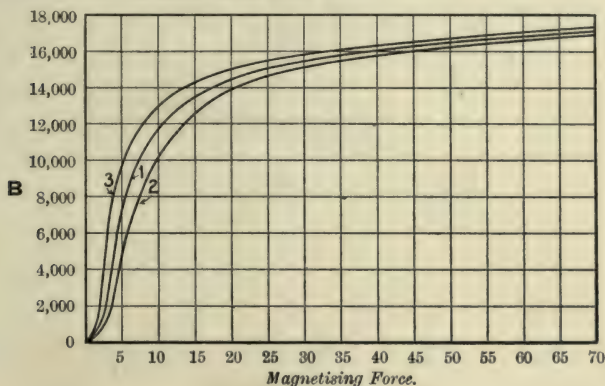


FIG. 23.

In a particular experiment described by Prof. Ewing, made by this method, rods of Low Moor iron were used 0.25in. diameter and 4in. long. The distance between the yokes in the first experiment was 6.28cm. ( $=2\pi$ cm.), and the magnetising coils had each 100 turns. Hence the quantity above designated  $H_1$  is found by multiplying the number of amperes by 20. In the second part of the experiment, where the clear length of the bars was reduced to one-half, magnetising coils of 50 turns were used, and hence  $H_2$  was found also by multiplying the number of amperes by 20.

In Fig. 23 is given a diagram showing magnetising curves obtained by this method. Curve 1 shows  $H_1$  in relation

to  $B$ , and Curve 2 shows  $H_2$  in relation to  $B$ . A third curve (3) was drawn by setting back points from the first curve through a distance equal to  $H_2 - H_1$ , and this curve gives the relation of the true effective magnetising force  $H$  to  $B$ . The diagram Fig. 24 gives the results obtained in the same way for a pair of mild steel rods of the same dimensions as the above-mentioned Low Moor iron rods tested with the same yoke-pieces. It will be seen that the correction  $K$  is greater for the steel rods, in consequence of the greater magnetic reluctance of the sunk ends. With the above method of applying a necessary correction the double bar and yoke method becomes an accurate one to employ.

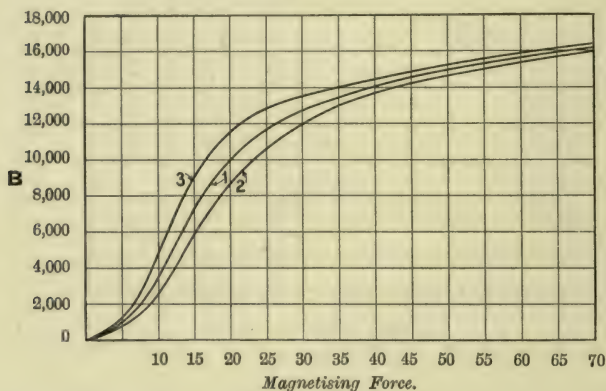


FIG. 24.

The general theory of the bar and yoke method is as follows :—Let  $L$  be the clear length inside the yoke and  $S$  the section of the test bar, and let  $\mu$  be its permeability corresponding to a flux density  $B$ . Let  $L_1$  be the mean length of the flux line in the yoke, and  $S_1$  the section of the yoke, and  $\mu_1$  the permeability of the yoke material. Let  $R$  be the magnetic reluctance of the two or more joints in the magnetic circuit. Let  $A$  be the ampere-magnetising current and  $N$  the turns on the coil. Then the impressed magnetomotive force is  $0.4\pi AN$ , and we may call  $0.4\pi AN/L$  the impressed magnetic force on the bar ( $=H_1$ ). By the fundamental equation for the magnetic circuit

$$0.4\pi AN = Z \left( \frac{L}{\mu S} + \frac{L_1}{\mu_1 S_1} + R \right),$$

where  $Z$  is the total flux up the test bar. But  $BS=Z$ , and hence we have

$$\frac{0.4\pi AN}{L} = H + Z \left\{ \frac{L_1}{\mu_1 S_1} + R \right\},$$

or

$$H_1 - H = Z \left\{ \frac{L_1}{\mu_1 S_1} + R \right\}.$$

By making the yoke of large section and of very pure soft iron we can make the value of the term  $L_1/\mu_1 S_1$  very small. There still remains, however, the unknown reluctance  $R$  at the joints. We can determine this as follows:— Let two experiments be made with various values of  $H_1$  (call them  $H_1$  and  $H_1'$ ), and two corresponding values of  $Z$ , say,  $Z$  and  $Z'$ . This can be done by varying the current in the coils. Assuming the yoke reluctance to be negligible in comparison with the "joint" reluctance, we have

$$H_1 = H + ZR$$

and

$$H_1' = H + Z'R.$$

Hence

$$R = \frac{H_1' - H_1}{Z' - Z}.$$

This "joint" resistance will not be identical for all bars, but we may, without sensible error, assume it to be so for the test bar and the standard bar.

Accordingly, the true magnetic force  $H$  operative in the bars is given by the equation

$$H = 0.4\pi \frac{AN}{L} - BS \left( \frac{H_1' - H_1}{Z' - Z} \right),$$

and the permeability  $B/H$  can then be obtained.

For additional information on the subject of "joint" reluctance the reader is referred to Prof. Ewing's book "Magnetic Induction in Iron and other Metals," pp. 273, 277, 279.

### § 10. Traction Methods of Determining ( $B/H$ ) Curves.—

If two bars of iron have their ends faced and are placed in contact with one another and then surrounded by a magnetising coil, the flux produced in the iron causes a magnetic attraction between the two faced ends. In order to pull these ends apart we have, therefore, to exert a force, and this (commonly called the "tractive force") can become a measure of the flux in the core. If  $B$  is the flux density at the contact-surface, it can be shown that this tractive force per square centimetre of surface in contact is equal to  $\frac{B^2}{8\pi}$  dynes.\* The

\* For a proof of this formula see Maxwell, "Electricity and Magnetism," Vol. II., §642, 2nd edition. See also Fleming, "Magnets and Electric Currents," p. 198.

traction method has been employed with success as an alternative method for determining the magnetisation curves of iron samples. Dr. Shelford Bidwell was the first to make measurements of the magnetic permeability of iron by this method. His mode of procedure was to cut an iron ring in half so as to form two semi-circular bars, the cut ends being carefully faced. These semi-circular bars were then included in two semi-circular solenoids, so as to magnetise the ring formed by the two bars placed in contact in the same direction throughout. One of the semi-circular bars was then fixed, and a weight attached to the lower one was increased until the two parts were separated.

The assumption has generally been made that, in the case of such a split ring wound over with a magnetising coil, the magnetising force applied to the iron is uniform in all parts and equal to  $0.4\pi AN/L$ ,  $L$  being, as usual, the mean perimeter of the ring,  $N$  the number of windings on the solenoid, and  $A$  the current sent through the coils. If  $S$  is the total square section of iron in contact in square centimetres, reckoning the area of both the two contact surfaces, and if  $W$  is the weight in grammes required to detach one semicircular bar from the other, then it is easy to show that the flux density  $B = 157 \sqrt{\frac{W}{S}}$ . Accordingly, traction experiments of this kind enable us to determine the values of  $B$  and  $H$ , and therefore of the permeability  $\mu$ , and, consequently, the magnetisation of the iron  $I$ .

Some results obtained by Dr. Bidwell for an iron ring cut as described are recorded in the Table X. below. In reducing his observations, Bidwell made use of the formula

$$F = Wg = (2\pi I^2 + HI)S,$$

where  $W$  is the weight required to detach the lower half-ring reckoned in grammes,  $S$  the surface of contact in square centimetres,  $g$  the acceleration of gravity = 981,  $I$  the magnetisation, and  $H$  the magnetic force in C.G.S. units.

It must be noted, however, that to assume the effective magnetic force to be strictly represented by the value of  $0.4\pi AN/L$  in the case of a cut ring is not quite legitimate. The two joints introduce a certain reluctance which absorbs magnetomotive force to overcome it, and the true effective magnetic force, as shown in § 9, is something less than that given by 1.25 times the ampere-turns per centimetre applied to the ring. The correction is small, but it is by no means negligible.

**TABLE X.**

*Data obtained and calculated from Tractive Force Experiments with a Split Iron Ring.*

(Bidwell.)

Magnetic Force. H	Detaching Weight. W.	Magnetisation. I	Susceptibility. k.	Permeability. $\mu$ .	Flux Density. B
4	1	2	6	5	3
3.9	2,210	587	151.0	1,899.1	7,390
5.7	3,460	735	128.9	1,621.3	9,240
10.3	5,400	918	89.1	1,121.4	11,550
17.7	7,530	1,083	61.2	770.2	13,630
22.2	8,440	1,147	51.7	650.9	14,450
30.2	9,215	1,197	39.7	500.0	15,100
40.0	9,680	1,226	30.7	386.4	15,460
78.0	11,550	1,337	17.1	216.5	16,880
115.0	12,170	1,370	11.9	150.7	17,330
145.0	12,800	1,403	9.7	122.6	17,770
208.0	13,810	1,452	7.0	88.8	18,470
293.0	14,350	1,474	5.0	64.2	18,820
362.0	14,740	1,489	4.1	52.7	19,080
427.0	15,130	1,504	3.5	45.3	19,330
465.0	15,275	1,508	3.2	41.8	19,470
503.0	15,365	1,510	3.0	38.7	19,480
557.0	15,600	1,517	2.7	35.2	19,630
585.0	15,905	1,530	2.6	33.9	19,820

Although the above traction method, making use of a cut ring, is a little less tedious than the use of the ballistic galvanometer for determining a  $(B/H)$  curve, it is not sufficiently expeditious for workshop purposes; moreover it necessitates the use of a ring-shaped specimen. Forms of commercial instruments for performing the above experiment in which straight bars are employed have, therefore,

been devised by Prof. S. P. Thompson\* and Mr. Gisbert Kapp†. These appliances are called permeameters. In Thompson's permeameter a spring balance is employed to measure the tractive force required to detach one faced end of the bar from a corresponding abutment on a soft iron yoke or ring, which completes the magnetic circuit. A modification of Thompson's permeameter is used in the test rooms of the Westinghouse and General Electric Companies in America, and is described in Foster's "Electrical Engineers' Pocket-Book," p. 68.

Another form of traction permeameter has been devised by Prof. J. A. Ewing, in which the joint which is pulled apart is between two faced surfaces of a yoke and not on the bar to be tested. The yoke is made in three parts, two of these constitute two collars, which are fitted on to the bar to be tested, and this itself is surrounded by the magnetising coil. A soft iron yoke with curved contact surfaces fits upon the circular collars, and it is at these curved surfaces that the detachment takes place. To standardise the instrument the relation between the pull required to remove the yoke and the magnetic flux in the bar is determined experimentally by testing a bar surrounded with a secondary coil, which allows ballistic measurements of the flux to be made. In an improved form of magnetic balance devised by Prof. Ewing the contact which is pulled apart is between the curved cylindrical surface of a test piece and the convex end of an electromagnet. The appearance of the instrument is shown in Fig. 25. The pull is measured by the position of a weight upon a steel yard-arm, and the sample tested is included in a coil of known magnetising power.‡

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\* *Jour. Soc. Arts*, Vol. XXXVIII., p. 885.

† "On Methods of Testing the Magnetic Qualities of Iron." *Jour. Inst. Elec. Eng.*, Vol. XXIII. p. 199, 1894; also *The Electrician*, Vol. XXXII., p. 498.

‡ For additional details of the construction and working of this magnetic balance the reader may consult *The Electrician*, Vol. XLI., pp. 110 and 148; or the *Jour. Inst. Elec. Eng.*, 1898, Vol. XXVII., p. 526; Prof. J. A. Ewing "On a Magnetic Balance for Workshop Tests of Permeability."

In apparatus of this type it is very necessary to remember, as already explained, that the presence of a joint renders the true effective value of the magnetic force less than  $0.4\pi$  times the ampere-turns per centimetre of the magnetising coil which is applied to the bar. In addition to the correction for the joint, a further one is required on account of the sensible reluctance of the yoke, and if the surfaces of the joint do not fit truly the amount of this correction is uncertain; also the distribution of the flux at the joint is irregular, and the pull consequently greater than that which would result from a uniform distribution of the same mean density.

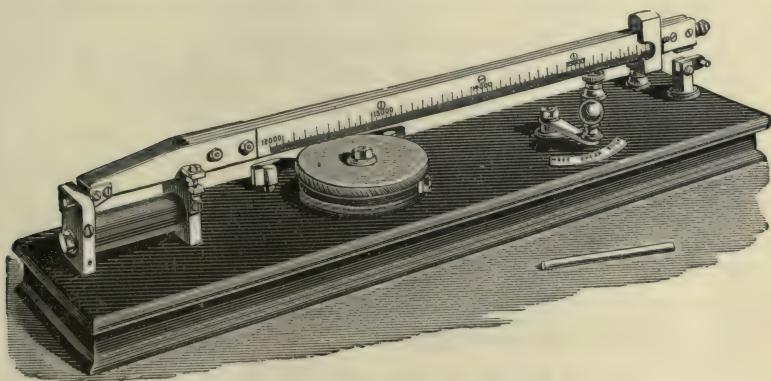


FIG. 25.—Ewing's Traction Permeameter.

Accordingly, traction methods never yield very satisfactory results by methods in which straight bars are employed as testing pieces, but if a cut ring is employed, and great care taken with the experiments, they may yield tolerably satisfactory results at an expenditure of time and labour almost equal to that which with the ballistic method would have secured results of unexceptionable character.

In order to overcome some of these difficulties a magnetic balance was devised by Dr. H. du Bois (*see* Fig. 26) in which the bar  $T$  to be tested, surrounded by a magnetising coil, is clamped between two end pieces,  $V_1$  and  $V_2$ , of soft iron. Over

these end pieces is balanced a semicircular piece of iron, *Y*, like the beam of a pair of scales. Its play is limited by two stops. This balance piece is not allowed to absolutely touch the ends of the terminal blocks, but a sliding weight, *W*, is employed to determine the torque which must be applied to the movable piece to cause it to tilt slightly over from one side to the other, against a stop. The test is made by moving this weight until the rocking piece is pulled away from a stop on the left hand side which determines the width of the gap. The details of the instrument have been worked out with much skill. For a fuller description the reader is referred to

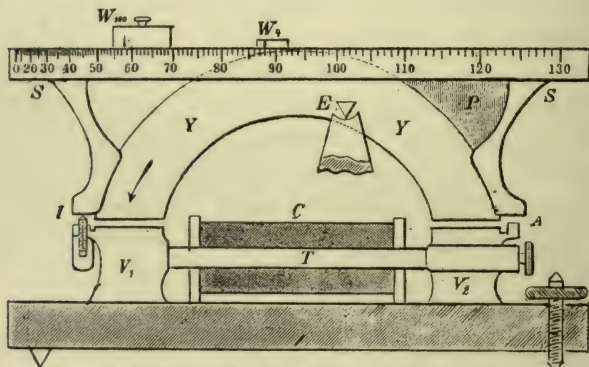


FIG. 26.—Du Bois' Magnetic Balance.

*The Electrician*, Aug. 26, 1892, Vol. XXIX., p. 448 ; or to the "Treatise on the Magnetic Circuit," by Prof. H. du Bois, p. 347.

§ 11. **Magnetic Bridge Methods for Determining Permeability.**—If time is no object, then there is no better method for determining the permeability of a sample of iron or steel than the method of employing the toroid or ring wound with two coils and the ballistic galvanometer. For commercial purposes, however, and the testing of iron and steel in the workshop, both the ballistic and the traction methods are too tedious to employ where many specimens have to be tested. In the testing room of a dynamo factory

a large number of samples of iron and steel have to be looked through rapidly with the object of determining, approximately, their permeability, and for this purpose it is not necessary, generally speaking, to take a complete magnetisation curve. The dynamo builder is concerned chiefly with the permeability corresponding to high magnetising forces; hence, where expedition and cost are important, and where a large number of specimens of iron and steel have to be tested, the methods which from the point of view of the scientific laboratory are the best, are almost impossible in the workshop.

Attention has, therefore, been directed to the invention of appliances for the rapid determination of the permeability of iron and steel when fashioned in the form of short cylindrical rods, yet without the necessity for facing the ends with great care. Methods have been developed for this purpose by Mr. Eickemeyer and by Prof. Ewing, which are analogous in their operation to the Wheatstone bridge. In this last instrument we measure the electrical resistance of a given wire by comparing it with that of a standard wire by the bridge method, and if these wires are cylindrical, circular-sectioned and of known dimensions, we can calculate from the resistance measurements the resistivity, and, therefore, the conductivity, of the material presented for test.

In the same manner an arrangement can be made by which the magnetic reluctance of a specimen of ferromagnetic material in the form of a bar can be compared with that of a standard bar, and from the dimensions the magnetic reluctivity or its reciprocal, the permeability, can be calculated, knowing that in the standard bar. The best arrangement of this kind is that due to Prof. Ewing, called a *magnetic bridge*. The rod to be tested is turned to the same dimensions as a standard rod of known permeability, and the two are made to form one magnetic circuit by joining short yokes to their ends. If these rods are surrounded with magnetising coils, and if the currents are so

adjusted that the magnetic flux up one bar is the same as that down the other, then there will be no difference in magnetic potential between the yokes. In that case, if the yokes are connected by a third magnetic conductor, say a soft iron bar, there will be no magnetic flux in it. The absence of magnetic flux in the last-named bar may be tested by making a gap in it and inserting there a small compass needle to serve as a detector. This magnetometer corresponds with the galvanometer used in connection with a Wheatstone bridge for measuring electrical resistance. In this last instrument we arrange two paths by which electric currents can flow along conductors, and if the currents and resistances are so adjusted that between two points on these two circuits there is no difference of electrical potential, as shown by connecting a galvanometer between the two points, then we are able to make an inference at once as to the electric conductivity or resistance of the two portions of each of the two circuits.

In the analogous magnetic instrument the test consists in varying the number of ampere-turns on one of the rods while the number on the other is kept constant until, on reversing the currents, we find no reversal of the direction of the flux in the gap in the bar connecting the two yokes. This variation of the ampere-turns is effected by varying the number of turns in the magnetising coil on one rod, the same current passing through the coils on both bars. Then, when a balance is obtained, as shown by the absence of movement in the compass needle when the current is reversed, the magnetising forces acting on the two rods are proportional to the respective number of turns on them, while the flux density is the same in both. This flux density is known, since its relation to the magnetic force in the standard rod is determined by the instrument maker and furnished with the instrument. Fig. 27 shows the instrument in detail. The yokes are in the form of soft iron rings, *bb*, held in place by three longitudinal brass rods, *f*, the cross yoke is formed

of iron pieces, *gg*, which are carried up so as to bring the detector compass needle far enough from the bars to avoid the influence on it of the magnetising current in the coils. There is also a directing magnet, *k*, placed below the compass needle for the purpose of directing the field around it. The standard bar *a* has on it a coil of 100 turns, and its clear length is 12.56cm.; hence, the corresponding magnetising force  $H_1$  is 10 C.G.S. units per ampere-turn. The bar under test is *a'*, and has a coil round it which can be varied from one to 300 turns by means of dial switches connected to points on the magnetising current circuit. One of these adds

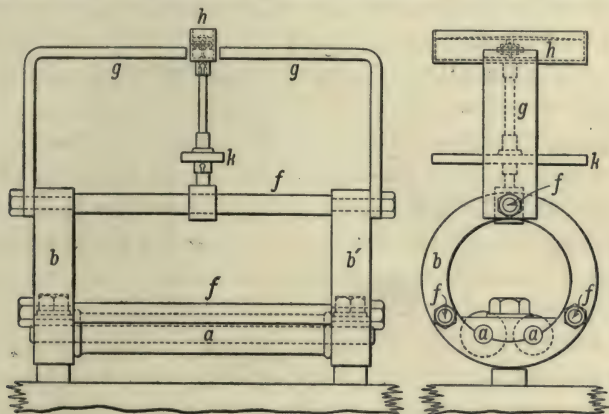


FIG. 27.—Ewing's Magnetic Bridge.

one turn for each step, a second adds 10 turns per step, and a third 100 turns. In each case the act of switching in more turns in the magnetising coil cuts out equal resistance by means of contacts on the other half of each dial, with the effect that the current is not changed though the number of effective turns on the magnetising coil is altered. The current is brought to any desired value by an external resistance and is read by an ampere-meter. It is then repeatedly reversed by means of a key, while the dial switches are manipulated until the reversal does not cause any permanent displacement of the compass needle *h*. The number

of turns on the dials is then read off, and gives the proportion to 100 which the magnetising force on the test-piece bears to the magnetising force on the standard bar. The magnetising force on the standard bar is 10 times the number of amperes passing through the instrument. By referring to a curve provided with the instrument the flux density in the standard bar can be found, but it is the same for both bars. Thus the relation of the magnetising force acting on the test bar to the flux density in it is determined, and this is the object of the test.

By using a series of currents of graded strength, as many points on the magnetisation curve are found as may be wished. The value of the magnetic force found in this way has to be corrected by the subtraction of a certain amount consequent on the magnetic reluctance of the yokes and of the joints not being negligible, and this can be done as described in §9. So long as the test-bar is not excessively different in magnetic quality to the standard bar, no great error is caused by treating the correction as the same for both. The dial switches give the means of increasing the number of turns on the test-piece over and above that on the standard bar by 210.

In cases where the test-piece is magnetically much worse than the standard, rather more than twice as many ampere-turns may be required to create an identical flux density in the two bars, and to provide for that there is a two-way key, by means of which the number of turns on the standard bar is readily reduced to 50 instead of 100, when the magnetising force becomes, of course, five instead of 10 per ampere, and the sample under examination may then have applied to it more than four times as much magnetising force as the standard bar.

When it is desired to test the permeability of sheet metal, the samples are formed by piling up a number of straight strips, giving a total cross-section equal to that of the standard, and a different form of yoke is used.

In the arrangement used with bars, the external appearance of which is shown in Fig. 27, the attachment of the bars

to the yoke is of the simplest kind. The bars slip through holes in which they are a loose fit, and are pressed against the middle portion of the yokes by a pair of set screws outside.

In practically working the instrument the current is provided for five or six cells of a storage battery, and an ampere-meter reading from 1 to 10 amperes is inserted in the circuit along an adjustable resistance. The bar to be tested is turned to a truly cylindrical form,  $\frac{3}{8}$  in. in diameter and about 6 in. long. The ends need not be faced or finished in any way. When this is done the procedure above described is followed. The current is increased step by step from 1 to 10 amperes. The directing magnet is manipulated so as to bring the compass needle into a central position before each reversal of the current. The switches are then manipulated, and the test consists in arranging the ampere-turns on the test-piece so that, when the current is reversed, the detector compass needle does not move permanently into a new position. There may, however, be, and generally is, a small temporary movement in the needle, due to the different rates at which the test-piece and the standard bar take up their magnetic state. The position of the switches then shows the number of ampere-turns acting on the test-bar, and from tables provided with the instruments the flux density and magnetising force operative are immediately deduced.

**§ 12. Forms of Commercial Instrument for Measuring the Permeability of Iron and Steel.**—The demand for instruments for use in the dynamo factory testing room to enable the permeability of iron and steel to be quickly tested, without the complications connected with the use of a ballistic galvanometer or the uncertainties connected with the traction method, have led to the invention of several forms of commercial permeameter. One devised by Messrs. Lamb and Walker\* is based upon the principle of balancing

\* See *Jour. Inst. Elec. Eng.*, Vol. XXX., p. 930, "An Instrument for Measuring the Permeability of Iron and Steel," by C. G. Lamb and M. Walker.

the reluctance of a bar of iron against that of an air-gap of variable length, taking advantage of the fact that the reluctance of an air-gap of given dimensions is invariable. The accuracy of an instrument based on this principle depends, therefore, only on the accuracy with which the dimensions of an air-gap can be measured and the delicacy with which the point of balance can be ascertained. The instrument is shown in Fig. 28. The specimen to be tested consists of a bar turned up to a diameter of  $\frac{3}{8}$  in. and 7 in. long, and is represented by A in the diagram. One end of this bar fits into a soft iron head, B, provided with a hole of the right size to take the specimen. This head

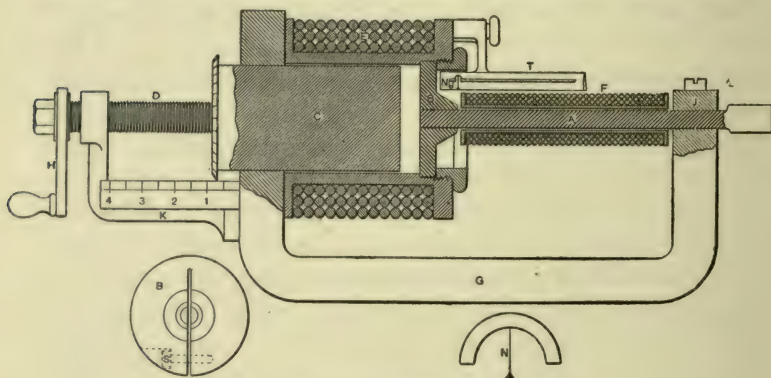


FIG. 28.—Lamb and Walker Permeameter.

is split and the two parts are drawn together by a screw so as to make a tight fit. Surrounding the specimen is a magnetising coil, F, and the head B fits into a recess turned in another magnetising coil, E, which is wound around an adjustable air-gap. The other end of the specimen is fixed to the yoke G by the block J and two set-screws. Opposite the end of the head B is the flat face of an iron cylinder, C, 2 in. in diameter and  $3\frac{1}{2}$  in. long. This slides freely, but with a good fit, through a hole in the yoke G, and can be moved parallel to its own axis by a screw, and its position ascertained by a scale, K, and a divided head on the screw D.

In the instrument described the coils E and F are wound with the same number of turns and placed in series in such a manner that the magnetising forces on the air-gap and specimen act in the same direction. Hence if the air-gap is so adjusted that its reluctance is equal to that of the specimen A, then no lines of flux will escape from the back face of B, but they will all pass round through the plug C and round back by the yoke G. The magnetic state of the head B is indicated by means of a suspended magnetic needle placed at N. It consists of a small curved steel bar pivoted on a jewel and needle point and carrying an aluminium pointer. It is protected from air draughts by being placed in a brass tube with windows covered over with mica to enable the position of the index to be seen. The operation of testing a specimen is as follows:—After being placed in the instrument and the coils being properly connected, a current is sent through the coils the value of which is read on an ammeter. The current is then adjusted until the ammeter reads a definite value corresponding to a magnetic force, say,  $H=5$ . The index needle will now be found hard over on one side, and if the current is reversed it will go hard over on the other. The handle H is then turned to lengthen or shorten the air-gap, until the needle points to a central position from which it is not disturbed by reversing the direction of the current. It is found that an alteration of even  $\frac{1}{1000}$  in. in the length of the air-gap is sufficient to disturb the balance. The inventors found that throughout the most important part of the magnetisation curve, namely, the part at fairly high flux densities, the instrument enabled permeability to be measured within a fraction of 1 per cent. With regard to tests at low portions of a magnetisation curve, it is doubtful if the use of small specimens is of much value for commercial purposes, since the behaviour of the iron under these low forces is so much dependent on its previous magnetic history, that we can never be sure that the specimen really represents the bulk of the material from which it

was taken in magnetic quality. One great objection to an instrument of this kind is that the user has generally to be dependent upon a calibration performed by the manufacturer. If the instrument is calibrated with extreme care by a competent person some value may be attached to its indications, but unfortunately this is not always the case with instruments made by manufacturers, and hence the user is

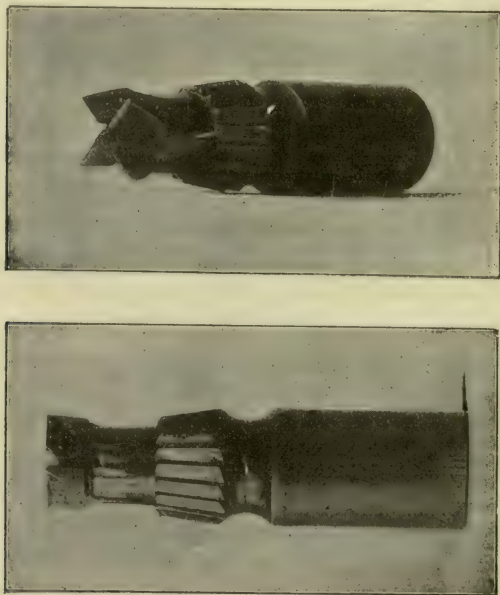


FIG. 29.—Drill for Drysdale Permeameter.

generally thrown back upon himself to devise some means by which the instrument may be checked.

Another form of permeameter for testing the magnetic qualities of materials in bulk has been devised by Dr. Drysdale.\* Dynamo manufacturers require some means of ascertaining the actual permeability of a casting or forging which is to be used in building a dynamo, and not merely

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\* *Jour. Inst. Elec. Eng.*, Vol. XXXI., p. 283.

tests made upon specially prepared bars. In the latter case we have not the security that the sample that is tested really represents the material in bulk. The method devised by Dr. Drysdale consists in drilling a hole either in the casting itself or into a boss cast upon it with a peculiar form of drill, which cuts a hole in the material, leaving in the centre a pin standing up. The measuring instrument itself consists of a

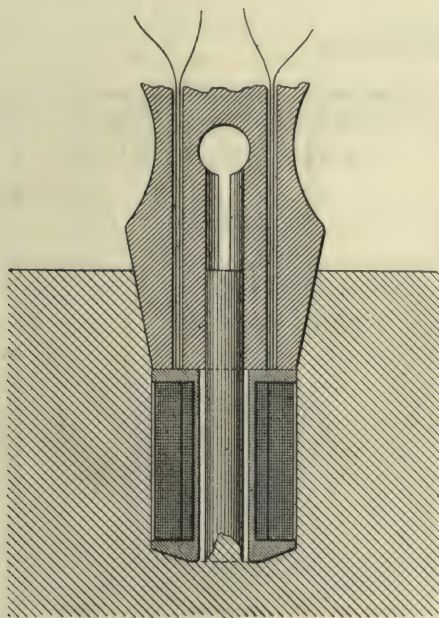


FIG. 30.—Section through Plug and Specimen.

handle having at the end a soft iron plug of a conical shape which fits into the conical mouth of the hole cut in the metal (*see* Figs. 29 and 30). This plug carries at its end a little coil on which are wound two circuits, and when it is placed in position this coil is, so to speak, buried in the interior of the metal to be tested, and the magnetic circuit is closed by the conical soft iron plug. By means of one of the wire

circuits the pin and surrounding metal is magnetised to any required degree of flux density, and on reversing this current a transient E.M.F. is produced in the secondary circuit which is allowed to act on a ballistic galvanometer.

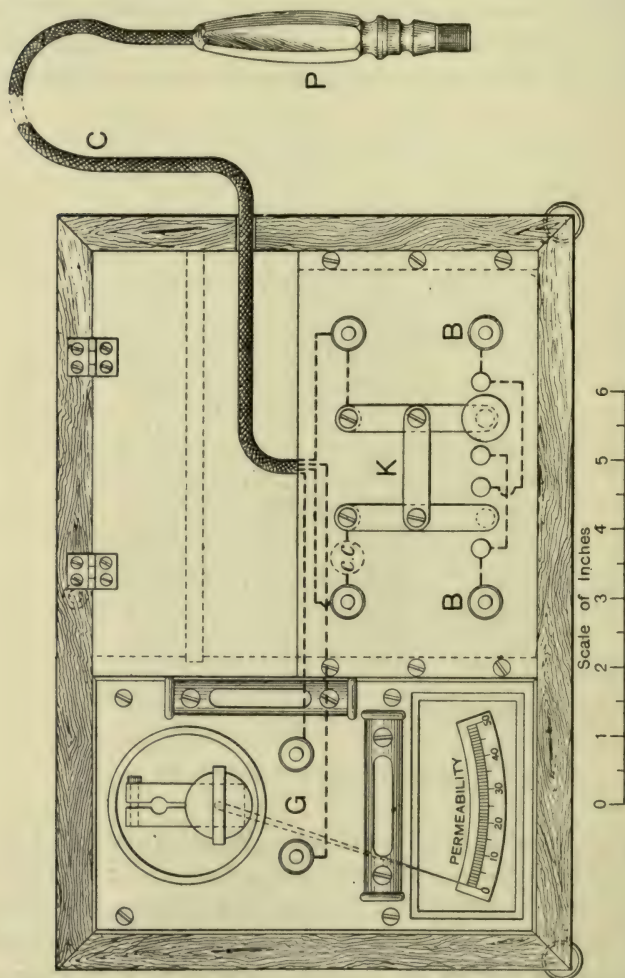


FIG. 31.—Drysdale Permeability Testing Set.

Dr. Drysdale has devised a convenient form of instrument in a box (see Fig. 31), in which the secondary circuit of the

coil on the plug is connected to a ballistic movable coil galvanometer, and the primary coil on the plug is connected to a battery consisting of a few dry cells, a reversing key being placed in this circuit with the battery. All that the user of the instrument has then to do is to place the plug and coil in the hole drilled out in the metal, and to move the reversing key so as to reverse the known current through the coil. The galvanometer then makes a sudden throw, and the scale is divided to show the permeability of the iron directly. If the circuit is then broken, and at once the second reading taken from the first, the value of the retentivity of the sample is given. The highest value of the magnetic force employed is 100 C.G.S. units, which corresponds to only  $\frac{1}{16}$ th ampere taken from the cells.

In a discussion which took place on the instrument at the Institution of Electrical Engineers, a criticism was made to the effect that the magnetic circuit contains two air-gaps, and that, although the conical plug may fit very well into the conical seat, and owing to the large surface may present a joint of negligible reluctance, yet, nevertheless, the pin standing up cannot possibly fit so well into the hole in the plug; there is, therefore, an uncertain amount of reluctance at that point. The inventor, however, claimed for it considerable accuracy. The inference to be drawn from these criticisms, however, is that in using all ready-made instruments of this kind the observer should not place implicit reliance upon their indications unless they have been tested to a considerable extent on materials of known permeability.

There is no difficulty, however, in testing by the ballistic method a mass of iron in the form of a ring, and then, having determined its permeability by a method which is certain to give correct results, to employ a portion of it for testing the commercial instrument in question. Otherwise, in the use of such ready-made instruments the user is really testing the accuracy of his instrument-maker as much as the quality of his material.

§ 13. **Measurement of Hysteresis.**—It has already been shown that if a cyclical magnetisation curve is plotted out for a sample of iron or steel, the area of this curve is a measure of the work done per cubic centimetre in carrying the metal through one complete magnetic cycle. A proof of this proposition has been given in the particular case of an iron ring, but the following argument shows in a more general way the connection between magnetisation changes in ferromagnetic materials and the absorption of energy in creating them. When any physical agency produces a corresponding physical change or operation which will disappear if the active cause is removed, it is generally usual to speak of this cause and effect as a *force* and a *flux*, or as a *stress* and a *strain*. Thus, for instance, if an elastic body is compressed, the compressing force is the stress and the compression is the strain. If a body is moved against a frictional resistance, the displacement constitutes the strain and the force overcoming the resistance the stress. In all these cases, if the stress is of constant magnitude, the work done or energy expended in making the change is measured by the product of the stress and the strain, or of the flux and the force. Thus, for instance, if a heavy body is lifted against gravity, the weight of the body constitutes the stress, the height through which it is lifted the strain; and the work done in making the displacement is, therefore, measured by the product of the weight of the body and the height through which it is lifted. Again, if a volume of gas is compressed, the reduction in volume constitutes the strain and the pressure per unit of area the stress; and the work done against constant pressure is equal to the product of the reduction of volume and the pressure per unit of area. If, however, the stress varies with the strain, and is proportional to it, then the work done in effecting the change is measured by half the product of the strain and the final stress. For example, if a spring is extended, and if the tension of the spring varies as the

elongation, then the work done in making a given elongation is measured by half the product of the elongation and the final tension.

Symbolically, this may be expressed as follows:—If  $dW$  stands for the work done in making a small displacement,  $dS$ , against a constant force,  $F$ , then  $dW = FdS$ ; but if the stress varies as the strain, and if  $F$  is the final value of the stress, then  $dW = \frac{1}{2}FdS$ .

Now, magnetic force and magnetisation stand to one another in the relation of cause and effect. Moreover, if the magnetic force is removed the magnetisation disappears partly or wholly. In consequence of this it follows that, when a constant magnetic force creates magnetisation, the work done by a given magnetic force,  $H$ , in increasing the magnetisation of a body in its own direction by an amount  $dI$  is equal to  $HdI$  per unit of volume. In this case the magnetisation is the strain and the magnetic force is the stress. The product  $HdI$  represents, therefore, the work done per cubic centimetre of the material in increasing the magnetisation by an amount  $dI$  in the direction of the magnetic force. It may be noticed that the measure of a quantity of work or energy is always the product of two factors, one of which is of the nature of a stress and the other of a strain. Thus the work done per unit of volume in diminishing the volume of a gas under constant pressure is measured by  $PdV$ , where  $P$  is the pressure per unit of area and  $dV$  is the change in volume. In those cases of magnetisation in which the magnetisation is proportional to the force, the work done per unit of volume in producing a final magnetisation,  $I$ , under the action of a magnetic force whose final value is  $H$ , is equal to  $\frac{1}{2}IH$  per unit of volume. If there were no hysteresis, and the changes in magnetisation always accompanied *pari passu* the changes in the magnetic force, then no work could be done in carrying a mass of ferromagnetic substance through a complete magnetic cycle in which the force was carried successively from zero to a certain

positive value, then back through zero to an equal negative value, finally returning to zero again; because in this case the work done on the material in producing a given magnetisation change would be given back by the material during the reverse movement. The action would resemble that of compressing a perfectly elastic substance and then removing the compressional force. The work done on the substance during compression would be given back again during expansion, and no permanent expenditure of work would result.

If, however, change in magnetisation does not accompany or correspond with change in magnetic force, but if the magnetisation change lags behind the magnetic force change, so that there is hysteresis, then it is clear that the result of carrying the substance through a complete magnetic cycle must involve expenditure of energy. This is at once seen from a consideration of the diagram in Fig. 16, which represents a cyclical magnetisation curve, the abscissæ being magnetic force and the corresponding ordinates magnetisation. Corresponding to any increment of magnetic force from  $H_1$  to  $H_2$  the magnetisation on the way up increases from  $I_1$  to  $I_2$ , but on the way down, corresponding to the decrease in magnetic force of  $H_2$  to  $H_1$ , the magnetisation decreases from  $I'_2$  to  $I'_1$ . The work done on the material on the way up in making the small increase in magnetisation is equal to the product  $H_2 - H_1$  and the mean value of the magnetisation between these limits of force. On the way down, owing to the hysteresis, the mean magnetisation is greater; hence the work actually expended on the ferromagnetic material corresponding to these limits of the magnetic force must be equal to the product of the mean magnetic force and the difference between the magnetisations—in other words, must be equal to  $\oint \mathbf{H} d\mathbf{I}$  per unit of volume.

This may be proved in a different manner. If an increase in potential energy accompanies any strain, then the rate at which potential energy increases with the strain is a measure of the force or stress resisting the strain. If  $dV$  is

the increase in potential energy accompanying a strain  $dx$ , then  $dV/dx$  is the stress  $F$  against which the strain is made. Consider the case of a permanent steel magnet placed in a uniform magnetic field of strength  $H$ . Let the uniform magnetisation of the magnet be  $I$ , and let its length be  $L$  and section  $S$ . Then its moment  $M = ILS$ . If it is inclined at an angle  $\theta$  to the field, then the torque tending to set it in the direction of the field is  $HM \sin \theta = HILS \sin \theta$ . If its potential energy is  $V$ , then

$$\frac{dV}{d\theta} = HILS \sin \theta.$$

Accordingly

$$V = -HILS \cos \theta,$$

or

$$\frac{V}{LS} = -IH \cos \theta.$$

That is, its potential energy per unit of volume is equal to the product of its magnetisation and the magnetic force in the direction of that magnetisation. Accordingly, if we suppose a bar to have a magnetisation  $I$  imparted to it when in a field  $H$ , the direction of  $H$  and  $I$  coinciding, the loss of potential energy per unit of volume of the material by being so magnetised would be measured by  $HI$ . We have only to realise that we should have to do work on the bar to take it out of the field to see that this is the case. Hence, if we consider the whole period of the cyclical change to be divided into small elements, represented by the slices into which we can divide the cyclical magnetisation curve it is obvious that the whole work done in carrying the material through one complete magnetic cycle must be equal to the value of the integral  $\int H dI$  per unit of volume throughout the cycle, and must therefore be equal to the area of the cyclical magnetisation curve when plotted in terms of magnetic force and magnetisation. Since, in every strongly ferromagnetic substance, the magnetic flux density  $B$  is equal to  $4\pi I$ , where  $I$  is the magnetisation, it follows that the work done per unit of volume in carrying the ferromagnetic

material through one complete magnetic cycle must be equal to  $\frac{1}{4\pi} \int \mathbf{H} d\mathbf{B}$ , as already shown. The energy so expended on the magnetic material, of course, ultimately takes the form of heat, and is called the *hysteretic loss* of the material.

There are several methods by which we may determine this hysteretic loss. First, we may draw a cyclical magnetisation curve for the metal by any of the methods already described, and we may determine the area of this curve with the planimeter. It is most convenient to plot the curve generally in terms of flux density  $\mathbf{B}$  or lines per square centimetre, and magnetic force  $\mathbf{H}$  in C.G.S. units. To do this some unit must be selected of appropriate magnitude for each of these quantities. It is usual to select one unit of length (centimetre or inch) to stand for one C.G.S. unit of magnetic force, and one unit of length to stand for 1,000 lines per square centimetre. To fix ideas, let us suppose that the cyclical curve of magnetisation is so drawn that 1in. in a horizontal direction represents one C.G.S. unit of magnetic force, and 1in. measured vertically represents a flux density of 1,000 lines per square centimetre. The area of the cyclical curve, measured in square inches multiplied by 1,000 and divided by 12.56, will, therefore, give the number of ergs per cubic centimetre of the material expended in carrying the material through one complete magnetic cycle. Instead of drawing the cyclical magnetisation curve and integrating its area by a planimeter, we may adopt some other method of measurement which gives us at once the work done in carrying the material through a magnetic cycle. Thus, for instance, we may measure the total amount of heat produced in one minute when the ferromagnetic metal is subjected to an alternating magnetic force having a known frequency, provided we can eliminate all causes of heat production other than that due to the hysteretic loss in the metal; or we may employ a properly constructed wattmeter to measure at one operation the work expended on the material in producing alternating magnetisation.

We have, however, to consider two cases of hysteresis: (1) When the magnetic field or force maintains the same direction in the ferromagnetic metal, but fluctuates periodically in magnitude or alternates through a cycle of values. This gives rise to *alternating hysteresis*. (2) When the field or force maintains the same magnitude, but revolves or periodically changes its direction, so that the ferromagnetic material is magnetised first one way and then the other. From this process results *rotating hysteresis*. These two magnetic operations are not quite the same in nature, and the resulting energy loss per cubic centimetre per cycle is not identical in the two cases, especially at high magnetisations.

The following facts have been established with regard to magnetic hysteresis:—As regards alternating hysteresis the work done per cycle is practically independent of the speed of the cycle, at any rate up to 400 cycles per second. This has been shown by experiments on soft iron wires by Prof. Tanakadate (*Phil. Mag.*, Sec. V., Vol. XXVIII., p. 207, 1889) and by experiments made by Prof. F. G. Baily. (The work of Messrs. Steinmetz and Siemens, in which a different result was obtained, has not been confirmed by later experimentalists.)

As regards the variation of hysteresis loss with flux density the empirical rule of Steinmetz has, however, been confirmed for alternating hysteresis up to a flux density of 8,000 or 9,000 C.G.S. units. Within these limits the loss varies as  $B^{1\frac{2}{3}}$ , where  $B$  is the maximum flux density during the cycle. Beyond that point the hysteresis does not increase according to the same law. Experiments made by Prof. F. G. Baily (see *Proc. Brit. Assoc.*, Ipswich, on "The Hysteresis of Iron in an Alternating Magnetic Field"; also *The Electrician*, Vol. XXXVI, p. 118), show that the hysteresis tends to a limiting value which corresponds with the point commonly called saturation. Prof. Baily's experiments were made by the heating method—that is to say, the hysteresis was determined by the amount of heat generated in a mass of very finely laminated iron placed in an alternating magnetic field, the

direction of which was parallel to the plane of lamination. He found that the curve representing the variation of hysteresis with flux density has a point of inflection or change of curvature somewhere about a value of  $B_{\max}$ , corresponding to 16,000 or 17,000 C.G.S. units; and that beyond a value of 20,000 the curve tends to become flat, and the hysteresis evidently approaches a maximum value, as shown in the diagram in Fig. 32, taken from Prof. Baily's Paper. These experiments were carried out with an alternating field having a frequency of 103, and the iron used was so finely laminated that the eddy current loss formed only 1.7 per cent. of the hysteresis loss, and was not greater than

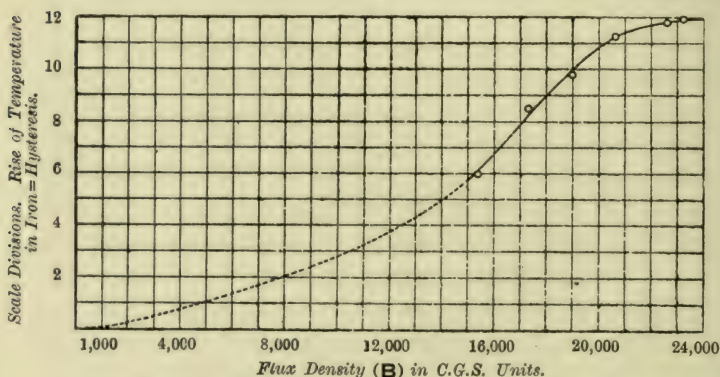


FIG. 32.—Alternating Hysteresis Curve for Soft Charcoal Iron.  
Frequency=103  $\sim$ .

the unavoidable inaccuracies of the calorimeter method of measurement.

The conclusion to be drawn from these experiments is that the alternating hysteresis in iron arrives at a limiting value coincidently with its limit of magnetisation, and hence that the hysteresis loss of the iron should be expressed as a function, not of the flux density, but of the magnetisation, which, as Prof. Ewing has shown, also tends to a limiting value. This conclusion has been emphasised by the Author in a Paper on "The Hysteresis of Cobalt,"

In the next place, with regard to the hysteresis of iron in a rotating magnetic field, it has been shown by Prof. F. G. Baily,\* that, if a mass of laminated iron is rotated in a magnetic field, the resistance which the iron experiences as the field increases in strength, rises up to a maximum value, and then decreases again practically to zero. This result shows that the *rotating hysteresis* loss at high magnetisations does not vary according to the same law as the *alternating hysteresis* loss. Experiments made by Messrs. R. Beattie and R. C. Clinker in the Pender Electrical Laboratory, University College, London, under the direction of the Author, have confirmed the above results.†

In the experiments by the last-named investigators the magnetic field produced by an electromagnet was rotated round a laminated iron specimen suspended from a torsion fibre in the field, the field moving in a horizontal plane and the laminations being horizontal. It was found that, as the strength of the magnetic field increased, the torque experienced by the iron increased up to a maximum and then fell down again practically to zero, as shown in the diagram in Fig. 33. The work expended, therefore, on the suspended iron, and therefore the hysteresis loss in it, reaches a maximum corresponding to a flux density of 15,000 or 16,000 C.G.S. units in the iron. In the diagram in Fig. 33, two curves are shown. Curve A represents the total loss in the specimen, and curve B the results corrected as far as possible for eddy current loss. In the case of the rotating hysteresis loss the maximum value is reached for a lower flux density in the iron than in the case of the alternating-current hysteresis.

These experiments show that the value of the alternating-current hysteresis at high flux densities cannot be deduced from experiments made with a specimen of iron or steel

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\* See *Proc. Roy. Soc.* 1896, Vol. LX., p. 182, "On the Hysteresis of Iron in a Rotating Field."

† See Messrs. Beattie and Clinker "On Magnetic Hysteresis in a Rotating Magnetic Field," *The Electrician*, Oct. 2, 1896. Paper read before Brit. Assoc., Liverpool, Sec. G.

rotated in a constant magnetic field, and that, therefore, appliances for determining the hysteresis loss in sheet iron to be used in the manufacture of transformers, which depend upon measurements made of the torque on a sample of laminated iron placed in a rotating field, must be accepted with some reservation in their application to the core losses in alternating-current transformers. Also, they show that the deductions from experiments made with alternating magnetic forces cannot be applied to the magnetisation of dynamo armature cores, especially at high flux densities.

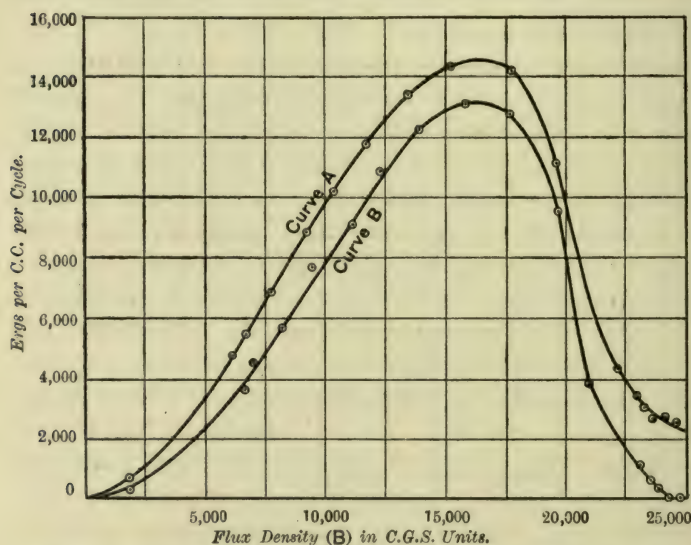


FIG. 33.—Curve showing the Variation in the Hysteresis of Iron in a Rotating Magnetic Field. (Beattie and Clinker.)

**§ 14 Delineation of a Steinmetz Curve.**—When the hysteresis loops or cyclic magnetisation curves have been determined corresponding to various maximum values of the magnetic force or magnetic flux density, we can calculate, as already explained from the areas of these loops, the work done in ergs per cubic centimetre, in performing each cycle. If we measure also the maximum flux density during each

cycle we can set out a curve from these data termed a Steinmetz curve. It is usual to plot the maximum value of the flux density in a horizontal direction and the energy absorbed in performing the magnetic cycle vertically. Thus, in Fig. 34 is shown the Steinmetz curve for a sample of transformer sheet iron. It has been shown by Mr. C. P. Steinmetz\* that for iron and steel of various compositions, subjected to alternating magnetic force, the relation between the maximum flux density **B** and the work done in ergs in

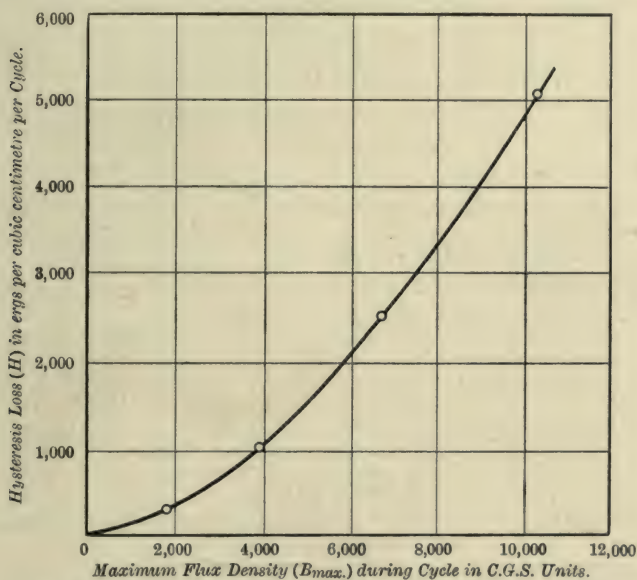


FIG. 34.—Steinmetz Curve for a Sample of Transformer Sheet Iron.

performing a cycle of magnetic change over any range of flux density lying between 0 and about 9,000 C.G.S. units, is expressed by an equation of the form

$$W = \eta B_{max}^{1.6}$$

where  $\eta$  is a constant called the *hysteretic constant* and  $B_{max}$

\* See *The Electrician*, Vol. XXVIII., pp. 384, 408, 425; also Vol. XXXII., p. 672.

is the value of the maximum flux density during the magnetic cycle. Whilst Steinmetz's law connecting  $W$  and  $B$  is an empirical one, and only moderately exact within certain limits of  $B_{\max}$ , it is nevertheless of great utility in enabling us to calculate the hysteresis loss at any assigned flux density within the limits for which the law holds good, when we know it at another standard flux density within the same range.

The following tables (XI. and XII.) contain the hysteretic constants for various kinds of iron and steel, and the values of  $B^{1.6}$  within the limits above mentioned.

TABLE XI.

*Hysteretic Constants for Different Materials.*

Material.	Hysteretic Constant. $\eta$
Best annealed soft transformer sheet iron.....	0.001 to 0.0015
Very soft iron wire .....	0.002
Thin, well-annealed soft sheet iron.....	0.0024
Thin, good sheet iron .....	0.003
Thick sheet iron.....	0.0033
Most ordinary sheet iron transformer cores after use for some time.....	0.004 or 0.0045
Soft annealed cast steel .....	0.008
Soft machine steel.....	0.0094
Cast steel .....	0.0120
Cast iron .....	0.0162
Hardened cast steel .....	0.025

TABLE XII.

*Hysteresis Losses in Iron calculated from the formula  $W = \eta B^{1.6}$  in ergs per cubic centimetre per cycle.*

Maximum Flux Density $B$ . $B_{\max}$ .	Value of $B^{1.6}$ .	Values of $\eta B^{1.6}$ .			
		$\eta = 0.002$ .	$\eta = 0.003$ .	$\eta = 0.004$ .	$\eta = 0.005$ .
1,000	63,100	126	189	252	315
2,000	191,300	382	573	765	956
3,000	365,900	731	1,096	1,463	1,829
4,000	580,000	1,160	1,740	2,320	2,900
5,000	828,800	1,657	2,486	3,315	4,144
6,000	1,111,000	2,222	3,333	4,444	5,555
7,000	1,420,000	2,840	4,260	5,680	7,100
8,000	1,758,000	3,516	5,274	7,032	8,790
9,000	2,122,000	4,244	6,366	8,488	10,610
10,000	2,511,000	5,022	7,533	10,044	12,555

The following data are also useful in connection with this subject :—

The specific gravity of sheet iron = 7.8 (water = 1).

1 lb. avoirdupois = 453.6 grammes.

1 lb. avoirdupois of iron occupies a volume of 58 cubic cm.

1 square inch = 6.45 square cm. (Hence flux density per square inch = 6.45  $\times$  flux density per square cm.)

$(6.45)^{1.6} = 19.74$ .

1 cubic cm. of iron weighs 7.8 grammes or 0.0172 lb.

According to Mr. Steinmetz, the exponent of  $B_{\max}$  in all cases has a value not far from 1.6. It has been shown by Dr. A. E. Kennelly\* that, in the case of the hysteresis loss in nickel, and by experiments conducted by the Author and Messrs. Ashton and Tomlinson† on cobalt, that the same exponential law holds good for these metals also.

Instead of plotting the maximum flux density of the work per cubic centimetre absorbed in completing the cycle, if we plot the logarithms of these quantities respectively, we shall obtain a straight line, as shown in Fig. 35, which represents the logarithmic form of the Steinmetz curve for the transformer iron referred to in connection with Fig. 34. The physical meaning of this exponent (1.6) has not yet been determined. (For a discussion of Steinmetz's Law of Hysteresis the reader is referred to *The Electrician*, Vol. XXVIII., pp. 384, 408, 425; Vol. XXIX., p. 671; and Vol. XXXII., p. 672.)

It is clear that, if the hysteresis loss  $W$  per cubic centimetre per cycle varies as a power of  $B_{\max}$  such that  $W = \eta B_{\max}^n$ , then

$$\log W = \log \eta + n \log B_{\max}.$$

Since this last equation is linear, we can find the value of  $n$  by plotting the values of  $\log W$  and  $\log B_{\max}$  and measuring the slope of the straight line so obtained. For electro-technical purposes we require to evaluate samples of iron and

\* *The Electrician*, Vol. XXVIII., p. 666.

† See Messrs. J. A. Fleming, A. W. Ashton and H. J. Tomlinson, "On the Magnetic Hysteresis of Cobalt," *Phil. Mag.*, Sept., 1899.

steel in regard to their hysteresis loss under given circumstances of frequency and maximum flux density. The usual plan adopted is to specify the hysteresis loss in watts per pound, corresponding to a frequency of 100—that is to say, to state the power taken up in the performance of 100 complete magnetic cycles per second with a maximum flux density of 2,500 or else of 4,000 lines per square centimetre. Thus

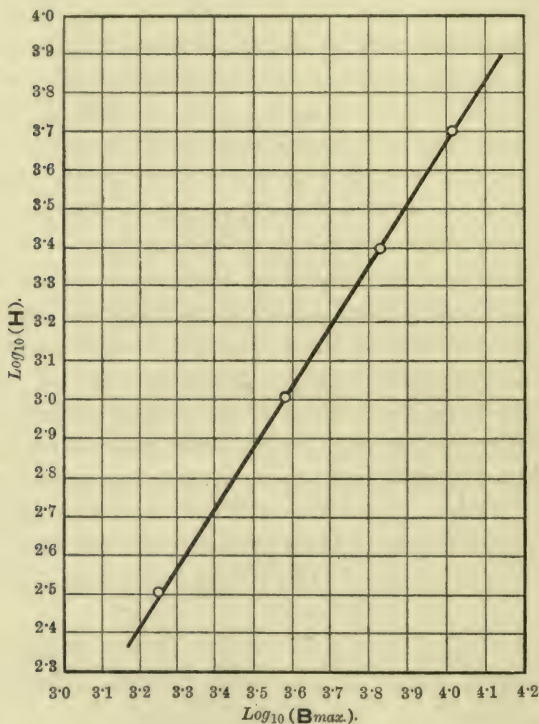


FIG. 35.

a particular sample of iron may be designated as taking 0.25 watt per pound per 100 cycles per  $B_{\text{max}} = 2,500$ . Otherwise the hysteresis loss may be expressed in ergs per cubic centimetre per cycle. The hysteresis loss expressed in ergs per cubic centimetre per cycle is obtained from the number

expressing the loss in watts per pound for 100 cycles per second by multiplying by 1,740.

The following table (XIII.) by Prof. Ewing\* gives the hysteresis loss of thin sheet Swedish iron, as employed in the manufacture of transformers at certain flux densities.

**TABLE XIII.**

*Hysteresis Loss in Strip of Annealed Transformer-plate rolled from Swedish Iron.*

$B_{\max.}$	Ergs per cubic centimetre per cycle.	Watts per lb. at frequency of 100.
2,000	200	0.129
3,000	410	0.242
4,000	640	0.376
5,000	910	0.535
6,000	1,200	0.710
7,000	1,520	0.890
8,000	1,900	1.120
9,000	2,310	1.360

The figures in the above table show the very great improvement that has been made of late years in iron and steel in respect of hysteresis loss. This is illustrated by a comparison of the foregoing figures with those given by Prof. Ewing (*loc. cit.*) in the following table (XIV.), representing the results of some hysteretic tests taken on a soft iron wire in Japan in 1881 :—

**TABLE XIV.**

*Hysteresis Loss in Soft Iron Wire taken in Japan in 1881.*

$B_{\max.}$	Hysteresis Loss. Ergs per cubic centimetre per cycle.	Watts per lb. at frequency of 100.
2,000	400	0.235
3,000	780	0.460
4,000	1,200	0.710
5,000	1,680	0.990
6,000	2,200	1.290
7,000	2,800	1.650
8,000	3,450	2.030
9,000	4,160	2.450
10,000	4,920	2.900
11,000	5,800	3.410
12,000	6,700	3.940

\* "On the Magnetic Testing of Iron and Steel," *Proc. Inst. Civ. Eng.*, Vol. CXXVI., 1895.

As  $B_{\max}=2,500$ , which is generally taken as a standard flux density, the hysteresis loss of the Swedish iron referred to in Table XIII. is 0.185 watt per pound at a frequency of 100. The lowest value recorded by Prof. Ewing in some tests on the hysteresis loss of sheet iron is 0.16 watt per pound at a frequency of 100 and a maximum flux density of 2,500.

Great variations, however, are found in ordinary commercial sheet iron as prepared for transformer manufacture, and it is not uncommon to find that various samples of iron, when tested as  $B_{\max}=4000$  will exhibit variations in hysteresis loss lying between 600 and 1,800 or 1,900 ergs per cubic centimetre per cycle.

The hysteresis loss of any particular sample of iron or steel is intimately connected with its chemical and physical constitution, but it appears as if the physical condition had more influence than the chemical composition. The following figures in Tables XV. and XVI., given by Mr. H. F. Parshall,\* show the hysteresis loss for six samples of iron of proved good quality and six samples of inferior quality:—

TABLE XV.

*Results of Hysteresis Tests of Samples of good Sheet Iron.*

Sample No. ... ..	1	2	3	4	5	6	Average.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Carbon ... ..	0.076	0.085	0.072	0.086	0.095	0.078	0.082
Phosphorus ... ..	0.058	0.052	0.057	0.084	0.075	0.056	0.064
Silicon ... ..	Trace	0.005	0.006	Trace	0.007	0.012	0.005
Manganese ... ..	0.297	0.021	0.417	0.320	0.322	0.026	0.246
Sulphur... ..	0.060	0.030	0.030	0.040	0.030	0.010	0.033
Watts per lb. per 100 cycles at a density of 4,000 lines per sq. cm. }	0.55	0.61	0.52	0.59	0.65	0.67	0.62

The hysteresis loss in good samples of transformer and armature sheet iron given by Mr. Parshall (*loc. cit.*), are shown

\* *Proc. Inst. Civ. Eng.*, Vol. CXXXVI., 1895.

TABLE XVI.

*Results of Hysteresis Tests of Six Samples of inferior Sheet Iron.*

Sample No. ... ..	1	2	3	4	5	6	Average.
	Per	Per	Per	Per	Per	Per	Per
	cent	cent	cent	cent	cent	cent	cent

**ERRATA.**

Page 474, line 11 from top :

*For  $B_{\max}=400$  read  $B_{\max}=4,000$ .*

Page 477, line 9 from bottom :

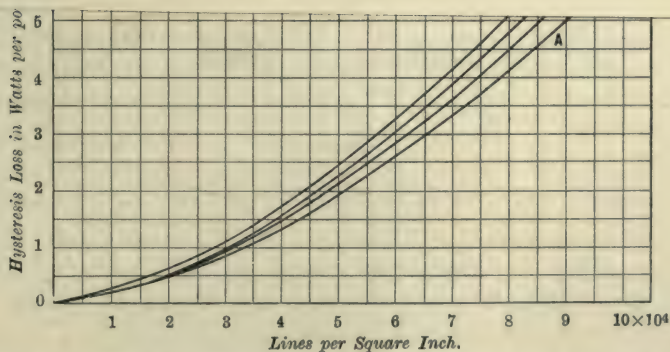
*For  $B = \frac{E_2}{4fnN_2S}$  read  $\frac{E_2 \times 10^8}{4fnN_2S}$ .**To face page 474.*

FIG. 35A.—Curves showing Hysteresis Loss in Transformer and Armature Iron.

§ 15 **Hysteresis Measurement by the Wattmeter.**—The most simple and direct method of measuring hysteresis loss in iron or steel is by the wattmeter, provided that the metal

As  $B_{\max}=2,500$ , which is generally taken as a standard flux density, the hysteresis loss of the Swedish iron referred to in Table XIII. is 0.185 watt per pound at a frequency of 100. The lowest value recorded by Prof. Ewing in some tests on the hysteresis loss of sheet iron is 0.16 watt per

TABLE XV.

*Results of Hysteresis Tests of Samples of good Sheet Iron.*

Sample No. ... ..	1	2	3	4	5	6	Average.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Carbon ... ..	0.076	0.085	0.072	0.086	0.095	0.078	0.082
Phosphorus ... ..	0.058	0.052	0.057	0.084	0.075	0.056	0.064
Silicon ... ..	Trace	0.005	0.006	Trace	0.007	0.012	0.005
Manganese ... ..	0.297	0.021	0.417	0.320	0.322	0.026	0.246
Sulphur... ..	0.060	0.030	0.030	0.040	0.030	0.010	0.033
Watts per lb. per 100 cycles at a density of 4,000 lines per sq. cm. }	0.55	0.61	0.52	0.59	0.65	0.67	0.62

The hysteresis loss in good samples of transformer and armature sheet iron given by Mr. Parshall (*loc. cit.*), are shown

TABLE XVI.

*Results of Hysteresis Tests of Six Samples of inferior Sheet Iron.*

Sample No. ...	1	2	3	4	5	6	Average.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Carbon ...	0·153	0·063	0·117	0·099	0·090	0·153	0·113
Phosphorus ...	0·032	0·075	0·101	0·135	0·101	0·031	0·088
Silicon ...	0·007	0·008	0·005	...	0·027	0·002	0·008
Manganese ...	0·430	0·036	0·401	0·449	0·449	0·486	0·375
Sulphur...	0·050	0·030	0·010	0·030	0·030	0·070	0·045
Watts per lb. per 100 cycles at a density of 4,000 lines persq. cm.	1·37	1·77	1·66	1·58	1·57	1·57	1·63

by the curves in Fig. 35A. These curves show the hysteretic loss in the metal as first received and also its increase after heating to 65°C. for six months.

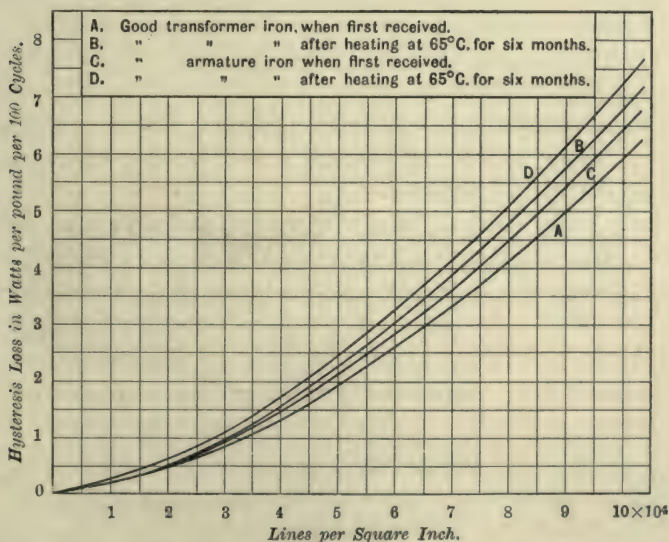


FIG. 35A.—Curves showing Hysteresis Loss in Transformer and Armature Iron.

§ 15 **Hysteresis Measurement by the Wattmeter.**—The most simple and direct method of measuring hysteresis loss in iron or steel is by the wattmeter, provided that the metal

can be obtained in sheets sufficiently thin to practically eliminate all heat production due to eddy currents. As this is not generally practicable, a correction has to be applied to the observed values. As the hysteresis loss in iron is generally only desired to be known in those cases in which the metal is employed in some construction in thin sheets, such as armature or transformer cores, it is always possible to obtain samples of the material of the thickness which will ultimately be used, stamped in the form of rings. A convenient size for such rings is a circle 9.5cm. interior diameter and 10.5cm. exterior diameter, and having, therefore, a mean diameter of 10cm. and a width of 1cm. The thickness of metal generally employed for the above-mentioned purposes will vary between 0.010in. and 0.014in.—that is, 10 to 14 mils. A sufficient number of these ring stampings should be varnished on both sides to prevent conducting contact, and then piled one above the other to make a ring core. Previously to varnishing, the thickness of each ring should be carefully measured, and the weight taken. A number of rings sufficient to make a core 2cm. or 3cm. in depth may be taken.

These rings, having been bound together with tape, are wound over with an endless solenoid forming a secondary coil. This first coil should be formed of silk-covered wire, No. 36 S.W.G. Over this wire is placed one or more layers of thicker cotton-covered wire, forming an endless primary magnetising coil. From the number of turns on this coil, the mean perimeter, and the current sent through it, we can calculate the magnetising force to which the iron is subjected. If such a coil is joined up with an alternating-current wattmeter of the type described in Vol. I., p. 508 of this HANDBOOK, and if an alternating current of known R.M.S. value is sent through the coils, we can, by means of the wattmeter when standardised as already described (*see* Vol. I., p. 480 of this HANDBOOK), determine the whole power taken up in the ring and solenoid. Of this power

a small portion is expended in heating the copper coils, and this can be calculated at once from the known or measured resistance of the wire forming the solenoid and the R.M.S. value of the current passing through it. For, if  $A$  is the value of the current as read on an alternating-current ammeter, and  $R$  is the resistance of the coil at the temperature at which it is at the time of the experiment, then the power wasted in heating the copper is equal to  $A^2R$  watts. Of the remaining portion of the power part is taken up in producing eddy currents in the laminated iron, and the amount of this can be very approximately calculated from the empirical formula,

$$E = \left( \frac{B^2}{10^{16}} t^2 n^2 \right) V,$$

where  $B$  is the maximum flux density in the core during the cycle,  $n$  the frequency or number of periods per second of the alternating current,  $t$  the thickness of the sheet metal in mils, and  $V$  the total volume of the iron in cubic centimetres. The value of  $B$  can be ascertained from the voltage produced upon the fine wire secondary coil. For, if  $S$  is the cross-section of the core,  $N_2$  the number of windings on the secondary coil,  $E_2$  the R.M.S. value of the voltage at the terminals of this secondary coil, and  $f$  the form-factor\* of the primary voltage curve, then we have  $B$  given by the expression

$$B = \frac{E_2 \times 10^8}{4fnN_2S}.$$

Hence the proper correction for eddy-current loss can be calculated from these measurements, and the reading on the wattmeter, after deduction of the power expended in heating the copper coils and in the production of eddy currents in the iron, is equal to the hysteretic loss in the iron. Accordingly, if  $W$  stands for the wattmeter reading and  $H$  for the hysteresis loss in the iron ring per cubic centimetre per cycle,

\* See Vol. I. of the Author's "Alternating-Current Transformer," Chap. VI, sec. 10, p. 585.

we have the following expression for the value of  $H$ , where  $V$  is the volume of the ring in cubic centimetres:—

$$H = \frac{W - A^2R}{V} - \left( \frac{B^2}{10^{16}} n^2 t^2 \right).$$

In connection with hysteresis measurement by the wattmeter, attention may be directed to an elegant method for its determination which is due to Mr. G. F. C. Searle, and has been fully worked out by Messrs. Searle and Bedford in a research described in the *Phil. Trans. Roy. Soc., Sec. A*, Vol. CXCVIII., p. 33.

In the above method a ballistic electrodymanometer is employed, just as in the ordinary determination of flux density a ballistic galvanometer is used.

For the details and theory of the method we must refer the reader to the original Paper. Suffice it to say that, by means of a suitable electrodymanometer, the hysteresis loss and the maximum value of the flux density during the cycle can be determined simultaneously far more simply than when an alternating-current wattmeter is employed. By this method Messrs. Searle and Bedford (*loc. cit.*) have conducted a valuable investigation on the effect of stress and strain on the magnetic hysteresis of steel and iron.

**§ 16. Workshop Methods of Measuring Hysteresis.**—The method of taking one or more cyclical curves of magnetisation with the ballistic galvanometer and then obtaining, by means of the planimeter, the area of these curves, is by far the most trustworthy method for determining the hysteresis loss corresponding to any given maximum flux density. It is, however, excessively tedious, and its employment in the workshop is out of the question when a large number of samples of iron have to be tested. The sheet iron or steel which is employed in the manufacture of the cores of transformers or the armatures of dynamo machines is furnished by the manufacturers in large sheets, which are generally from 12 to 16 mils in thickness (one mil = 0·001in.).

Samples of these sheets have to be tested for hysteresis loss at certain flux densities, in order to evaluate the iron for the purpose for which it is to be used. Hence some method is required by which small samples of any sheet can be rapidly tested for hysteresis loss. Several appliances have been devised for this purpose. One of the simplest and most convenient is the hysteresis tester of Prof. Ewing. This instrument depends on the principle that if a number of thin sheets of iron in a symmetrical form are piled one above another and placed with the plane of the sheets parallel to the lines of flux of a magnetic field, then, if this field is made to rotate round an axis perpendicular to the plane of the sheets, a torque will be brought into operation tending to drag round the metal in the same direction as the field, and the counter torque which must be applied to keep it at rest is a measure of the hysteresis loss in the metal. To apply this method the sheets must be sufficiently thin to prevent the creation in the metal of any eddy currents which would otherwise involve the performance of work to create them, and therefore vitiate the purity of the test. Assuming, however, that the sheets do not exceed in thickness 10 or 12 mils, the eddy current loss produced in them by the moving field will not be large. As, however, the reversal of the direction of the magnetisation through the mass due to the hysteresis involves the performance of work which is not annulled by any amount of diminution in the thickness of the sheets, it can easily be seen that the rotation of the laminated metal in the field must bring into existence a resistance, and the movement against this resisting force is the work done, the equivalent of which is found in the hysteresis loss in the metal. This hysteresis loss of energy, of course, ultimately takes the form of a rise of temperature in the metal; hence, rotating a mass of laminated iron in a magnetic field causes it to be heated, in virtue of the magnetic hysteresis in the metal, apart altogether from any dissipation of energy by eddy currents. If the sheets or laminations

are thicker than 12 or 14 mils, then, in addition to the rise of temperature caused by the magnetic hysteresis, there will be a further rise created by the eddy currents generated in the iron, which will involve the performance of work to create it. If, therefore, the magnetic field created by a permanent magnet is made to revolve round whilst traversing a mass of laminated iron placed in it, it will drag with it the mass of metal. If this mass is suspended by an elastic wire, or is constrained by a spring, it will be rotated against the torque due to the elasticity of the spring through a certain angle, until the restoring couple or torque balances that due to the revolving field. On the other hand, the magnet creating the field may be suspended, and the mass of laminated iron may be revolved, and in that case the magnet itself would be moved through an angle which may be limited by a couple or torque due either to the elasticity of a spring or to gravity. In the form of hysteresis tester devised by Prof. Ewing (illustrated in Fig. 36), the latter method is adopted.\* The magnetic specimen takes the form of a small bundle of laminated iron prepared by cutting or stamping a number of discs from the sheet to be tested. These are in the form of a rectangle 3in. long and  $\frac{5}{8}$ in. wide. Six or seven of these steps are placed in a carrier, *a* (see Fig. 36), covered with an ebonite washer, and secured by clamping screws, *bb*. The carrier is made to revolve by a friction pulley, *c*, and a hand-wheel, *d*. The remainder of the appliance consists of a curvilinear magnet which is supported by a knife-edge in line with the axis of the carrier. This magnet has a pointer attached to it working over a curved scale and a weight, *g*, which is adjustable, in order to give the magnet stability and provide a small restoring force if it is displaced. The magnet is therefore free to oscillate about the knife-edge, but if deflected through a small angle a restoring torque is brought into resistance proportional to the angle of deflection, which

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\* See Prof. J. A. Ewing, "A Hysteresis Tester," *Jour. Inst. Elec. Eng.*, Vol. XXIV., p. 400, 1895.

tends to bring the magnet back into the vertical position. The work done on the magnet is, therefore, proportional to the square of this angle of deflection. The magnetisation of this magnet is so adjusted that the flux density in the specimen placed between its poles is about 4,000 C.G.S. units.

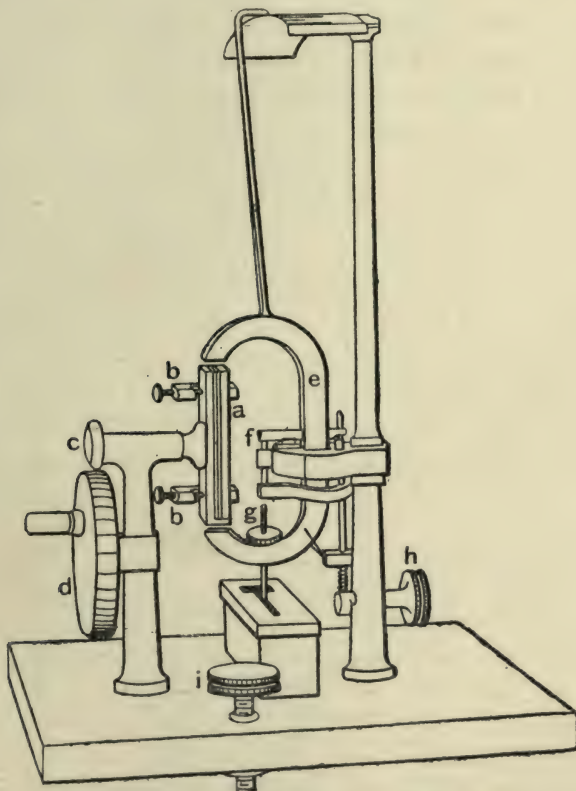


FIG. 36.—Ewing's Portable Hysteresis Tester.

A considerable air space is left between the ends of the sample and the magnet poles, with the result that the total fall of magnetic potential from pole to pole takes place almost wholly in the air spaces, and consequently such variations of

permeability as are met with in different samples are almost without influence on the total flux through the iron. This is important, since, in comparing the hysteresis of one specimen with another, the flux density should be the same in both.

The process of making a measurement is as follows:— Each instrument is provided with a sample specimen, the hysteresis loss in which, in ergs per cubic centimetre per cycle, has been already determined by the instrument maker.

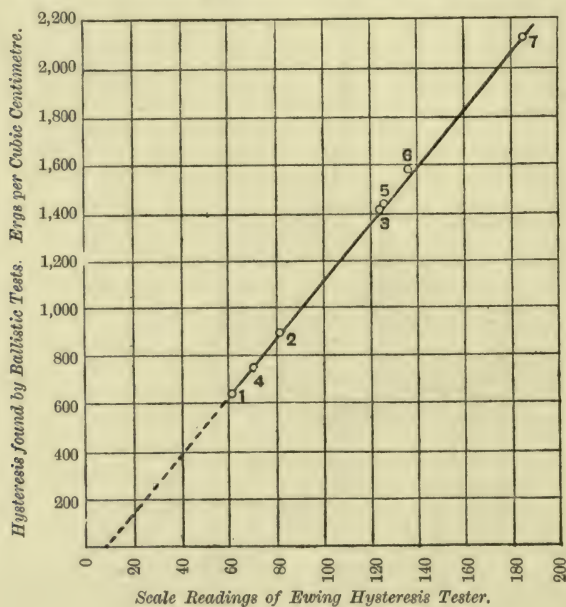


FIG. 37.

The observer places this standard specimen in the clamps, and rotates it at a speed sufficient to make the magnet deflect steadily. The exact speed is of no consequence. In order to assist in the irregular impulses on the magnet running together into a steady torque, the steel knife-edge which is attached to the magnet rests on agate planes, and from the bottom of the magnet a metal vane hangs down which moves

in a vessel of oil and acts as a dash-pot. Having noticed the deflection of the magnet produced by the standard specimen, the observer then makes up a bundle of strips of the iron to be tested, of the same kind, making it as nearly as possible the same weight as the standard sample. This being rotated as before, causes a deflection in the magnet, which may be different from that given by the sample, but the scale over which the magnet pointer moves can be graduated in such a manner as to show at once the hysteresis loss in the sample. As a matter of fact, it was found by Prof. Ewing that the deflection given by a series of samples for which the hysteresis have been determined by ballistic tests, showed that the calibration curve of the instrument is a straight line (*see* Fig. 37). The hysteresis of the standards is stated for  $B=4,000$ , and the comparison of deflection gives results applicable to that value of  $B$ . If it be desired to infer the hysteresis for any other flux density, this is readily done by the aid of the following table, in which  $B_{\max}$  stands for the maximum flux density during the cycle, and the hysteresis loss is stated in comparison with that at  $B_{\max}=4,000$  taken as unity:—

TABLE XVII.

Value of $B_{\max}$ in C.G.S. Units.	Relative Hysteresis Loss.
2,000	0.33
2,500	0.47
3,000	0.63
4,000	1.0
5,000	1.41
6,000	1.89
7,000	2.41
8,000	3.00

The advantages of this apparatus are that no special skill is required on the part of the observer, that the sample prepared consumes only a few square inches of sheet iron, and at the same time there is no winding of coils or necessity to use electric currents.

A similar instrument for determining the hysteresis in sheet iron by means of a rotating magnetic field was designed by Mr. F. Holden for the General Electric Company in the United States. In this instrument the sheet iron to be tested was punched into rings, and a number of these clamped together were placed between the poles of a rotating electromagnet. The rings were held in a fibre frame so as to be concentric with a vertical shaft which worked freely on a pivot bearing at its lower end. The rings had a width of 1cm., and an outside diameter of 9cm., and enough were used to make a cylinder nearly 2cm. high. The shaft which carried the rings had a pointer moving over a circular divided scale, and was connected to a helical spring, the other end of which was fastened to the fixed frame. Hence any angular deviation of the rings was indicated by the pointer, and was resisted by the torsional elasticity of the spring. When the electromagnet was made to rotate, the work expended on the metal in hysteresis and eddy current loss was represented by a torque twisting the iron rings round against the restoring couple due to the spring. The spring having been calibrated previously, the indications of the pointer gave at once the work measured in ergs spent on the rings per cycle.

In order to determine the maximum flux density a coil of wire was so arranged that it surrounded but did not touch the rings, and made contact at its ends with two fixed brushes that rested in diametrically opposed positions on a two-part commutator, which revolved with the magnet. These were connected by means of brushes and connector rings with a voltmeter; hence the deflections of this voltmeter could be made to indicate the maximum flux density in the rings when corrected for the speed of the magnet. In reducing the observations a correction was applied for the eddy current loss in the iron due to its thickness and maximum flux density. In this experiment the hysteresis loss at various flux densities per cubic centimetre per cycle was found to be equal to a constant multiplied by a power of

the flux density varying between 1.4 and 1.7. Rings tested in this manner for hysteresis were afterwards wound, and tested with the ballistic galvanometer, using the step-by-step method of determining the cyclical magnetisation curve. There were discrepancies of as much as 4 per cent. between the two results, but the average of 10 tests showed that the ballistic galvanometer method gave results about  $2\frac{1}{2}$  per cent. lower than the method by rotation in a magnetic field.

The measurement of hysteresis loss by the rotation of a laminated specimen in a constant magnetic field is unquestionably the right method to employ in order to test sheet iron for hysteresis loss which is to be used in the manufacture of dynamo armature cores. When, however, sheet iron or sheet steel is to be tested which is intended for use in the cores of transformers, it is advisable that the hysteresis loss should be tested directly by means of the wattmeter.

**§ 17. Variation of Hysteresis Loss in Iron with Time.**  
**Ageing of Iron.**—It has been found that when iron or steel is subjected to heat the hysteresis loss in the iron is gradually increased. This appears to have been first brought to notice by experiments made by Mr. G. W. Partridge (see *The Electrician*, December 7, 1894, Vol. XXXIV., p. 160), and the subject was immediately afterwards studied by several other observers, including the Author (*The Electrician*, December 14, 1894, Vol. XXXIV., p. 190; and October 22, 1897), Mr. W. M. Mordey (*The Electrician*, December 21, 1894, Vol. XXXIV., p. 219; February 22, and March 1, 1895, Vol. XXXIV., pp. 498 and 547), Dr. D. K. Morris (*The Electrician*, May 28, 1897, Vol. XXXIX., p. 143), Mr. S. R. Roget (*The Electrician*, June 3, 1898, Vol. XLI., p. 182, and February 10, 1899, Vol. XLII., p. 530). It was found by Mr. Mordey that the slow heating of iron to very moderate temperatures was sufficient to increase the hysteresis loss by very notable percentages.\* The following

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\* See W. M. Mordey "On Slow Changes in the Magnetic Permeability of Iron," *Proc. Roy. Soc.*, Vol. LVII., p. 224, 1894.

conclusions were arrived at by him. First, that the effect was not due merely to magnetic reversals; secondly, that neither electric nor magnetic action is necessary to its production, but that it is a physical change resulting from long continued heating at very moderate temperatures, and that it tends to be greater if pressure is applied during heating. Thirdly, it was found to be similar to the effect produced by hammering, rolling, or heating and cooling quickly the iron, and also that it would be almost completely annulled by re-annealing. These results were confirmed by experiments made by Mr. S. R. Roget.\* He found that the effects

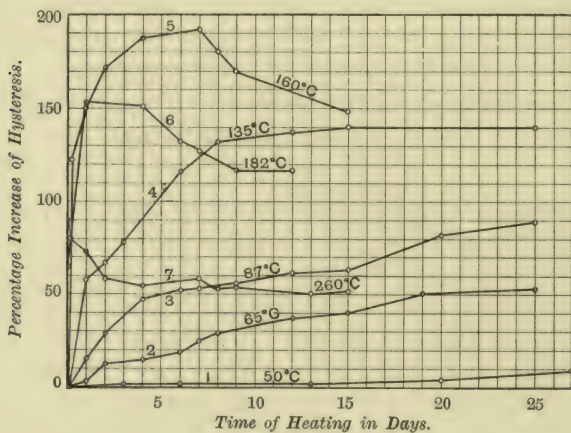


FIG. 37A.

produced by heating the iron differed widely at different temperatures. If the heating did not exceed 40°C. no change in the hysteretic quality of the iron was produced. If heated to between 40°C. and 135°C., the hysteresis increases with time, being relatively rapid at first but slow as time goes on. He found, however, that at higher temperatures than 135°C. the hysteresis reached a maximum value in

\* See S. R. Roget, on the "Effects of Prolonged Heating on the Magnetic Properties of Iron," *Proc. Roy. Soc.*, May 12, 1898, Vol. LXIII., p. 258; also *The Electrician*, Vol. XLI., p. 182, and Vol. XLII., p. 530.

a comparatively short time, after which any further heating caused a marked decrease in hysteresis instead of further increase.

The effect on the increase in hysteresis of exposing iron to different temperatures for various periods of time in days or hours is well shown by a series of curves taken from Mr. Roget's Paper. In these curves (*see* Figs. 37A, 38, 39 and 40) the abscissæ represent the time in days or hours during which the sample of iron was heated, and the ordinates the corresponding hysteresis or percentage increase in hysteresis.

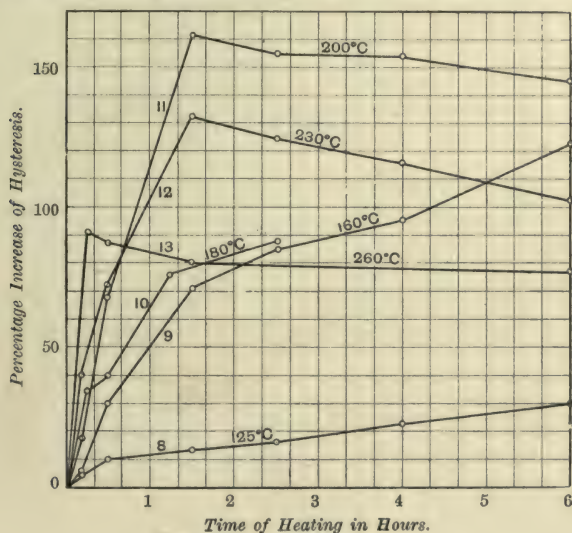


FIG. 38.

esis. Actual experiments of the variation of the core loss in transformers when heated have been made by Mr. A. H. Ford.\* Eight transformers were tested, being connected to a supply circuit for 120 days, and the core loss measured at intervals. In two cases the rise in core loss did not exceed 4 per cent., in one case it mounted to 113 per cent. The time

\* See A. H. Ford, "On Hysteresis in Sheet Iron and Steel," *Proc. Amer. Inst. Elec. Eng.*, April, 1900.

variation of core loss in the individual cases was found to be as follows :—A large number of experiments were also made by

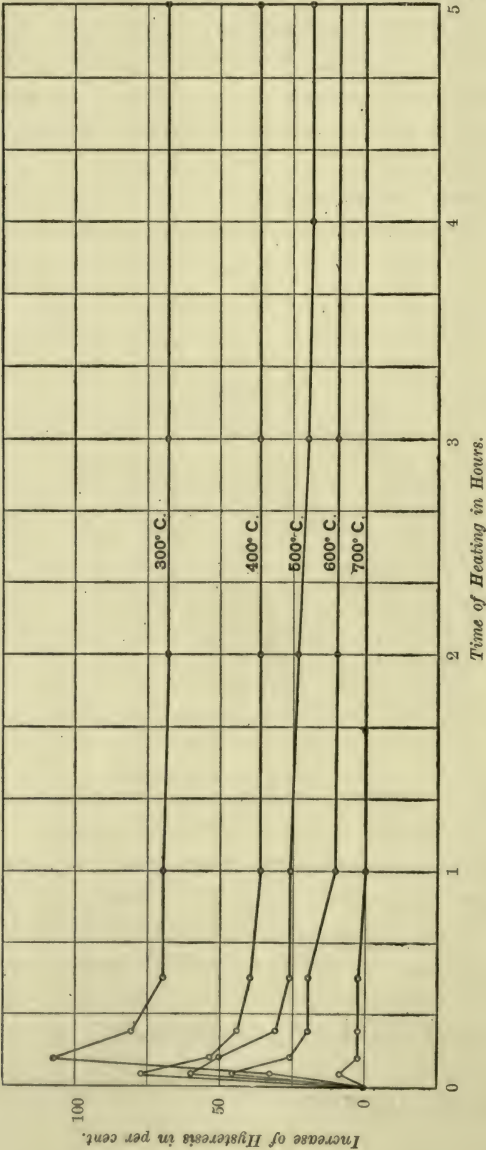


FIG. 39.—Soft Swedish Transformer Iron.

Mr. Ford on the time increase in hysteresis loss in samples of sheet iron and steel. The chemical composition of these samples was taken, and from a comparison of the results of chemical analysis with the hysteresis tests, it was found that neither a single impurity nor any pair of impurities had any preponderating effect on the absolute values of the hysteresis constant or on its increase by heating. It appears that two specimens of iron and steel may have the same chemical composition but differ widely as to hysteresis constant and ageing. It is found that the hysteresis constant for iron of considerable purity, chemically speaking, is not so much

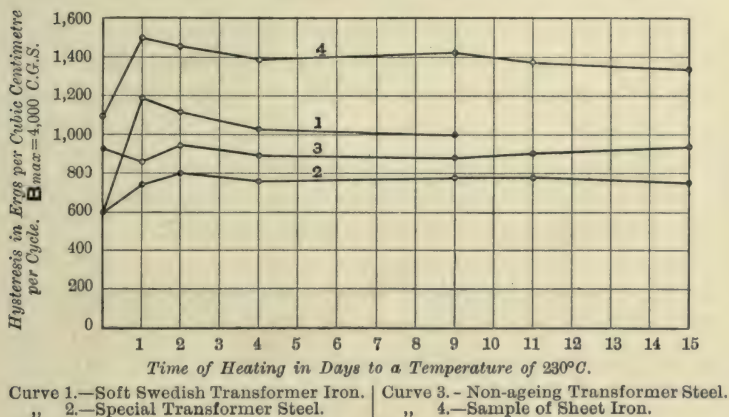


FIG. 40.

dependent upon the chemical composition as upon the physical structure of the material, and in particular, upon the way in which it has been annealed. Generally speaking, a very small initial hysteresis loss must be looked upon with suspicion, as indicating that the iron will probably suffer a considerable percentage increase by ageing. The measurement of the magnetic ageing of iron is at best a tedious process, but it can be very conveniently carried out by means of a Ewing magnetic hysteresis tester, because the samples of iron required for this instrument are small in size and can be

quickly tested. A large number of these samples can, therefore, be prepared from different sheets of iron, and after testing can be heated in a hot-air oven kept at a constant temperature by a gas burner regulated by a thermostat, by means of which the iron can be kept night and day for any required period of time at the same temperature. It cannot be said, however, that any research on this subject has yet revealed a single criterion by which we can judge whether a given sample of iron is likely to possess a large or small ageing quality in respect of hysteresis. The experience of transformer manufacturers, however, has enabled them to select certain qualities of mild steel which possess sufficiently small hysteretic constant to be used for the manufacture of transformers, and which yet do not show any marked percentage increase in hysteresis loss by such heating as they are subjected to when used as the cores of transformers. Nevertheless, in drawing up transformer specifications it is well to introduce some clause guarding the purchaser against the use of "ageing" iron by the manufacturer.

**§ 18. The Measurement of the Magnetic Susceptibility of Feebly Magnetic Substances.**—The methods which are applicable for the measurement of the permeability of the ferro-magnetic bodies, such as iron, nickel and cobalt, are quite inapplicable for the measurement of the permeability of feebly magnetic substances such as oxygen, palladium, manganese, &c. The permeability of these last-named substances is best determined by making a measurement directly of the susceptibility ( $k$ ), and then determining the permeability  $\mu$  by the relation  $\mu=1+4\pi k$ .

When a ferro-magnetic substance is placed in a magnetic field it greatly disturbs the distribution of the original field. From one point of view we may say that it creates a number of fresh lines of flux. If, however, the susceptibility of the body is so small that it does not sensibly disturb the distribution of the field, the susceptibility may be measured by

a method which depends upon the determination of the mechanical force acting upon the substance. If a small mass of a substance of which the susceptibility is  $k$  and volume is  $V$  cubic centimetres is placed in a non-uniform magnetic field, then, if the body is feebly magnetic, it experiences a force tending to move it in the field from the weak to the strong parts of the field in the direction in which the force is increasing most rapidly. On the other hand, if diamagnetic, it is urged in the opposite direction. If  $dx$  represents a displacement in any direction, and if  $H$  is the magnetic force at that point, then the mechanical force reckoned in dynes acting on the body in the direction  $x$  is represented by the expression  $f = k V H \frac{dH}{dx}$ , where  $V$  is the volume of the body in cubic centimetres and  $H$  the magnetic force at the centre in C.G.S. units.

The value of  $k$  in the above expression is a difference value—*i.e.*, it is equal to the difference between the susceptibility of the body and that of the medium in which it is immersed. If, therefore, the experiment is performed in a vacuum, we have an absolute measure of the susceptibility of the body. If the experiment is performed in air, the value of  $k$  is the difference between the susceptibility of the body and that of the air. In order to create a non-uniform magnetic field an electromagnet has to be employed of particular form.

In some experiments carried out by the Author and Prof. Dewar on the magnetic susceptibility of liquid oxygen, an electromagnet of tubular form was employed. It consisted of a solid soft iron core,  $C$ , surrounded by a tubular shell,  $S$ , this cylindrical sheath being closed top and bottom by thick circular plates of iron having holes in them. The wire windings surrounding the core were inside the sheath. The electromagnet actually employed had the following dimensions:—The wire coil consisted of double cotton-covered copper wire No. 14 S.W.G. The coil was

30cm. long,  $18\frac{1}{2}$ cm. outside diameter, and  $9\frac{1}{2}$ cm. inside diameter, the total weight of wire being 71lbs. and the number of turns on the bobbin 2,478. The total resistance of the wire was very nearly  $5\frac{1}{2}$  ohms at  $20^{\circ}\text{C}$ . This bobbin was enclosed in a cylinder of mild steel of the same height as the bobbin, the walls of which were  $2\frac{1}{2}$ cm. in thickness. The end plates were also  $2\frac{1}{2}$ cm. in thickness, and the core was 9cm. in diameter and 37cm. in length. The end surface of the inner core was  $7\frac{1}{2}$ cm. below the upper surface of the wire coil. By means of a small secondary bobbin and a ballistic galvanometer the flux density or field at various distances  $x$  from the pole along the axial line was determined. Having in this manner determined the variation in the magnetic flux along the axial line for any given exciting current, we have at once the means of determining  $d\mathbf{H}/dx$ , or the rate of change in flux density or field in the direction of the axis at any point, and therefore the value of  $\mathbf{H}\frac{d\mathbf{H}}{dx}$  at any point in the field. The manner in which the measurement of the force is best made is to enclose the body to be tested in a very thin glass bulb, if a powder, or, if solid, to shape it approximately to a spherical shape. This body is then suspended by a very fine platinum wire from the pan of a balance, and the volume of the body is determined in the ordinary manner by weighing it in water at a known temperature. The loss in weight of the body when weighed in water gives us the weight of the displaced water, and from the tables of the density of water at different temperatures the volume is, therefore, immediately deduced, for the volume of the body  $V$  is equal to the loss in weight when hung in water divided by the density of water at that temperature. The position of the centre of the body must then be accurately determined with reference to the pole face of the magnet over which it is suspended, with its centre in the axial line of the magnet, and from the previous experiments the value of the quantity  $\mathbf{H}\frac{d\mathbf{H}}{dx}$  at that point is known.

If the body is suspended in air over the magnet pole, and if the body is feebly magnetic, it will then be found that on exciting the magnet the body will be attracted to the pole and a weight  $W$  will have to be placed in the opposite scale pan to maintain the body at the original position. If this weight, reckoned in grammes, is  $W$ , then the force in dynes acting on the body is  $981W$ , and the relative susceptibility of the substance is expressed by the formula

$$k = \frac{981W}{VH \frac{dH}{dx}}.$$

The absolute magnetic susceptibility of air at ordinary pressures and temperatures is equal to  $0.024 \times 10^{-6}$ .

Employing the above method, the Author and Prof. Dewar found the following susceptibilities for the substances mentioned in the Table:—

TABLE XVIII.

Substance.	$k10^{-6}$ .
Silver.....	1.73
Bismuth .....	- 13.75
Water .....	- 0.74

Having obtained values for the susceptibility of a silver ball in air, this body can be used to make a determination of the magnetic susceptibility of any liquid. Thus the Author and Prof. Dewar made measurements of the susceptibility of liquid oxygen by suspending the silver ball in a vacuum vessel full of liquid oxygen over the pole of the magnet. The ball was first balanced in the liquid oxygen at a known position in the field, and then, the magnet being excited, the ball was found to be strongly repelled by the magnet, exhibiting a spurious diamagnetic quality, in virtue of the magnetic quality of the liquid oxygen. The value of the apparent diamagnetic susceptibility of the silver ball in the liquid oxygen having been found, the true diamagnetic susceptibility of the silver in vacuo was deducted from it,

leaving a value for the magnetic susceptibility of liquid oxygen ( $k$ ) such that  $k=326 \times 10^{-6}$ .

Similar experiments made with glass balls and copper balls confirmed the above number, thus showing that the permeability  $\mu$  of liquid oxygen is equal to 1.0041. Further experiments on the susceptibility of various magnetic and ferro-magnetic substances showed that, for feebly magnetic bodies, the magnetic susceptibility ( $k$ ) varies inversely as the absolute temperature. The diamagnetic susceptibility of bismuth at 15°C. was found to be equal to  $13.7 \times 10^{-6}$ , and at the temperature of liquid oxygen ( $-182^\circ\text{C}.$ ) it was found to be equal to  $15.9 \times 10^{-6}$ , an increase of 16 per cent. The magnetic susceptibility of gaseous oxygen as determined by Faraday, E. Becquerel and others, is  $0.13 \times 10^{-6}$ , and the susceptibility of liquid oxygen as shown above was found by the Author and Prof. Dewar to be approximately  $326 \times 10^{-6}$ . These numbers are in the ratio of 1 to 2,420, which is very nearly equal to three times 806. The density of liquid oxygen is 806 times that of gaseous oxygen at 0°C. and 760mm., and the absolute temperature of liquid oxygen is one-third of that of gaseous oxygen at the melting point of ice. Hence the above figures show that for liquid oxygen, and probably for other feebly magnetic bodies, the magnetic susceptibility varies directly as the density and inversely as the absolute temperature.

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## CHAPTER V.

### DYNAMO, MOTOR AND TRANSFORMER TESTING.

§ 1. **Definition of the Term Efficiency.**—The term *efficiency*, when used in connection with any device for transforming power denotes the ratio which the power given out or given up by it bears to that taken in or absorbed. Since this ratio generally varies with the output of the machine, its values corresponding to various outputs are best represented by an

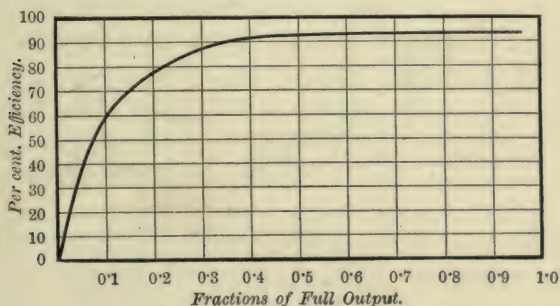


FIG. 1.

*efficiency curve.* In plotting the curve, the work done by the appliance is generally represented as a fraction of the full load output taken as unity, whilst the efficiency is expressed as a percentage. Hence, an efficiency curve generally takes some such form as the curve shown in Fig. 1, the efficiency increasing with the load.

We may represent the results of the energy transformation in another manner. Every energy-transforming device, such as a dynamo or motor, has in it certain sources of dissipation

of energy, such as friction, magnetic hysteresis or electrical resistance, and these cause a power loss called the *internal losses*. Hence, if we represent by  $O$  the power output, by  $I$  the power taken in, by  $L$  the power losses, and by  $e$  the efficiency of the machine, we have

$$e = \frac{O}{I} = \frac{O}{O+L} = \frac{I-L}{I},$$

and 
$$\frac{1}{e} = 1 + \frac{L}{O}, \quad \text{or} \quad L = \frac{1-e}{e}O.$$

We can, therefore, set out a *loss curve* in terms of the output, which gives us the loss of power corresponding to the various fractions of full load output. From this *loss curve* we can construct the *efficiency curve*, if required.

This internally lost power is often called the *stray power*, and it is sometimes subdivided into *copper losses*, depending upon resistance, *iron losses*, depending upon magnetic hysteresis, and eddy currents and *frictional losses*, dependent upon mechanical movement or rubbing of surfaces.

The efficiency test of any electrical appliance resolves itself, therefore, into a measurement of the power given out and power taken in, or else of the power losses in the machine.

The efficiency measurements with which we are concerned in this Chapter are those of dynamos, motors and transformers. In all these cases, the power output and intake must be expressed in the same units—that is, in kilowatts, or watts, or horse-power, and if one of these measurements is made in horse-power it may be converted to its equivalent in kilowatts, by the rule that 1 H.P. = 0.746 kilowatt.

**§ 2. Dynamo Testing.**—The testing of a dynamo-electric machine involves measurements made to determine the efficiency of the machine as a transforming device, as well as an examination of its behaviour under trial runs of various lengths, its power to carry overload, its temperature rise in

use, and the variation of the voltage of the machine with load. Measurements have also to be made of the internal losses, such as those due to friction, hysteresis and eddy currents. As a large dynamo is a costly machine, it should not be purchased except on a precise and carefully drawn specification, and the nature of the tests to which it is to be subjected should be fully set out. In reporting on a dynamo submitted for testing, the number of the machine, the maker's name, and the type should be noted. The details of the windings of the armature and the field, and the dimensions of the armature core and of the field magnets should, if possible, be ascertained. The resistance of the armature and the field magnet windings should then be taken. This is done as follows:—If the machine is a continuous-current machine, the commutator is first carefully cleaned with fine glass-paper and the brushes are set to touch exactly at two opposite sections on the commutator. A pair of insulated wires with well cleaned ends are then placed under the tips of the brushes, so as to be pressed firmly against two commutator sections exactly opposite to each other. As the resistance of a continuous current armature is generally rather low, the best method to adopt for measuring it is by the fall of potential method (*see* section 13, Chapter II., Vol. I. of this HANDBOOK). A known low resistance must be provided of the strip form (*see* section 5, Chapter I., Vol. I. of this HANDBOOK). A convenient magnitude for this resistance is one-hundredth of an ohm. This low resistance should be connected in series with the armature, and a continuous current sent through the two together by means of one or two cells of a secondary battery, an additional regulating resistance being interposed. Pairs of potential wires are then taken from the ends of the armature and from the ends of the low resistance, and by means of galvanometer deflections, as described in section 13, Chapter II., Vol. I., a measurement is made of the resistance of the armature. The same process can be applied in the case of low resistance armatures of alternators.

When, however, the armature resistance exceeds 1 or 2 ohms, and also in the case of the field magnet coils, the resistance is best measured by means of a portable Wheatstone bridge, which may be of the type shown in section 5, Chapter II., Vol. I. It is important to bear in mind that, in measuring a resistance of a field magnet or dynamo armature, we are dealing with a highly-inductive circuit, in which the bridge current creates a slowly-changing counter E.M.F., due to the slow rise of the magnetic flux in the iron core. As already explained in the previous chapter, the magnetisation in large iron cores begins at the surface and penetrates slowly inwards, when the core is magnetised by an electric current circulating round its surface. Hence, as long as this magnetic flux is changing in the core, and until it becomes practically constant in disposition, there is a changing counter E.M.F. in the magnetising coil, and, unless the precaution is taken to wait until this has died away, very great errors may be made in the bridge determination of the resistance of the coil.

The proper method of procedure is as follows:—Having joined up the armature circuit or the field magnet circuit to the bridge, which should be placed at a sufficient distance from the dynamo to prevent any direct effect of the dynamo field upon the galvanometer needle of the portable bridge, the bridge circuit is closed by means of the battery key. This key should be held down not merely for one moment but for five or six seconds before attempting to close the galvanometer circuit and to take a bridge reading. It is not until the current flowing in the bridge circuits has had time to create a constant magnetic state in the field magnet core or armature core that the galvanometer circuit should be closed and the bridge reading taken. The same precaution must be used after making any change in the relative value of the bridge measuring arms or of the ratio arms, and when once the bridge balance has been obtained it should be carefully examined to ascertain that it is really due to the ratio of the resistances of the bridge arms, and not disturbed by any

small counter E.M.F. in the circuit being measured. When these measurements of resistance are made, the temperature of the copper coils should be noted by means of a chemical thermometer laid upon the coils and covered with a pad of cotton waste. In order to calculate the true copper losses in these coils when the dynamo is at work, these measured values of the resistances have to be corrected for temperature. If we assume that the final temperature which will be permitted when the dynamo is at work is, say,  $40^{\circ}\text{C}.$ , and that the measurements are made when the dynamo is at  $15^{\circ}\text{C}.$ , we have then to correct the measured resistance as follows:—The temperature coefficient of high conductivity copper may for this purpose be taken as 0.004 per degree Centigrade. Hence, if  $R_0$  is the resistance of the copper at  $0^{\circ}\text{C}.$ , and  $R_t$  is the resistance of the copper at  $t^{\circ}\text{C}.$ , these quantities are related as follows:—

$$R_t = R_0(1 + 0.004t).$$

Hence it follows that, if the resistance is measured at two temperatures,  $t$  and  $t'$ , and if  $R_t$  and  $R_{t'}$  are the resistances at these temperatures, then we have

$$\frac{R_t}{R_{t'}} = \frac{1 + 0.004t}{1 + 0.004t'}.$$

From the above expression we can calculate, therefore, the resistance at  $40^{\circ}\text{C}.$  when we know that of  $15^{\circ}\text{C}.$ , and in this manner determine what is called the *hot resistance* of the armature or field magnet coil. We have then all the materials for calculating the copper losses—or, as it is sometimes called, the  $C^2R$  loss—in the armature and fields at the temperature of the machine when working.

**§ 3. Dynamo Efficiencies and Losses.**—The term efficiency has been applied in several different senses in relation to the energy transformations taking place in a dynamo. Thus we have the terms (1) *Conversion Efficiency*, or *Gross Efficiency*; (2) *Electrical Efficiency*; and (3) *Commercial or Net Efficiency*, in common use.

The term *Conversion Efficiency* signifies the ratio of the total electrical power produced, to the mechanical power applied to the shaft of the machine. The *Electrical Efficiency* is the ratio of the electrical power given to the external circuit to the total electrical power created. The phrase *Commercial Efficiency* means the ratio of the electrical power given to the external circuit to the total mechanical power applied to the machine.

In this section we shall denote the various measurable quantities by symbols, as follows :—

$P$  = Power applied to dynamo pulley in watts.

$W$  = Output of dynamo in watts on external circuit.

$A$  = Current in external circuit in amperes.

$a$  = Field current in amperes.

$V$  = Volts at terminals of dynamo.

$E$  = Total E.M.F. in armature.

$R$  = Resistance of external circuit in ohms.

$R_a$  = Armature resistance in ohms.

$R_f$  = Field-magnet resistance in ohms.

$e$  = Commercial or net efficiency.

$f$  = Electrical efficiency.

$g$  = Conversion or total efficiency.

Then we have

$$\text{The Conversion Efficiency } = g = \frac{AE}{P}.$$

$$\begin{aligned} \text{The Electrical Efficiency } &= f = \frac{AV}{AE} \text{ (for the series dynamo),} \\ &= \frac{AV}{(A+a)E} \text{ (for the shunt} \\ &\quad \text{dynamo).} \end{aligned}$$

$$\text{The Commercial Efficiency } = e = \frac{AV}{P}.$$

(1) Consider the series dynamo. In this case  $a=A$  and  $e=f \times g$ . Also

$$f = \frac{AV}{AE} = \frac{A^2 R}{A^2 R + A^2 R_f + A^2 R_a + x},$$

where  $x$  is the loss in friction, magnetic hysteresis, eddy current loss and windage, reckoned in watts. If we assume  $x$  can be negligible in comparison with the other terms, then

$$f = \frac{R}{R + R_f + R_a} = \frac{V}{E}.$$

Again, for the series dynamo,

$$g = \frac{AE}{P} \quad \text{and} \quad f = \frac{V}{E}.$$

Hence, 
$$e = fg = \frac{AV}{P}.$$

(2) For the shunt dynamo we have

$$f = \frac{AV}{(A + a)E} = \frac{A^2 R}{A^2 R + A^2 R_a + a^2 R_f + x},$$

or, if  $x$  may be neglected, we have

$$f = \frac{1}{1 + \frac{R_a}{R} + \frac{R_f}{R}}.$$

If we vary  $R$ , then  $f$  will have a maximum value when  $\frac{df}{dR} = 0$ , or when  $\frac{1}{R_f} - \frac{R_a}{R^2} = 0$ ; that is, when  $R = \sqrt{R_a R_f}$ .

Under that load we have  $f$  a maximum, and

$$f = \frac{1}{1 + 2\sqrt{R_a R_f}}.$$

For the shunt machine,  $g = \frac{(A + a)E}{P}$ ,  $f = \frac{AV}{(A + a)E}$  and  $e = fg$ .

Therefore 
$$e = \frac{AV}{P}.$$

(3) In the case of the compound-wound dynamo we have

$$f = \frac{A^2 R}{A^2 R + A^2 R_a + A^2 R_s + a^2 R_f + x},$$

where  $R_s$  is the resistance of the series field coil. Neglecting  $x$ , we have

$$f = \frac{1}{1 + \frac{R}{R_f} + \frac{R_a}{R} + \frac{R_s}{R}}.$$

Hence, putting  $\frac{df}{dR} = 0$ , we have

$$\frac{R_a + R_s}{R^2} - \frac{1}{R} = 0, \quad \text{or} \quad R = \sqrt{(R_a + R_s)R_f},$$

as the value of the external resistance which makes  $f$  a maximum, and then

$$f = \frac{1}{1 + 2\sqrt{(R_a + R_s)R}}.$$

The internal power losses in dynamos may be divided into (1) the iron losses, (2) the copper losses, (3) the mechanical losses. Under (1) we have losses  $a$ ,  $b$ ,  $c$ , as below; under (2)  $d$  and  $e$ ; and under (3)  $f$  and  $g$ , viz. :—

- (a) Eddy current loss in the armature core.
- (b) Hysteresis loss in the armature core.
- (c) Eddy current and hysteresis loss in the pole-pieces, due to armature reaction.
- (d) Resistance or copper loss in the armature and field circuits.
- (e) Eddy current loss in the copper of armature.
- (f) Frictional losses at the bearings and brushes.
- (g) Windage or power absorbed in air churning.

Generally speaking, in well-designed machines the sources of waste (c), (e) and (g) will be small—at any rate, in continuous-current machines. The chief losses are due to (a), (b), (d) and (f), and these should be determined separately and *in toto*, as explained in the following sections.

#### § 4. The Practical Measurement of Dynamo Efficiency.—

Two modes of driving dynamos are in use—viz., by belts or ropes from a prime motor, and by direct coupling to the shaft of a steam-engine, gas or oil engine, steam or water turbine. In the case of dynamos coupled directly to a steam engine, which are shortly termed *steam dynamos*, the station engineer does not generally concern himself very much about the commercial efficiency of the dynamo apart from the engine; but he is concerned to know the ratio between the electrical

output of the dynamo and the indicated horse-power (I.H.P.) of the engine, or the number of pounds of steam consumed by the engine per hour, or per kilowatt output of the dynamo. In the case of the smaller belt-driven machines it is desirable to know the commercial efficiency of the dynamo as well as the relation between output of the dynamo and the indicated horse-power of the engine, so we are then able to determine independently the efficiency of the engine, since the efficiency of the engine and dynamo taken together is the product of the efficiencies of the two portions separately. The two classes of efficiency tests that have most frequently been taken in connection with dynamos are, therefore, the measurement of the commercial efficiency of a single machine—*i.e.*, the ratio between the electrical output in kilowatts and the power, also expressed in kilowatts, applied to the pulley to turn the machine round at its standard speed; or, the engineer is called upon to determine the steam consumption of a steam dynamo—*i.e.*, the weight of steam consumed per hour per kilowatt output of the dynamo at full load or at certain assigned fractions of full load. We have, therefore, in both cases to make provision for taking up the electrical output of a dynamo and for measuring it, and means have also to be provided for measuring the mechanical power applied to the pulley in the case of a dynamo alone, or the steam supplied to the engine in the case of a steam dynamo.

The most direct method of measuring the commercial efficiency of a dynamo is obviously to determine the mechanical power supplied to the pulley by means of a transmission dynamometer, and the ratio to the latter of the electrical power given out. Unfortunately, however, a transmission dynamometer for large power is not only an expensive, but a somewhat untrustworthy appliance, and in any case the accurate measurement of the torque communicated to a rotating pulley with any degree of accuracy is not an easy matter. Hence, although great ingenuity has been spent in designing transmission dynamometers, we shall not in this chapter occupy

space with any discussion of them, but the reader must be referred to special treatises on Applied Mechanics for a description of the various forms that have been devised. Happily for the electrical engineer, methods have been invented by which the commercial efficiency of a dynamo can be measured with great accuracy by methods which are altogether electrical, and therefore there is no need to describe the various attempts that have been made to devise efficient transmission dynamometers.

The thing we are really desirous of knowing is the internal loss of power in the machine. Hence, those methods are to be preferred which give us this quantity directly, rather than as the difference of two much larger magnitudes of the same kind.

**§ 5. Arrangements for the Absorption of the Electrical Power in Dynamo Tests.**—In arrangements for dynamo testing, provision must be made for taking up electrical power in any required quantity. The methods that are employed are as follows:—In the case of continuous current machines of small size, the power may be taken up on incandescence lamps. For this purpose old or blackened glow-lamps, which have ceased to be useful for electric lighting, can be employed very conveniently in the construction of a bank of lamps for dynamo testing. On the other hand, if new lamps have to be obtained there should always be a large margin of voltage, so that the risk of breaking the filaments may be diminished. Thus, in testing a 135-volt dynamo, 150-volt lamps may be used, or in testing a 100-volt dynamo 120-volt lamps may be employed. In any case, the lamps should be attached to sockets or suspended in parallel between service wires, and it is convenient to divide the whole bank of lamps used into 10 sections. For instance, if 100 lamps are put up, they may be arranged in sets of 10, each set on a separate switch, so as to be able to turn on the various decimal fractions of the full load. In taking up moderate powers the high candle glow lamps are very convenient, because they take up relatively

less space. We may employ 100 c.p. lamps for this purpose, each taking up 400 watts. Five of these lamps will, therefore, take up 2kw. and by means of 25 such lamps a 10kw. dynamo can be conveniently tested.

In taking up larger powers than 10kw. it is necessary to use special resistances made of wire. In so doing some material must be used which has a high melting point and yet a small temperature coefficient. Iron wire is, therefore, not so good as nickel-steel or manganese-steel wire, because the resistance of the latter alloys change much less with temperature than pure iron. Whatever material is used should be employed in the form of wires about the size of No. 36 S.W.G., three or four of these wires being bunched together into a strand, but not twisted. In constructing a resistance of this kind, arrangements must be made for taking up the slack of the wire, as it expands during heating, and for this purpose the form of resistance devised by the Author, called a cage resistance (*see* Vol. I. of this HANDBOOK, page 80), is very convenient.

A rheostat which has been long in use in the dynamo room of the Pender Electrical Laboratory, University College, London, which satisfies the requirements of a power-absorbing resistance for dynamo testing, is made as follows :—A number of wires of nickel-steel are held a little way in front of a glazed brick wall, being carried at the top by a support let into the wall, and at the bottom being connected to a set of switches which enables the required number of sections of the wire to be put in parallel. The resistance consists of 20 strands, each consisting of four single wires, No. 32 gauge S.W.G. Each individual wire is 50ft long, and the whole of the 80 wires are connected at the bottom to a strip of copper forming an omnibus bar. They then pass up and down in front of the wall four times to a support fixed on the wall 14ft. above the omnibus bar. To this support are fixed a number of porcelain insulators which carry steel bell springs, and these springs take up the slack of the wire as it expands.

The other ends of each strand of four wires are connected to blocks of metal set in a semicircle, and so arranged that, by means of a rotating semicircular disc (*see* Fig. 2), one or more of these strands can be placed in parallel. The arrangement, therefore, forms a device by which a resistance can be formed of nickel-steel wires 50ft. in length, added in parallel four at a time as required, so as to form a variable resistance. This

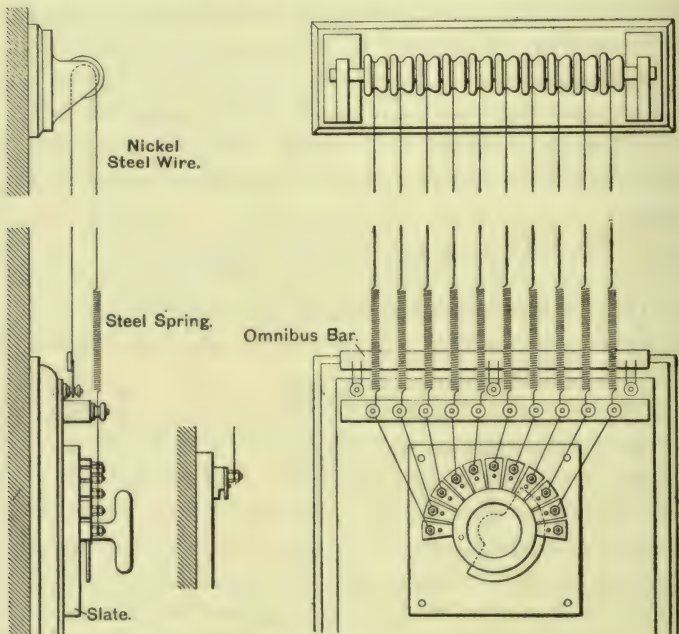


FIG. 2.—University College Dynamo Rheostats.

resistance can sustain a voltage of 100 volts at its extremities, each strand of four wires carrying a current of 2 amperes. If, then, the terminals of a dynamo are connected to this resistance by putting in the required number of the strands in parallel, a power-absorbing resistance is provided of which the dissipative power can be varied by 20 steps as required.

In testing large dynamos a convenient and cheap resistance can be made of strips of iron wire netting, which is wound

backwards and forwards in a zigzag fashion over iron bolts fixed in a wall, each bolt being threaded through a short length of porcelain tubing. The strip of netting may be conveniently 6in. wide, and a length must be taken such that, when the voltage of the machine to be tested is applied to the ends, the current passing through the strip will not make it visibly red-hot in the dark. If properly erected, such a resistance can be worked at a temperature of about  $400^{\circ}\text{C}.$ , and an economy of material is effected by working the power-absorbing resistance at as high a temperature as possible. In the arrangements for taking up the power of large dynamos in dynamo factories, where tests are continually being made, the wire which is employed as a power-absorbing resistance should be erected in a place perfectly fire-proof, where it can be kept cool by a current of air passing through it. To economise space the wire may sometimes be wound in spirals, but experience shows that a thin wire placed in a vertical position has a higher current-carrying capacity than the same wire wound into a spiral as a greater surface is offered to the air for radiation and the removal of heat by convection. In some cases wire spirals may be immersed in a tank through which water is kept running, and so a higher current-carrying capacity still be secured, and the heat more readily removed, than if the wire were simply exposed to the air. If, however, such a water-cooled resistance is employed in testing continuous current machines, the water should be as pure as possible, otherwise electrolysis will be set up, which will destroy the resistance wire near the positive terminal.

In calculating the necessary size and length of wire for constructing a power-absorbing resistance of any required magnitude, we must first ascertain the resistance of known lengths of wires of different diameters at stated temperatures made of the material selected. These data can generally be obtained from the tables published by the manufacturers of the wire. The next point to be considered is the highest temperature at which it is safe to work the wire. If the

resistance is being worked in the open air, and entirely away from any inflammable material, and if the melting point is sufficiently high, the current density may be pressed up to a point at which the wire even reaches a dull red heat. It is well, however, not to carry the temperature above  $200^{\circ}\text{C}.$ , for above this temperature most of the high resistance alloys become rapidly oxidised and are damaged. Moreover, at high temperatures the cooling effect of currents of air makes a considerable variation in the resistance of the wire, and, therefore, affects the current passing through it. If the resistances are used in the interior of a building, in contiguity to wooden walls, they should never be allowed to rise in temperature above  $200^{\circ}\text{C}.$  The wire tables will generally give the resistance of each size of wire at a certain temperature, and from the known temperature coefficient of the wire the resistance can be ascertained at any required temperature; and the same tables will generally also furnish the approximate current which, passed through open spirals of the wire, will bring them to a final state of temperature as selected. Hence, knowing the resistance of the wire at that temperature, and the current which will bring it to that temperature, it is easy to calculate the length of wire required, which must be placed across a circuit of given voltage, and will then take up a known power.

Thus, if it is required to take up the power of a dynamo having an output of 10kw. at 100 volts, this necessitates taking 100 amperes out of the machine. Suppose we select for use in the manufacture of a power-absorbing resistance a wire made of one of the special trade alloys, such, for instance, as the material called *Beacon*, which is a high resistance alloy; and that we choose a wire of No. 20 S.W.G. size, which, when wound into a spiral of rather open turns, is brought to a temperature  $220^{\circ}\text{C}.$  in the open air by a current of 4 amperes. The manufacturers' tables show this wire has a resistance of  $1\frac{1}{2}$  ohms per yard. Accordingly, each spiral may consist of a wire having a resistance

of 25 ohms—in other words, it must have a length of about 50ft. If, therefore, 25 wires, each having a length of 50ft., are wound in rather open spirals, the ends being joined in parallel between two omnibus bars, such a wire resistance would take up and dissipate the whole power from a 10kw. dynamo without rising in temperature much above 220°C.

Since the dissipation of energy by wires is effected by radiation and convection from the surface, the economical problem in making a power-absorbing resistance consists in getting as much *surface* as possible for the money paid for it. If all wire were sold at the same price per pound, the finest wire made would be the one to select, because it has the largest surface in proportion to its cross-section. But as fine wire costs more per pound than thick wire, and moreover, since a fine wire is more easily damaged and destroyed by oxidation than a thick wire, the above conclusion has to be corrected by the following considerations:—Taking any particular metal, wires drawn from this metal have varying prices per square inch of surface. Thus, for instance, taking platinoid, the following results have been obtained for the number of square inches of surface obtained for a penny in the case of different sized wires\* :—

Diameter of Wire in Inches.	Surface in Square Inches obtained for One Penny.
0·002	12·2
0·004	12·9
0·006	18·0
0·010	16·0
0·020	12·5
0·040	7·8
0·080	4·3
0·160	2·25

The above table shows that, in the case of platinoid wire, we get most surface for our money if we select a wire whose diameter is 0·006. This corresponds with No. 38 S.W.G. Such

\* See *The Electrician*, Vol. XXXVIII., p. 16, 1896, "On the Dissipation of Heat by Wires,"

a wire can carry half an ampere without being dangerously hot; but 1.2 amperes makes it red-hot. No. 38 S.W.G. is, however, rather too small a wire to employ, for mechanical reasons, being too easily broken; and therefore we may say that the most economical size to use, taking all circumstances into consideration, is No. 36 S.W.G., and this conclusion holds good for the generality of wires which are used for power-absorbing purposes. In the case of platinoid, a No. 36 S.W.G. wire has a resistance of 4 ohms per foot, will carry half an ampere without reaching a temperature above  $200^{\circ}\text{C}$ ., and will therefore dissipate 1 watt per foot. A 4ft. length of this wire has a surface of nearly 1 square inch, and will dissipate 4 watts without rising in temperature above  $200^{\circ}\text{C}$ . Hence, a quarter of a square inch of surface dissipates 1 watt, and, as a practical rule, we shall not be far wrong in saying that from the surfaces of wires of this size of most of the metals used for making power-absorbing resistances we can dissipate energy safely at the rate of 1 watt per square centimetre of surface. Two No. 36 platinoid wires, each 50ft. long, laid in parallel, will therefore safely carry 1 ampere and dissipate 100 watts. They have a resistance of 100 ohms, and can therefore take a voltage of 100 volts. If we lay in parallel four or five such platinoid wires, the dissipating power is not quite so great as from four or five times one wire, because the aggregate radiating surface is less.

In regard to the choice of a wire and of material for a power-absorbing resistance, the cheapest material to use is, therefore, one for which the product of the number of square inches per penny on its most economical gauge, multiplied by the number of degrees of temperature through which it can be raised safely, is a maximum. If there is any considerable difference in the emissivities, then this product must be corrected with reference to it.

The specific resistance of a material has no direct importance, but, for any given gauge of wire, its variation affects the current-carrying power; although it cannot in any

way affect the total surface of the wire which must be used, it will affect the number of parallel spirals or parallel strands into which the required length must be divided. This may be proved as follows:—Suppose we have to absorb a given amount of power ( $W$ ), reckoned in watts, and that we elect to work the resistance wire at an emissivity at which 1 watt is dissipated per square centimetre of surface, then, if  $S$  is the total surface of the wire in square centimetres, we must have  $W=S$ . The surface  $S=\pi DL$ , where  $D$  is the diameter of the wire and  $L$  the length, both reckoned in centimetres. Hence,

$$W=\pi DL.$$

Again, if  $R$  is the resistance of the wire in ohms and  $V$  the potential difference of its ends in volts, then the power wasted in it by a current  $A$  is  $RA^2=V^2/R=W$ , and the resistance  $R=4\rho L/\pi D^2$ . Hence

$$W=\frac{V^2\pi D^2}{4\rho L},$$

and, therefore, equating these values of  $W$ , we have

$$V^2D=4\rho L^2.$$

Next, suppose we divide up the total length  $L$  into  $N$  equal parts, and arrange these in parallel, but not touching; we have then a total resistance equal to  $4\rho L/\pi D^2N^2$ , where  $L$ , as before, is the total length of the  $N$  wires. Hence, if  $V$  is the potential difference (P.D.) at the ends of these  $N$  wires in parallel and  $W$  is the power wasted in them, then

$$W=\frac{V^2\pi D^2N^2}{4\rho L},$$

and hence, since  $W=S=\pi DL$ , we have

$$N^2V^2D=4\rho L^2.$$

Now  $V$  is a fixed quantity, being the E.M.F. of the dynamo tested, and  $D$  may also be considered to be fixed, as it is the economical diameter of the wire. Therefore, also,  $L$  is a fixed quantity, because, numerically speaking, we have  $\pi DL=S=W$  and  $W$  is the known and fixed power to be taken up. Accordingly, when we have to take up a stated amount of power  $W$

at a stated voltage,  $V$ , in  $N$  wires of a material of which the resistivity is  $\rho$ , then the value  $N^2/\rho$  is also a constant or fixed quantity; therefore

$$N \propto \sqrt{\rho}.$$

For example, suppose we have two materials—iron and platinoid—for which the resistivities in C.G.S. units are 10,000 and 40,000 respectively. Then the square roots of these are in the ratio of 1 : 2. Hence, to take up and dissipate any given amount of electrical power by resistance wires made of these materials of the same diameter, we need the same total length of wire in each case, but there will be twice as many strands in parallel in the case of the platinoid resistance.

An illustration may make this clearer. Let us suppose we wish to take up 100 watts at 70 volts, and that we have settled that a No. 36 S.W.G. wire ( $=0.02$ cm. in diameter) is the economical size. We must then, in either case, employ a wire having a surface of 100 square cm. This means that a length of 1,590cm. altogether will be required, since  $\pi \times (0.02) \times 1,590 = 100$  nearly.

Now, platinoid wire No. 36 S.W.G. has a resistance of 1 ohm per 7.5cm. length, and an iron wire of the same size has a resistance of 1 ohm per 30 cm. Hence, if we use the 1,590cm. of iron wire in *one* length, we have a resistance of 53 ohms, and a voltage of 70 volts on the ends of this will dissipate 4,900/53, which is not far from 100 watts. In the case of the platinoid wire we must arrange the wire in *two* parallel strands of equal length, and its total resistance will then be half of 798/7.5, or 53 ohms, and the power wasted by a P.D. of 70 volts the same as in the iron wire. Accordingly, the number of strands varies as the square root of the resistivity.

In testing alternating-current machines we have to provide arrangements for non-inductive and also for inductive loads. In the first case, for small powers, wire resistance may be employed as for continuous-current machines, but in testing large alternators it is now always the custom to employ water resistance as a non-inductive load. The arrangements which are most convenient are as follows:—A number of wooden beams forming a frame have porcelain insulators fastened to them, and from these porcelain insulators iron plates are suspended. These iron plates may be about 6in. or more apart, according to the voltage of the machine to be tested. Alternate plates are then connected together by a well-insulated cable, and a pair of well-insulated leads run from these two sets of plates to the alternator to be tested. The wooden frame carrying the plates must be suspended from insulated pulleys so that it can be lowered more or less into

a tank or pond, or a number of insulated butts of water. In the case of large powers, it is convenient to arrange a flow of water round the plates, so that the hot water is continually being removed and replaced by cool water. In testing high tension alternators from 2,000 to 10,000 volts, great care must be taken in insulating both the plates and the pulleys and ropes by which they are lowered, so that the attendants who manipulate them may not receive shocks. The power absorption is then tested by lowering the plates more or less into the water, and by doing this gradually the load can be very accurately adjusted. Such a water resistance is perfectly non-inductive, and, accordingly, when tests have to be made on alternators with inductive load the water resistance must be supplemented by a special form of adjustable choking coil.

The construction of a suitable inductive resistance may be carried out as follows:—A number of circular coils of insulated wire are prepared which may be 8in. internal diameter and 10in. external diameter. These coils should be formed of double cotton-covered copper wire well shellaced and bound together in the manner of transformer coils. The size of wire which is employed will depend upon the output of the machines to be tested. These coils should be arranged one above the other round a fibre tube placed in a vertical position, and through which a good draught of air can be sent. Through the tube is passed a number of bands of well-annealed soft sheet iron of the kind used in the manufacture of transformer coils. These strips of iron should be long enough to pass up the tube, and, when bent over, to lap over each other on the outside. Two sets of bands should be employed and bent over in the manner of an old-form Ferranti transformer (*see* Fig. 3). These bands are held together so as to complete the magnetic circuit by an appropriate brass clamp. By adding or removing bands, and by adding or removing coils as required, it is possible to vary the inductance of the wire circuit, the

coils being joined up in series or parallel as required, over wide limits, and in this manner an inductive resistance can be made which, when added in series with the water resistance, enables an inductive power-absorbing resistance to be constructed with any assigned power-factor within certain limits. If the inductance coil has negligible resistance, then, when used in series with a water resistance, the power-factor of the arrangement is measured by the ratio of the drop in voltage down the water resistance to the total voltage drop when any current of the right frequency is flowing through them.

In testing alternators destined to be used for the supply of power, it is essential to know the behaviour of the

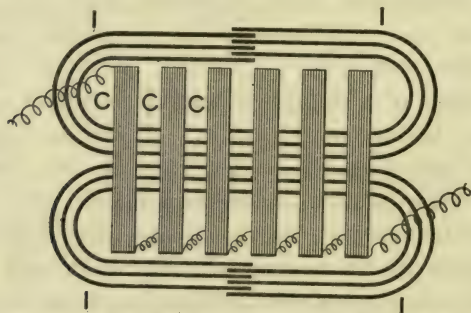


FIG. 3.—An Inductance Coil.

alternator on an inductive load having a power-factor varying between 0.75 and unity, and it is therefore necessary to be able to build up an inductive resistance, as above described, on which the machines can be tested.

In constructing an inductive wire resistance as described above, the ohmic resistance should be kept as small as possible, so that the heat production in the coils of wire may be kept down. The removal of the heat unavoidably generated can be facilitated by placing under the coils a blower which supplies a current of cool air. In the absence of a special inductive resistance of the above kind, the necessary inductance can be obtained by placing in series with the alternator a number of transformers arranged in

parallel, the secondary circuits of which are more or less closed by water resistance formed of plates of metal immersed in water standing in insulated tubs. The power-factor of an ordinary closed iron circuit transformer, when on open secondary circuit, is about 0.75, but it can be increased up to unity by closing the secondary circuit. Hence, by an appropriate number of transformers arranged with the primaries in parallel, the secondary circuits of some being open and others being short-circuited by water resistances, it is possible to construct an inductive power-absorbing resistance having any power-factor required between 0.75 and unity. Whatever arrangement is adopted

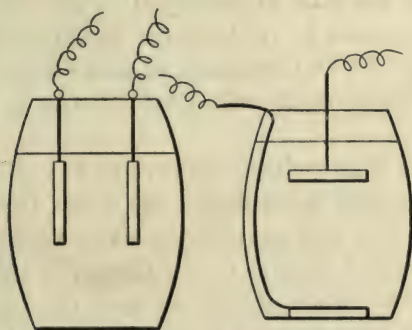


FIG. 4.—Forms of Water Rheostats.

for the power absorption of a dynamo under test it should be capable of being varied in amount, and its resistance and inductance adjusted, without stopping the machine under test, as in some cases the test would be vitiated as regards the temperature determinations if such stoppage took place.

In the construction of water resistances for testing small alternators of not very high voltage, a single insulated tub of water may be employed in which two lead plates are placed. These plates may be either vertical or horizontal (see Fig. 4). For voltages between 200 and 500 a single tub may be employed with two plates in it; for voltages between 500 and 1,000 two such tubs may be joined in series. For

large currents it is best to regulate by immersing more or less of the electrodes in the water, and hence the vertical arrangement of the plates is then most convenient. But for the exact regulation of smaller currents the horizontal arrangement of the plates is preferred. As a guide to the power to be taken up, it may be noted that an ordinary beer-barrel filled with water, having in it two lead plates about 6in. apart, can be employed to take up about 20kw. As a rough guide to the resistance of such a water resistance it may be noted that 1 cubic ft. of ordinary fresh water has a resistance of 100 ohms between opposed faces; but if 1 per cent. of sulphuric acid is added to the water the resistance will fall to about 0.5 or 0.75 of an ohm per cubic foot. If the voltage of the machine to be tested is less than 1,000, then it may be necessary to increase the conductivity of the water by adding to it a little sulphate of soda; but if voltages from 1,000 to 5,000 are dealt with, then it is best to use pure water alone.

If the power of high voltage alternators has to be taken up, then it is necessary to circulate the water through the tank or barrel, and for that purpose the water must be led in by means of a rubber hose-pipe to the bottom of the tank or barrel, and the warm water drawn off near the top surface by another rubber tube acting as a syphon. If these tubes are 10ft. or 12ft. long, and not more than 1in. internal diameter, there will be no danger of shock being communicated through the column of water.

As an instance of what can be done in the taking-up of large powers with a water resistance the following illustration may be given. In some tests of a three-phase 450kw. 500-volt plant three water-tanks were employed. The tanks were made of hard wood dovetailed together, the size of each being 4ft. by 4ft. by 6ft. Two sheet-iron plates were placed in each of such a size as to give 5in. clearance all round when in the tank. One of these plates in each tank were joined together, and the other three plates formed the terminals to which the leads of the three-phase machine were attached. Each tank could absorb 150kw. A small quantity of sulphate of soda was put in each tank, which enabled the water rheostat to carry 500 H.P. for three hours. In another instance a similar wooden tank, 8ft. by 3ft. by 4ft., was mounted on

an insulated platform, the electrode consisting of two sheets of steel  $\frac{3}{8}$ in. thick of such a size as to give a clearance of 1in. on all sides. The current was supplied at a pressure of 2,300 volts. The tank could absorb 250kw. The water was continually flowing through rubber pipes as above described.\*

Another form of power-absorbing resistance can be made with electrodes of carbon placed in water. Two plates of battery plate carbon 0.5in. thick and 10in. square are perforated with 40 or 50 holes to facilitate the circulation of water. If two such plates are placed in a long wooden insulated box through which water flows it will be found that, with a pressure of 2,300 volts, a separation of 10in. between the carbon plates permits approximately a current of 20 amperes, or takes up nearly 50kw., whilst double this distance reduces the current 10 amperes. One advantage of a water resistance made in any of the forms described is that it constitutes a perfectly non-inductive resistance, at any rate for E.M.F.s of anything like 1,000 or 2,000 volts, and hence the power absorbed in the water is exactly measured by the product of the readings of an ammeter and voltmeter giving respectively the R.M.S. values of the current and the potential difference between the plates. Hence, wherever an inductive resistance is required for alternator testing it can be obtained by adding together in series an inductionless water resistance and a highly inductive coil, having a small ohmic resistance but a sufficient current-carrying capacity.

**§ 6. Differential Methods for Testing Shunt-wound Continuous-current Dynamos and Motors.**—In the case of dynamos of large power the expense of making efficiency tests when the power is wholly dissipated in resistance is very considerable, as, generally speaking, a full load test for six hours is required prior to the delivery to the purchaser of such machines. The expenditure of steam during the run is at least 120lbs. per kilowatt output of the machine tested. When making tests of alternating-current machines the employment of a water

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\* See *The Electrician*, Vol. XL., p. 696, "Water Rheostats."

resistance enables a considerable power to be taken up without any costly arrangements, but in the case of continuous-current machines, unless they are conducted in a dynamo factory where such tests are habitually being made, and where all the power-absorbing resistances are already installed, the provision of the necessary resistance generally involves a not inconsiderable outlay. Furthermore, it has already been pointed out that the direct measurement of the power supplied to a single dynamo, when it has to be tested for commercial efficiency apart from an engine, is not one which can easily be made with accuracy. Methods have, therefore, been devised by which two dynamos can be tested together, or one dynamo tested by the aid of others, which are called differential methods, and in which the actual expenditure of power is not that transformed by the dynamo, but only equal to that which is dissipated in the dynamo or dynamos by electrical and magnetic sources of waste.

The first suggestion of a method of this kind was made by Drs. J. and E. Hopkinson in a Paper on "Dynamo Electric Machinery."\* (See *Phil. Trans. Roy. Soc.*, Vol. II., p. 347, May, 1886.) These authors proposed in 1886 to test two identical machines for efficiency by coupling together their shafts in one line and driving the combined shaft by means of a belt working on a pulley attached to it, the belt passing through a transmission dynamometer of the Hefner-Alteneck form, and the power being supplied from any steam engine or prime motor. One of the machines was to act as a dynamo and the current from it was put back into the other machine, which was to operate as a motor, the field of the motor being weakened by appropriate resistances, so as to make the current pass from the armature of one machine through that of the other in the right direction. By a proper adjustment of the fields of the two machines it is then possible to make a current circulate through the two armatures

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\* See "Original Papers on Dynamo Machinery," by Dr. J. Hopkinson, F.R.S., p. 112.

equal to the full load current of either machine. When this is the case the external motor has only to furnish power to the shaft sufficient to overcome the internal losses in the two machines. This power is measured by the dynamometer, and half the reading of the dynamometer is taken to correspond to the internal loss in each machine. The output of the dynamo can be determined as usual by measuring the current with an amperemeter placed in the armature circuit and a voltmeter placed across its terminals. Hence the efficiency of the machine can be determined, as already explained, from its output and loss. The advantage of this method is that a very large percentage of error, say even 10 per cent., in the reading of the dynamometer does not affect to anything like the same extent, perhaps only to the extent of 1 or 2 per cent., the resulting measured efficiency of the dynamo or motor.

Drs. J. and E. Hopkinson put their method into practice in 1886 at the works of Messrs. Mather and Platt. Shortly afterwards Lord Rayleigh suggested\* that it would be possible to supply the difference of the power electrically by means of a secondary battery, and so make the whole of the measurements electrical.

Major P. Cardew† then pointed out that he had been for some time previously, in fact since 1882, employing a method of dynamo testing in which all the measurements were electrical. Cardew's method consisted in coupling together by a belt the pulleys of two identical machines. One of these was employed as a dynamo and the other as a motor. The motor had electrical power given it from a third larger engine and dynamo, and this electric power was measured by an ammeter and a voltmeter as usual. The output of the dynamo was also measured electrically, and the square root of the ratio of the power given out by the dynamo to that taken in by the motor was taken as the efficiency of either

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\* *Electrical Review*, Vol. XVIII., p. 242, 1886.

† *The Electrician*, Vol. XVII., p. 410, 1886 ; also Vol. XXI., p. 273, 1887 ; also *Electrical Review*, Vol. XVIII., p. 242, 1886 ; also Vol. XIX., p. 464, 1886.

machine. Although this method had the advantage that all the measurements were electrical it was not a differential method, since the power passing through the dynamo was all dissipated on a resistance.

Messrs. Trotter and Ravenshaw described, however, a method in which three dynamos were employed which was a true differential method.\* One of these machines was the machine the efficiency of which was required (A), the other two machines, (B) and (C), were machines of about equal output but not necessarily of a high efficiency. These machines were joined up with their armatures in series. One of the machines, called the third machine, was driven by a motor as a dynamo, and the other two machines had their shafts coupled together or their pulleys united by a belt. The machines were joined up electrically so that one of them, call it the dynamo, produced a current which passed through the armature of the other machine, call it the motor, and the E.M.F. of the third machine was so arranged in direction as to assist the current coming out of the dynamo to flow through the motor armature. The voltage at the terminals of each machine was then measured, and the ratio of the voltage at the terminals of the dynamo to that of the terminals of the motor may be called the combined efficiency of the dynamo-motor combination. Each machine in turn was made the third machine, the dynamo and the motor.

Then let  $E_1, E_2, E_3$  be the combined efficiencies in the three sets of experiments, and let  $e_1, e_2, e_3$  be the individual efficiencies of each machine. If, then, they are tested, coupled pair and pair, the combined  $E_1, E_2, E_3$  efficiencies are related to the individual efficiencies, as follows :—

$$E_1 = e_2 e_1, E_2 = e_2 e_3, \text{ and } E_3 = e_1 e_3.$$

Hence, 
$$e_1 = \sqrt{\frac{E_1 E_3}{E_2}},$$

and similar expressions for  $e_2$  and  $e_3$ .

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\* See *Electrical Review*, Vol. XIX., pp. 424 and 437, 1886.

The following figures give the results of a test made in this manner, published by Mr. A. P. Trotter.\* The three machines are denoted by A, B and C :—

MOTOR.			DYNAMO.			
Volts.	Amps.	Watts.	Volts.	Amps.	Watts.	Efficiency in per cent.
<b>B</b>			<b>A</b>			
125	64	8,000	66	56	3,700	46·2
120	67	8,050	54	68	3,680	45·7
<b>C</b>			<b>B</b>			
95	53	5,030	60	37	2,440	48·4
93	51	4,750	63	36	2,260	47·5
<b>C</b>			<b>A</b>			
118	68	8,000	89	56	5,000	62·5
120	70	8,400	91	57	5,200	62·0
<b>A</b>			<b>C</b>			
125	65	8,100	81	63	5,100	63·0
125	66	8,250	70	74	5,180	62·8
<b>A</b>			<b>B</b>			
111	62	6,900	66	62	4,080	59·0
108	58	6,380	66	53	3,500	55·0
104	56	5,800	64	52	3,330	57·5

Then, 
$$e_A = \sqrt{\frac{62.5 \times 57.1}{46.8}} = 87.3 \text{ per cent.}$$

Another mode of differential testing was described by Mr. G. Kapp in 1892.† Mr. Kapp's method consisted in coupling together the shafts of two machines, one of which was to act as a dynamo and the other as a motor. The current from the dynamo was put back into the armature of the motor and the current from another small machine was added to it, the small machine being joined in parallel with the motor and dynamo. The field of the motor was then weakened to make it take up the current from the dynamo and make it drive the combination. This method may be described as a parallel differential test, whereas the method

\* "The Commercial Efficiency of Dynamo Machines," *Electrical Review*, Vol. XIX., p. 424, 1886.

† See "The Determination of the Efficiency of Dynamos," *Electrical Engineer*, Vol. IX., p. 87, 1892.

originally suggested by Lord Rayleigh may be described as a series differential method of testing.

These differential methods of testing have also been applied in the case of alternators, motors and transformers, as explained in subsequent sections.

The methods of testing in which the whole of the measurements are electrical are so superior to the original Hopkinson method, in which the difference of power was supplied and measured mechanically, that the latter is now never used. The modern methods of dynamo testing involve only electrical measurements, which can be made with great accuracy.

**§ 7. The Differential Measurement of the Efficiency of Continuous-current Shunt-wound Machines by the Series Method.**—The efficiency test of a pair of identical machines with simple shunt fields can be conducted as follows:—The method involves the use of two dynamos of equal size and construction, and a third dynamo the armature of which has an equal current-carrying capacity to that of the other two. This last need not be of the same power, as it has only to supply the internal losses in the other two machines. In place of a third dynamo a secondary battery can be employed, provided that the number of cells can be varied and that the current which can be passed through the battery without damaging it is equal to the full load current of the machines to be tested. The first procedure is to bed down the two machines on a temporary bedplate and couple their shafts in one line. It is necessary to employ a flexible coupling in order to avoid the necessity for setting the shafts in one line with great accuracy. An effective form of flexible coupling may be made as follows:—The machines should be set down on a stout timber foundation, with their pulleys facing one another and the shafts as nearly as possible in one line. Round the adjacent edge of both of the pulleys must then be drilled half a dozen small holes to take  $\frac{1}{4}$  in. bolts, these holes being made through the flange of the pulley on the outside. A thick

piece of leather, which may be part of an old wide belt and should be about  $\frac{1}{4}$  in. in thickness, is then cut of a length sufficient to wrap completely round both of the pulleys, which must, of course, be of the same size. A series of holes are drilled in the edge of this piece of leather to match the holes in the flanges of the pulleys. The leather is bent round the pulleys and fastened to them by small bolts in such fashion that the leather forms a tube, which is secured at one end to one pulley flange and at the other end to the other. This forms a very effective arrangement for transmitting the torque of one machine to the other, yet at the same time it is slightly flexible and avoids the necessity for setting the shafts with extreme accuracy.

The machines having been so set that, when one of them is run as a motor, it will drive the other as a dynamo, it will then be found that, if the machines are identical in every way, the brushes of one machine need to be reversed, so that when the two armatures turn together in the same direction, the commutators of the two machines being on the outside, the commutators of each will run under their own brushes in the proper direction. When this arrangement has been made, one machine should be run as a motor, to ascertain if it turns the other machine round properly and causes it to give current as a dynamo.

If the arrangement is satisfactory, then the test can proceed as follows:—The two armatures are coupled in series (*see* Fig. 5) by means of pieces of thick cable capable of carrying the full load current of each machine, and an ammeter is inserted to give the value of this current. If the ammeter is a magnetic instrument, it should be placed far enough away from the fields of the machines not to be affected thereby. In series with the two armatures is then inserted either the armature of the third dynamo or the secondary battery, and the connections must be so made that the current flows through one machine and drives it as a motor, and that the current from the coupled machine

acting as a dynamo passes back through the battery and through the armature of the motor in the same direction. To make the coupled machines rotate it is necessary to regulate the fields of both machines, and for this purpose it is better to excite the fields either off the armature of one machine or else separately off the terminals of the secondary battery or of the third dynamo. In Fig. 5 the fields of both machines are shown as excited off the auxiliary battery. If, however, the battery or third dynamo at disposal has not sufficient voltage for this pur-

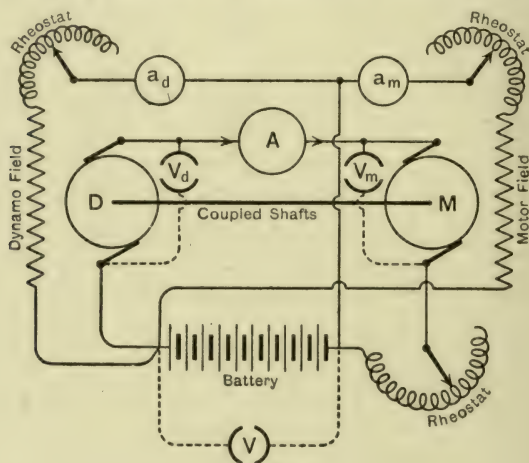


FIG. 5.

pose, then the fields of both machines must be excited from the armature terminals of that one which acts as a dynamo in the test. Adjustable regulating resistances must then be inserted in these field-circuits, so as to regulate the field-currents. The flow of current through the armature circuit is regulated and adjusted by means of these field rheostats. If the machines coupled together are identical, then, by inserting the resistance in the field circuit of the machine which acts as a motor, its E.M.F. can be regulated so that it takes up current from the other machine which acts as a dynamo. By regulating these field currents the armature current can be

controlled so as to be equal to the full load current of either machine, or to any decimal fraction of it. We can thus adjust the arrangement so that the coupled armatures revolve, and any required current passes through them. When this is the case, the battery or the third dynamo must be supplying the total internal losses of the two machines and also the field excitation, or not, according to the manner in which the circuits are coupled.

The assumptions made in working out the results of the test are that, after deducting the power taken up to excite the fields and heat the armatures of both machines, the residue of that supplied by the external source is equally divided between the dynamo and the motor—in other words, we assume that the losses in these two machines, due to hysteresis, eddy currents and friction, are the same. This assumption is not absolutely true, either in practice or under the conditions of the test, but it is so nearly true for machines of good efficiency, if identical in every way, that, for all practical purposes, it enables us to calculate very easily the efficiency of either machine without making any other measurements than electrical ones. The measurable electrical quantities are denoted by the following symbols:—

$A$  = Armature current.

$W_D$  = Dynamo output in watts.

$W_M$  = Motor intake in watts.

$V_d$  = Volts at the brushes of the dynamo.

$V_m$  = " " " " " motor.

$a_d$  = The field current of the dynamo.

$a_m$  = " " " " " motor.

$V$  = The P.D. of the battery terminals or the brushes of the third machine.

$R_D$  = The resistance of the dynamo armature.

$R_M$  = " " " " " motor armature.

$r_d$  = Resistance of the field circuit of the dynamo.

$r_m$  = " " " " " " " motor.

$L$  = The loss in each machine due to hysteresis, eddy currents and friction.

Then we have the following theory of the arrangement:—If we assume the fields of both machines to be excited from a separate source, then the power which has to be supplied from the battery or third dynamo or external source to the combination is evidently equal to the difference between that given out by the dynamo and that taken in by the motor, or to  $W_D - W_M$ . We shall denote this difference by  $X$ . Then the assumption made is that the amount by which  $X$  exceeds the copper or resistance losses in the armatures of the dynamo and motor is equally divided between the two machines. This is equivalent to the assumption that the sum of the frictional loss ( $K$ ), the hysteresis loss ( $H$ ) and eddy current loss ( $F$ ) is the same in each machine.

Let  $L$  denote the sum of these losses in each machine. Then

$$L = K + H + F.$$

Hence

$$L = \frac{1}{2}X - A^2R_D - A^2R_M.$$

Then the commercial efficiency of the dynamo ( $e_d$ ), if a shunt machine, is given by

$$e_d = \frac{W_D}{W_D + A^2R_D + a_d^2r_d + L}, \quad \dots \dots \dots (1)$$

and that of the motor by

$$e_m = \frac{W_M - A^2R_M - L}{W_M + a_m^2r_m}. \quad \dots \dots \dots (2)$$

Instead of assuming the frictional, hysteresis and eddy current core losses to be identical in the two machines we might assume the efficiencies to be the same. In that case we should have  $e_m = e_d = e$ , and

$$e = \frac{W_D}{P + w_d} = \frac{P}{W_M + w_m},$$

where  $w_d$  is the exciting power in watts given to the field of the dynamo,  $w_m$  that given to the motor field, and  $P$  the power transmitted through the mechanical coupling. Then, eliminating  $P$ , we have

$$e^2(W_M + w_m) + ew_d - W_D = 0.$$

or

$$e = \frac{-w_d + \sqrt{w_d^2 + 4W_D(W_M + w_m)^2}}{2(W_M + w_m)}. \quad \dots \dots \dots (3)$$

This last formula is troublesome to use in calculations, and the former assumption is the one generally made in practice. The above formulæ need slight modification if the fields of both machines are supplied from the armature of one of them, but there is no difficulty in constructing the right expressions by the rule that efficiency is *output/intake*, and the assumption that the iron and frictional losses are the same in each machine.

The power  $X$ , reckoned in watts, supplied from the battery or third dynamo is obtained at once by taking the product of its terminal voltage and the current flowing out of it. A

this current is the same in each armature circuit, and is denoted by  $A$ , we have for the value of  $L$  to be inserted in the equations the expression

$$L = \frac{1}{2}(AV - A^2R_D - A^2R_M), \quad . \quad . \quad . \quad (4)$$

where  $V$  is the terminal P.D. of the battery or dynamo used as the third machine for supplying the loss in the two machines.

### § 8. The Differential Measurement of the Efficiency of Continuous-current Shunt-wound Machines by the Parallel Method. Kapp's Method.

—A method of differential testing devised by Mr. G. Kapp has some advantage over the series method described in the previous section. In Mr. Kapp's method, in addition to the two identical machines to be tested as dynamo and motor, the only other requirement is a small machine or a battery having the same or, rather, greater E.M.F. than the machines to be tested, and which is capable of supplying power equal to the total losses in these two machines. In this method, the two machines to be tested are put down on a temporary bedplate and coupled together, as already described in the previous section. The third machine or battery has its terminals joined to the terminals of the machine under test, which is to act as a motor. This third machine, therefore, supplements the current of the dynamo, and so supplies the motor with the current which is required to enable it to drive the other coupled machine as a dynamo, whereas in the previous test the third machine supplements or raises the voltage. Since the third machine is coupled in parallel with the armatures of the two machines under test, this arrangement is called the *parallel method* of differential testing. Rheostats must be provided to act as regulating resistances in the fields of both the machines under test, and the third machine must be capable of being regulated as regards its E.M.F. so as to vary the current it is giving. The parallel method of differential testing has the great convenience that it can be carried out with current

which is drawn from the supply mains of a public electric lighting station. The connections are as shown in Fig. 6.

In both the series and the parallel method of differential testing instruments have to be provided for measuring the current passing through the armature of both machines under test, and also for measuring the voltage at the terminals of both machines and the field current of both machines. These electrical measurements may, of course, be carried out with electrical instruments, such as Weston ammeters and voltmeters, which are properly standardised, or may be compared with standard instruments before and

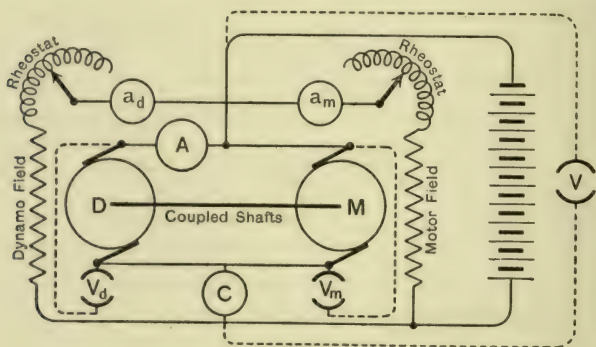


FIG. 6.

after the test. But it is very much better in both cases to employ the potentiometer method of measurement directly, as it enables all these currents and voltages to be measured with great accuracy and very quickly, and refers the whole of them to the E.M.F. of a Clark cell and the value of a standard resistance. For this purpose, resistance strips must be provided which have appropriate current-carrying capacities, of the kind shown in Vol. I. of this HANDBOOK, p. 50, a low resistance strip being placed in the circuit of the armatures having a resistance of, say, 0.01 or 0.001 ohm, according to the current output of the dynamo under test. Thus, for instance, if the full load current of the machine

under test does not exceed 100 amperes, then it will be best to employ a low resistance of  $\frac{1}{1000}$ th ohm; but if it exceeds 100 amperes then a low resistance strip of  $\frac{1}{10000}$ th ohm should be employed. In the same way, appropriate resistance strips should be inserted in the field circuits, the resistance of the strip to be used being calculated from the rule that the drop in volts down the strip should be something between  $\frac{1}{2}$  volt and 1 volt. Divided resistances must then be provided for the measurement of the voltage. These resistances should consist of a high-resistance wire divided into two sections, in the ratio of 99 to 1, as described in section 5, Chapter IV., Vol. I. of this HANDBOOK. Three of these divided resistances will be required to be placed across the terminals of the motor, the dynamo, and the third machine respectively. From the terminals of these low-resistance strips, and from the terminals of the smaller sections of the divided resistance pairs of insulated wires, must then be taken to a potentiometer, which may be of the Crompton, Elliott or Nalder form, and the potentiometer apparatus, as described in Chapter IV. of Vol. I. of this HANDBOOK, should be set up so that an observer can read quickly the P.D. between the ends of these strips or between the ends of the similar sections of the divided resistances upon the potentiometer wire or scale. In the case of the parallel method of testing an additional low resistance strip will be required for measuring the current coming out of the third machine, or from the supply mains. These arrangements having been completed and the test started, the observers are able to read quickly on the potentiometer the value of the armature current and of the field currents and the several P.D.'s as required, and to take a number of observations corresponding to one another in which the armature current of the two coupled machines is gradually reduced by steps from its full load value down to zero, preferably being taken at the decimal fractions—that is, of  $\frac{1}{10}$ th,  $\frac{2}{10}$ ths,  $\frac{3}{10}$ ths, &c., of full load up to full load.

The theory of the parallel method of differential testing may be stated as follows:—As in the series test, we make the assumption that the external source of power or third dynamo supplies the total losses in the two machines, and that the frictional, hysteresis and eddy current loss in the two machines are identical. Then, if, as before,  $W_M$  is the power given to the motor and  $W_D$  the power given out by the dynamo, then, as in the previous case, we have  $W_M - W_D = X$ , where  $X$  is the power supplied by the external source. Then also we have  $L = K + H + F$  as the expression for the frictional and core losses in each machine.

Let  $V$  be the terminal P.D. of the external supply or third dynamo, and  $C$  the current in amperes given out by it. Then, if  $A$  is the current coming out of the dynamo, the current going into the motor is  $A + C$  amperes. Hence we have

$$L = \frac{1}{2}(VC - A^2R_D - (A + C)^2R_M), \dots \dots \dots (5)$$

and if  $e_d$  is the efficiency of the dynamo and  $e_m$  that of the motor, then

$$e_d = \frac{W_D}{W_D + A^2R_D + a_d^2r_d + L}, \dots \dots \dots (6)$$

and

$$e_m = \frac{W_M - (A + C)^2R_M - L}{W_M + a_m^2r_m}.$$

The following figures give the results of a series and a parallel differential test on two enclosed or iron-clad 7 H.P. motors which were coupled together on the same bedplate, one of them being used as a dynamo and the other as a motor.

*Differential Test of a pair of Enclosed Shunt-wound Electric Motors 5kw. size, wound for 220 volts and 27 amperes output at 970 revolutions per minute.*

The machines were numbered No. 367 and No. 325. They were tested differentially by the parallel and by the series methods, each machine being in turn the dynamo and the motor. They were coupled together for the purposes of the test on one bedplate with shafts united. Calibrated Weston ammeters and voltmeters were employed for the readings of current and voltage. The resistances of armature and fields were taken and found as follows:—

No. 367.—Armature resistance = 0.271 ohm.

Field magnet coils = 231 ohms.

No. 325.—Armature resistance = 0.271 ohm.

Field magnet coils = 224 ohms.

The figures on pp. 532 and 533 give the results of the observations made in one parallel test and one series test, but it is not necessary to give the similar figures in the other pair of tests with the machines exchanged in function. The efficiencies are worked out from the figures of observation by means of the formulæ given on page 526 for the series test, and page 530 for the parallel test. The figures given are in every case the corrected currents and voltages.

The following is an extract from the report sent in with the results of this test:—

Two 7 H.P. motors, Nos. 325 and 367, constructed by Messrs. Blank & Co., Limited, were submitted for test as to efficiency, construction, power to bear overloading, general performance and value in comparison with other motors of the same horse-power. These two motors were identical, and constructed to work at 220 volts pressure, taking 27 amperes at 970 revs. per min., and denominated 7 H.P. motors. The two machines were bolted down on one frame, and were submitted to a differential test by two processes. In the first set of tests both the machines were placed in parallel with one another and with the external supply, and in the second set of tests the two machines were placed in series with one another and with the external supply. Each machine was used alternately as a motor and generator, and accordingly there were four complete sets of tests of efficiency. In every test the fields were excited separately at 220 volts, variations in load being effected by the use of a regulating resistance placed in one of the fields. The electrical measurements were all made with Weston standard ammeters and voltmeters, which were afterwards most carefully checked by the potentiometer against a standard Clark cell, which was in agreement with a Kelvin ampere-balance that had been checked by the Board of Trade; hence the electrical measurements may be relied on as being very accurate.

In reducing the observations one of two assumptions has to be made—either that the external power supplied to drive the combination is equally divided between the two machines, or else that their efficiencies as transforming devices at any one load are equal. These two assumptions do not lead to quite the same result, but yet, in the absence of any accurate transmission dynamometer, the only way to determine the efficiency of a machine is by making one or other of these assumptions; hence efficiencies of dynamo machines or motors cannot be calculated to a very small fraction per cent., although figures are sometimes given stating efficiencies to the second decimal place.

At full load the machine used as a motor has an efficiency just under 90 per cent.; at half load an efficiency of about 81 per cent.; and at quarter load an efficiency of close upon 70 per cent. These values are exceedingly good for machines of this calibre, and moreover, if the machine is overloaded by 20 per cent., the efficiency does not fall but rises slightly still more, so that at 20 per cent. over full load the efficiency is still just under

## Test I — BY PARALLEL METHOD.

No. 367 run as Dynamo. No. 325 run as Motor.

DYNAMO No. 367.

No. of Experiment.	Speed of Machine. R. P. M.	Voltage of Supply Circuit.	Armature Current in amperes.	Field Current in amperes.	Output in watts.	Watts lost in Armature.	Watts lost in Field.	Percentage load.	Efficiency.
1	1,050	230.8	0.47	1.044	108.5	0	251.8	2.08	16.12
2	1,040	231.7	1.59	1.034	368.4	0.7	246.9	7.05	40.07
3	1,050	230.8	3.32	1.024	766.1	3.0	242.3	14.7	58.09
4	1,072	231.2	8.66	1.017	2,003	20.3	238.8	38.35	76.73
5	1,116	225.1	12.11	0.984	2,726	39.7	223.6	52.5	82.00
6	1,062	228.9	13.97	1.009	3,197	53.1	235.1	61.2	84.06
7	1,078	226.0	16.83	1.011	3,804	79.4	236.0	72.8	85.94
8	1,108	226.0	18.81	0.964	4,252	95.8	214.5	81.4	86.71
9	1,092	222.3	23.04	0.974	5,121	143.7	219.0	98.0	88.10
10	1,100	223.5	25.31	0.946	5,656	173.6	206.7	108.3	88.82

## MOTOR No. 325.

No. of Experiment.	Speed of Machine. R. P. M.	Voltage of Supply Circuit.	Armature Current in amperes.	Field Current in amperes.	Output in watts.	Watts lost in Armature.	Watts lost in Field.	Percentage load.	Efficiency.
1	1,050	230.8	3.19	1.070	736.1	2.7	256.5	8.06	42.41
2	1,040	231.7	4.23	1.039	980	4.8	241.7	12.9	55.09
3	1,050	230.8	6.04	0.989	1,394	9.8	219.1	20.6	66.74
4	1,072	231.2	11.92	0.944	2,756	38.5	199.5	45.4	80.20
5	1,116	225.1	15.60	0.893	3,511	65.9	178.5	59.4	84.03
6	1,062	228.9	17.32	0.913	3,965	81.3	186.7	68.3	85.91
7	1,078	226.0	20.38	0.903	4,607	112.5	182.6	80.2	87.41
8	1,108	226.0	22.87	0.855	5,168	141.7	163.7	89.8	87.86
9	1,092	222.3	27.55	0.837	6,124	205.7	157.0	107.1	89.00
10	1,100	223.5	30.14	0.810	6,756	246.2	146.9	117.9	89.47

## Test II.—BY SERIES METHOD.

No. 325 run as Dynamo.

No. 367 run as Motor.

## DYNAMO No. 325.

No. of Experiment.	Speed of Machine. R.P.M.	Armature Current in Amperes.	Field Current in Amperes.	Terminal Voltage of Machine.	Added Voltage of External Supply.	Output in Watts.	Watts lost in Armature.	Watts lost in Field.	Percentage Load.	Efficiency.
1	985	6.25	0.465	134.35	87.65	839.8	10.6	49.9	16.0	72.17
2	980	8.35	0.551	150.65	71.8	1,258	18.7	70.1	24.1	77.28
3	984	11.85	0.646	168.55	55.0	1,967	38.0	96.4	38.2	82.54
4	986	15.00	0.696	175.75	47.35	2,636	61.0	111.8	50.5	84.95
5	960	18.70	0.739	175.90	42.0	3,289	94.8	126.1	63.0	86.37
6	1,044	22.85	0.772	181.30	39.2	4,143	141.5	137.6	79.3	87.61
7	991	26.72	0.798	185.90	37.0	4,967	193.5	147.1	95.1	88.56
8	1,004	30.22	0.810	187.55	36.85	5,667	247.5	151.5	108.5	88.88

## MOTOR No. 367.

No. of Experiment.	Speed of Machine. R.P.M.	Armature Current in Amperes.	Field Current in Amperes.	Terminal Voltage of Machine.	Added Voltage of External Supply.	Output in Watts.	Watts lost in Armature.	Watts lost in Field.	Percentage Load.	Efficiency.
1	985	6.25	1.074	222.3	87.65	1,390	10.6	257.6	21.2	67.73
2	980	8.35	1.074	222.3	71.8	1,857	18.7	257.6	29.8	73.63
3	984	11.85	1.059	220.4	55.0	2,612	38.0	251.1	43.8	79.85
4	986	15.00	1.029	222.7	47.35	3,340	61.0	237.3	57.1	83.44
5	960	18.70	1.017	216.7	42.0	4,042	94.8	231.6	70.1	85.42
6	1,044	22.85	1.004	220.1	39.2	5,029	141.5	225.7	87.7	87.18
7	991	26.72	.991	222.0	37.0	5,532	193.5	220.0	104.1	88.39
8	1,004	30.22	.982	223.5	36.85	6,754	247.5	216.0	118.7	88.91

90 per cent. The tables herewith give these observed and calculated efficiencies at the various fractions of full load when the machines are run respectively as motor and as dynamo.

In addition to the efficiency tests, a series of tests was made for temperature. The machines were run at full load, No. 367 running as motor and No. 325 as dynamo. The temperature of the room at the beginning of the test was  $17^{\circ}\text{C.}(=62.6^{\circ}\text{F.})$ . The full-load run lasted for  $4\frac{1}{2}$  hours, and the temperatures were taken immediately after stopping by noting the maximum rise of temperature of a thermometer inserted into the armature winding and that of a thermometer laid on the commutator, the bulb being covered with a plug of cotton waste.

The temperature of the dynamo armature at the end of the run was  $132.3^{\circ}\text{F.}$ , the rise of temperature above that of the room  $69.7^{\circ}\text{F.}$  The temperature of the commutator at the end of the run was  $125^{\circ}\text{F.}$ , the rise above the temperature of the room  $62.4^{\circ}\text{F.}$

The temperature of the motor armature at the end of the run was  $141^{\circ}\text{F.}$ , the rise of temperature above that of the room  $78.4^{\circ}\text{F.}$  The temperature of the motor commutator at the end of the run was  $131^{\circ}\text{F.}$ , the rise of temperature above that of the room  $68.4^{\circ}\text{F.}$  The mean efficiency of the motor during the run was 89.5 per cent. at 105.5 per cent. of full load. The mean efficiency of the dynamo during the run was 88.5 per cent. at 97 per cent. of full load.

The above figures show that in a  $4\frac{1}{2}$  hours' run at full load the machines do not increase in temperature above the limit which is usually assigned as a limiting temperature for open dynamos of large size running on a six hours' full-load test—viz.,  $70^{\circ}\text{F.}$  above the atmosphere. The machines, therefore, can safely be used on a six hours' full-load service without dangerous heating.

In the next place, a test was made for general behaviour at much overload. The machines were run and overloaded to the extent of nearly double the normal current—that is to say, about 50 amperes were allowed to circulate through the armatures, but no dangerous heating occurred during the half hour in which the test lasted, neither did the machines spark at the brushes. Several other tests of the same character were made at lesser currents, showing the capacity of the machine to bear overloading considerably for a short time without injury.

As regards general construction, the machines are substantially made and mechanically well designed; the lubricating arrangements are good and suitable for prolonged runs. The brushes are used in a fixed position independent of the load, and there is an almost entire absence of sparking. When running rather under full load it is impossible, by looking at the brushes, to tell whether current is passing through the armature. Electrically, therefore, they are well designed so that there is no shift of field or sparking when the load changes, and mechanically they are substantially constructed for hard and continuous wear. Taking all these facts into account, it may be said that the machines do their designer great credit, and they place the makers in possession of a motor which is not inferior to any of the same class now in the market.

**§ 9. The Use of a Tested Shunt-wound Continuous-current Dynamo or Motor as a means for measuring the Efficiency of another Machine.**—The reader should notice that the method described in § 7 and § 8 is only applicable to the efficiency testing of simple shunt-wound dynamos and motors. It is not applicable for testing a series-wound or compound-wound dynamo or motor.\* A shunt-wound motor which has had its efficiency taken by one of the previously described differential methods can, however, be employed to drive another machine, either a direct-current or an alternator, the efficiency of which is required. Thus, for instance, supposing that we have an alternator the efficiency of which we require to know, and we have a direct-current dynamo the efficiency of which, as a motor, has been already obtained at various fractions and at full load. This direct-current machine can be coupled by a flexible coupling to the shaft of the alternator and be employed to drive it. Having ascertained the efficiency of this machine as a motor, we know, therefore, the driving power upon its shaft when it is absorbing a certain measured electrical power; hence, if it is direct-coupled to an alternator, we know the power which is being supplied to the shaft of the alternator. We can then take up the electrical output of the alternator on water resistance or in any other way, and measure this output by means of a proper alternating-current ammeter and voltmeter. We have, therefore, the power applied to the shaft of the alternator and the electrical output, and hence the efficiency.

**§ 10. Efficiency Testing of Sets of Direct-coupled Engines and Dynamos.**—In the case of engines and dynamos coupled directly together on the same bedplate, the chief measurements which the engineer has to make in taking an efficiency test are the steam consumption of the engine, the indicated horse-power of the engine, and the electrical output of the

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\* For a method of finding the efficiency of series motors, see a Paper by Prof. E. Wilson, *The Electrician*, Vol. LI., p. 891.

dynamo. These figures enable the steam consumption of the engine per indicated horse-power to be stated, and also the steam consumption of the combined set per kilowatt output or per electrical horse-power, 1 E.H.P. being equal to 0·746 of a kilowatt.

In the case of dynamos coupled directly to steam turbines, or rotatory engines of any kind, of a class in which the steam indicator cannot be employed, then the only measurements which can be taken are the steam consumption and the electrical output. In most large dynamo factories or engine works, such as those of Messrs. Willans and Robinson at Rugby, and Messrs. C. A. Parsons & Co. at Newcastle-on-Tyne, the arrangements for measuring the steam consumption of the engines tested are very perfect. The engine and dynamo set to be tested is bedded down upon a temporary foundation, and the terminals of the dynamo or alternator are connected through proper measuring instruments with a power-absorbing resistance, either of water or wire, by means of which the electrical output of the dynamo can be determined. The steam consumption of the engine is best measured by weighing, or taking the weight of, the exhaust steam delivered in a known time when condensed in a surface condenser. The exhaust steam passes into a surface condenser, and the water formed is pumped into a tank having a known capacity, or else into a tank standing on a platform balance. In the first case the tank must be graduated so that the volume of the water formed from the condensed steam can be ascertained by inspection, and from this volume and the density of the water at its known temperature its weight can be ascertained; or else the weight is directly ascertained from the readings of the platform balance. In making a test of an engine-dynamo set it is usual to arrange for a six hours' run at full load without stopping, and during that time readings are taken every quarter of an hour of the output of the dynamo and the volume or weight of the condensed steam. In this manner the figures are provided for calculating the mean value of the pounds of steam

consumed per kilowatt-hour. The test is usually supplemented by another run of two hours, varying from three-quarters to quarter load, so as to give the values for the steam consumption at these various loads, and the values of the intermediate loads being then ascertained by setting off a curve the ordinates of which represent steam consumption and abscissæ the fractions of full load. Another run is then often taken at 25 or 33 $\frac{1}{3}$  per cent. overload.

If a surface condenser is not at hand, or water cannot be obtained for it, then the weight of steam used in any time can be ascertained roughly from the volume of the water pumped into the boiler measured by a feed-water meter, provided the boiler is supplying no other engine; or it may be obtained also somewhat roughly by condensing the steam in cold water in a tank standing on a platform balance, the weight gained by the tank in any given time being taken as the weight of the condensed steam passed into it at that time.

Both these measurements, however (the measurement of the feed water and the measurement of the steam condensed in water), are inferior in accuracy to the method employing a surface condenser. If the engine is a reciprocating engine, then at the same time that the steam consumption tests are taken the engine indicator diagrams can be taken and the pounds of steam per horse-power as well as per kilowatt output can be reckoned out. The indicator cards also furnish an indication of the general performance of the engine.\*

In specifications prepared by consulting engineers for direct-driven engine-dynamo sets it is usual to specify the pounds of steam which shall be taken per kilowatt at full load, half load and quarter load when the engine is running non-condensing, and also when it is running condensing, with a certain vacuum in the condenser, and the engine and dynamo builders are required to give guarantees for the

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\* For information on the steam engine indicator, the reader is referred to the special treatise on the "Steam Engine Indicator," by W. Worby Beaumont, published in "The Electrician" Series.

fulfilment of these undertakings. Hence, the steam test made as above described is a very important one, and no large engine-dynamo set should be purchased except from makers of high reputation, without distinct guarantees on this point, or without arrangements being made beforehand for conducting such a trial. As a minor matter it should be definitely stated in the specification which party, whether purchaser or manufacturer, shall bear the cost of making such trial both for steam consumption and for the expert assistance necessary in taking and superintending the processes of measurement.

§ 11. **Temperature Tests.**—It is usual to insert clauses in the specification of a dynamo defining the temperatures which certain portions of the machine shall not exceed at the end of a full load run as above described for a certain number of hours. In the Admiralty specifications for the supply of dynamo-electric machines it is required that, at the conclusion of a run of six hours at full load, no part of the machine shall, at the end of one minute after stopping, show a greater rise than  $30^{\circ}\text{F.}(=16.6^{\circ}\text{C.})$  above the surrounding air. This, however, is a rigorous test. It is quite sufficient for most purposes to specify  $60^{\circ}\text{F.}$  or  $70^{\circ}\text{F.}$  above the surrounding air as the temperature which shall not be exceeded by any part of the machine at the end of one minute after stopping the machine after an uninterrupted run of six hours at full load. It is also not unusual to specify the temperatures which shall not be exceeded at the end of a certain run on, say,  $33\frac{1}{2}$  per cent. overload. In the case of alternators the nature of this load should be clearly specified—that is to say, whether it is to be an inductive or non-inductive load, and, if inductive, what shall be the power-factor. Thus, for, instance, in the case of three large Parsons turbine sets, each of 300kw. size, tested by the Author in 1902, the specification provided that, at the end of an eight hours' run with a load having a power-factor 0.9, no electrical part of the machine was to be at a temperature of

more than  $40^{\circ}\text{C}$ . above the air, and after two hours' run at  $33\frac{1}{3}$  overload no part of the alternator was to rise more than  $55^{\circ}\text{C}$ . above the air, these temperatures being taken with a thermometer. As a matter of fact, no part of the alternator did rise more than  $20^{\circ}\text{C}$ . above the temperature of the atmosphere, and the part of the machine which became hottest was the field circuit of the exciter.

In a specification for a dynamo machine it is, therefore, quite reasonable to specify that no part of the dynamo, whether continuous current or alternator, shall, at the end of an uninterrupted full load run of six hours, be found to exceed the temperature of the air by more than  $40^{\circ}\text{C}$ . It is usual to take these temperature tests by means of a chemical mercury-in-glass thermometer graduated on the stem. This thermometer is laid upon the armature or part to be tested and the bulb is covered by a pad of cotton wool. Note is then taken of the highest temperature which the thermometer reads. It is very questionable, however, whether such a rough method gives the true temperature of the metal against which the thermometer is placed. The thermometer itself has a large capacity for heat; the material of which it is made—viz., glass—is a bad conductor of heat, and the heat, therefore, passes slowly into it. A better arrangement is to take the temperature with a platinum thermometer, consisting of a fine silk-covered wire of platinum which is attached to a piece of felt. To this platinum wire are attached conducting leads and a pair of compensating leads, and by means of a Wheatstone's bridge the resistance of the platinum can at any moment be ascertained. If the resistance at various known temperatures has been previously ascertained and set down on a chart, the resistance reading gives at once the temperature of the platinum wire. This platinum wire can be laid against the armature or the field magnet coils and be covered with a thick pad of wool; the wire takes up, therefore, very quickly the temperature of the part against which it is placed.

**§ 12. Efficiency Tests by the Direct Measurement of Lost Power.**—Mr. James Swinburne suggested in 1887\* that the efficiency of a dynamo machine could be measured by measuring the stray or internal losses directly, and then, on adding this loss to the known output of the machine, we obtain the total power taken in. In order to make these measurements electrically, he suggested the following procedure:—Suppose that the machine to be tested is a continuous-current shunt-wound dynamo, there are in it four sources of energy loss: (1) The losses due to the resistance of the armature and the field, (2) the magnetic hysteresis loss in the armature core, (3) the eddy current losses in the armature core, (4) the frictional loss at the bearings and brushes and that due to windage. These losses may be divided into two parts, which may be respectively called the resistance losses and the rotational losses, since these last are due to the rotation of the armature. The resistance losses can be determined at once when we know the resistance of the armature and the field and the currents which are passing through these circuits. The problem of the measurement of the efficiency of the machine therefore resolves itself into the experimental determination of the rotational losses. Mr. Swinburne proposed to effect this in the following manner:—The dynamo to be tested, which we shall assume is a shunt machine, has its field magnets separately excited so that the field magnets carry the same magnetic flux as at full load. The armature is then caused to rotate by passing into it a small current, either from another dynamo, or from a secondary battery, or from the circuit of a public supply station. This current is adjusted in strength until the machine rotates at its normal speed, and the field current is also regulated by external resistance until the volts on the brushes of the dynamo under test are equal to the total E.M.F. of the said dynamo when working in the ordinary manner. This latter figure is determined as follows:—Let us

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\* See *Electrical Review*, Vol. XXI., pp. 181 and 215.

suppose that  $A$  is the full-load normal-current of the dynamo,  $R_a$  the armature resistance,  $R_f$  the field magnet resistance, and  $V$  the voltage at the brushes of the dynamo when working as a dynamo in the ordinary manner. Then the internal electromotive force  $E$  in the armature is given by the equation

$$V + (A + a)R_a = E,$$

where  $A$  is the current in the external circuit and  $a$  is the current in the field coils. If, then, the source of current is regulated so as to give a terminal voltage equal to  $E$  on the dynamo under test, and if the current which then flows into the dynamo armature is represented by  $a'$ , the power supply to the dynamo is  $Ea'$ , and this is the value of the rotational losses in watts. Accordingly, the whole of the internal losses of the dynamo when working in the normal manner must be equal to

$$A^2R_a + a^2R_f + Ea'.$$

If the normal full load voltage of the machine when running as a dynamo is  $V$ , the commercial efficiency of the dynamo at full load ( $e$ ) is given by the expression

$$e = \frac{AV}{AV + A^2R_a + a^2R_f + Ea'}$$

The values of  $V$  and  $A$  can be obtained from the maker or are marked on the machine, and the values  $R_a$  and  $R_f$  can easily be measured. Hence, also, we know or can find the value of  $a$ , for in a shunt machine  $a = V/R_f$ . If, then,  $a'$  is measured, we have all the quantities required to determine  $e$ .

In the above method of measuring the efficiency of a dynamo, the assumption is made that the iron losses in the armature due to the magnetic hysteresis and the eddy current loss in the core plates are the same under the circumstances of the test as when the machine is working as a dynamo or motor in the ordinary manner. This assumption, although not strictly true, does not lead to any very great errors, but a

modification of the process has been devised by M. Routin in which the proper correction is made for this source of error.\*

**§ 13. Routin's Method of the Determination of Dynamo Efficiencies.**—In those cases in which one large dynamo machine is submitted for test, and where the duplicate machine does not exist or is not available for testing by the differential method, the plan proposed by M. Routin for the direct determination of the internal losses may be employed with advantage. For the sake of example, let it be supposed that the machine to be tested is a direct-current shunt-wound dynamo. Let  $W$  represent the full load normal output of the machine, and let  $P_1$  represent the power absorbed in field excitation,  $P_2$  the power losses in the copper circuits of the armature, and  $P_3$  the power losses in hysteresis in the armature core, eddy currents in the core and friction at the bearings, and windage. Having measured the resistance of the armature and of the field circuit, and corrected these values to bring them to the value which they would have at the limiting temperature when the dynamo is working, and, furthermore, knowing the exciting current ( $a'$ ), the full load current ( $A$ ), and the voltage ( $V$ ) at the terminals of the brushes, we have all the means for calculating the values of  $P_1$  and  $P_2$ , for  $P_1 = V^2/R_f$ , where  $R_f$  is the hot resistance of the exciting coils of the field magnets, and  $P_2 = (A + a)^2 R_a$ , where  $a$  is the field current and  $A$  is the current of the external circuit, and  $R_a$  is the hot resistance of the armature circuit. The efficiency of the machine ( $e$ ) is given by the fraction

$$e = \frac{W}{W + P_1 + P_2 + P_3}.$$

Hence the problem of determining the efficiency resolves it into the determination of the internal loss  $P_3$ . Since these losses are due to the rotation of the machine, they may be

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\* See the Appendix to the French translation of Dr. J. A. Fleming's "Electrical Laboratory Notes and Forms," by J. L. Routin, p. 141 "Méthode Routin pour la détermination des rendements."

called, as above, the *rotational* losses, and  $P_3$  may be further divided into two parts, one which we shall represent by  $p$ , which is the loss due to mechanical friction, and another part represented by  $q$ , which is the loss due to magnetic hysteresis and eddy currents. M. Routin has suggested the following method by which the values of  $p$  and  $q$  can be separately determined:—

First, as regards  $p$ . Suppose the machine is made to act as a motor, and is set in revolution at a higher speed than its normal speed,  $N$ , when running as a dynamo. If then the current is switched off from both the armature and field magnets quickly, and the machine is allowed to run on in virtue of the momentum of the armature, the speed will gradually die down owing to the frictional resistance. If, therefore, by means of a tachometer, we take observations of the rate at which the speed dies away, taking observations say every four or five seconds, we can plot a curve representing the variation of speed with time as the momentum of the armature decreases. At every instant the work absorbed by friction and windage is equal to the diminution of the angular energy of the armature. Suppose  $I$  represents the moment of inertia of the armature, and  $\omega$  the angular velocity of the armature at any instant, then the energy stored up in the armature at any moment is measured by  $\frac{1}{2}I\omega^2$ , and the power  $p$  taken up by friction must therefore be equal to the time rate of change of this angular energy. In other words,

$$p = \frac{d}{dt}(\frac{1}{2}I\omega^2) = I\omega \frac{d\omega}{dt}.$$

The direct determination of  $I$  is difficult, but we can determine  $p$  in the following manner:—Supposing we have determined the variation of speed with time, as shown by the curve in Fig. 7, where the ordinates represent angular velocity and the abscissæ time. Let the curve  $C_1$  be drawn to represent the results of observation on the decrease of angular velocity with time. Draw a horizontal (dotted) line through the

diagram having an ordinate equal to the normal angular velocity ( $\Omega$ ) of the machine when acting as a dynamo. If  $N$  is the number of revolutions per minute proper to the machine when giving its rated voltage,  $V$ , as a dynamo, then  $2\pi\frac{N}{60}$  is its normal angular velocity  $\Omega$ . Then, corresponding to this normal angular velocity, denoted by  $\Omega$  at the place where this horizontal line cuts the curve  $C_1$ , draw a tangent to the curve and measure the angle  $\alpha$  which this tangent makes with the horizontal. The tangent of this angle therefore represents the value of  $\frac{d\Omega}{dt}$  at that moment. Hence we have

$$p = I\Omega \tan \alpha.$$

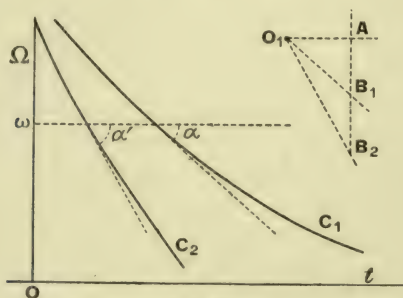


FIG. 7.

To eliminate  $I$  from this equation we take another observation of the mode in which the speed of the machine dies away with time when a certain known additional retarding power is applied to the shaft. This can be achieved by the use of a small rope belt, made on the plan described in a subsequent section under "Motor Testing." Let the diameter of the pulley of the machine be  $d$ , and let  $k$  be the difference of the tensions on the two sides of the rope brake as given by the spring balance and weight employed. Then, if the machine is set rotating with an angular velocity  $\Omega$ , the work taken up by the brake equals  $f = \pi dk\Omega$ , and will be given in *foot-pounds* if  $d$  is measured in

feet, or fractions of a foot, and  $k$  in pounds, or in kilogramme-metres if  $k$  is measured in kilogrammes and  $d$  in metres. If, then, we take another observation of the same kind by starting the machine as a motor and running it up to a higher speed than its normal speed ( $N$ ) as a dynamo, then switching off the current and letting the speed of the machine die away with the rope brake on the pulley, we shall be able to delineate another curve  $C_2$  (see Fig. 7), representing the

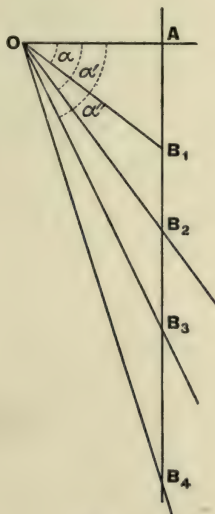


FIG. 8.

manner in which in this second experiment the angular velocity ( $\omega$ ) dies away with time. At the point where the horizontal line, corresponding with the normal angular velocity ( $\bar{\Omega}$ ) of the machine, cuts this second curve we again draw a tangent and measure the angle  $\alpha'$  which it makes with the horizontal. We have then the following value for  $p+f$ , viz.:—

$$p+f = I\bar{\Omega} \tan \alpha'.$$

Hence, eliminating  $I$  from this and the previous equation, we

have

$$p = f \frac{\tan \alpha}{\tan \alpha' - \tan \alpha}$$

$$= \pi dk \Omega \frac{\tan \alpha}{\tan \alpha' - \tan \alpha}.$$

We can then set off a tangent diagram (*see* Fig. 8) in which a line OA is taken as unity and lengths AB<sub>1</sub> and AB<sub>2</sub> are taken proportional on the same scale to the tangents of the angles  $\alpha$  and  $\alpha'$ , and represent the value of  $p$  as follows :

$$p = \pi dk \Omega \frac{AB_1}{B_1B_2}.$$

In the next place, to determine the value of  $q$  we proceed thus :—The circuit of the armature is closed by a low resistance, which must include an ammeter suitable for measuring the full-load current of the machine when acting as a dynamo, and also a voltmeter must be provided connected across the brushes, to measure the voltage at the ends of this external resistance at any moment. A third experiment is then made as follows :—The machine is run up as a motor to a speed rather above its normal speed  $N$  as a dynamo. The driving current is then switched off from the armature, but the field magnets are kept excited with a voltage equal to the normal brush potential difference  $V$  of the machine when running as a dynamo. The armature circuit is then connected to the low external resistance, and by trial a value must be determined for this such that, when the machine drops to its normal speed as a dynamo, the current which will be flowing through this external resistance is equal to the normal current of the dynamo. At that moment we must determine the difference of potential of the brushes and also the rate at which the angular velocity of the armature is dying away. This is best achieved as follows :—Suppose that the normal full-load current of the machine is  $A$ , and the normal voltage at the brushes is  $V$ , when the machine is acting as a dynamo, then the field magnet must be excited by the independent source of voltage equal to  $V$ , and the external resistance must be so adjusted that, as the machine

is dying down in speed, at the moment when its velocity is equal to the normal velocity as a dynamo, the current through this external resistance is equal to  $a$ . This can be nearly achieved by a few trial experiments. Having done this, we take a third curve showing the decay in speed with time, and determine the tangent of the angle  $\alpha''$  which the geometrical tangent of this curve makes with the horizontal line whose ordinate corresponds with the normal angular velocity  $\Omega$  of the machine. We then set off on the tangent diagram a third line,  $OB_3$ , such that the distance  $AB_3$  represents  $\tan \alpha''$ . Then, if at that moment  $v$  represents the terminal voltage of the dynamo, the product  $vA$  represents the work taken up in the external circuit, and from the previous equations it is easily seen that we must have

$$\frac{q+p+vA}{p} = \frac{AB_3}{AB_1},$$

or

$$q = \pi dk \Omega \frac{B_1 B_3}{B_1 B_2} - vA.$$

In this manner, from the results of three experiments we can obtain separately the value of  $p$  and  $q$ , and, moreover,  $q$  is obtained under the same circumstances that pertain when the machine is actually running as a dynamo. The field magnets and the armature have the same magnetic flux through them, and the armature coils carry the normal full-load current of the dynamo; hence the magnetic state of the armature is identical with that of the machine when acting as a dynamo under full load, and the only distinction between the circumstances then and when being used as a dynamo under full load is that the frictional losses are probably rather less under the circumstances of the test than under actual full load working, because in the former case there is the side pull on the bearings due to the belt. Nevertheless the magnetic condition of the armature is identical with its condition under actual working.

It has been maintained that the hysteresis loss in the armature of a continuous-current dynamo can be neglected,

owing to the fact that the flux density is in general very high. It should, however, be noted that experiments by Prof. F. Baily,\* and also by Messrs. Beattie and Clinker,† have shown that rotational hysteresis does not vanish or even become insignificant until much higher mean flux densities are reached than are generally found in the cores of armatures of dynamo machines. Moreover, in toothed armatures there are *changes* in the distribution of the magnetic flux which render any such conclusions invalid, since the reduction in hysteresis at high flux densities only take place when the latter is perfectly constant.

§ 14. **Separation of the Hysteresis and Eddy Current Losses.**—The method already described—suggested by M. Routin—enables us to separate out the frictional losses from the magnetic hysteresis and eddy current losses taken together, but it is desirable to determine the proportion in which the core loss is divided between magnetic hysteresis and eddy current losses. Methods for doing this have been suggested by Mr. Mordey,‡ Mr. G. Kapp,§ and also Mr. R. H. Housman.¶ All these methods essentially depend upon the fact that for the same excitation the eddy current losses vary as the square of the speed, whereas the hysteresis losses vary as the speed simply; hence, if we denote by  $L$  the loss in the armature core due to these causes alone, if  $N$  is the speed of the dynamo, and if  $H$  is the hysteresis loss and  $F$  the eddy current loss for a standard speed, then we have the equation

$$L = ANH + BN_1^2 F,$$

where  $A$  and  $B$  are certain constants determined by the field and the form of the core. Mr. Mordey was probably the first to make use of this equation to determine and separate

\* See *Phil. Trans. Roy. Soc. Lond.*, 1896; also *The Electrician*, Vol. XLIV., p. 323, 1899. Mr. F. G. Baily "The Testing of Hysteresis in Armature Cores."

† See *The Electrician*, Vol. XXXVII., 1896, p. 726.

‡ See *The Electrician*, Vol. XXIII., p. 121.

§ See *The Electrician*, Vol. XXVI., p. 699, 1891.

¶ See *The Electrician*, Vol. XXVI., p. 700, 1891.

the losses arising in the armature core. His method was as follows:--The armature of the machine under test is driven by means of a belt from a motor which has already had its efficiency determined, and hence the mechanical power employed to turn round the armature of the machine under test can be determined from the electrical power given to the motor. By running the armature under test at various speeds with its field magnets excited with the full normal current required to create its proper voltage when used as a dynamo, we can measure the power required to turn the armature round at different speeds in the same field. Hence, we can obtain equations involving the four unknown quantities A, B, H and F; the value of N being different in each case, and the value of L being determined experimentally, since it is the power in watts required to turn round the armature after deducting the loss due to mechanical friction. This last can be determined by an independent experiment in which the armature is rotated at a known speed with the field magnets unexcited. Thus, suppose we denote, as before, the frictional loss by K, the hysteresis loss by H, the eddy current loss by F, and the total loss by L at the normal speed, then we have

$$L=K+H+F. \quad \dots \dots \dots (1)$$

We may determine K, H and F as follows:—Let the speed be made greater or less in the ratio of  $n$  to 1. Then, if  $L$  is the total loss at the new speed, we may assume that

$$L_1=nK+nH+n^2F. \quad \dots \dots \dots (2)$$

For the hysteresis loss  $H$  varies as the speed and the eddy current loss  $F$  varies as the square of the speed, and we may *assume* that for moderate changes of speed the frictional loss varies as the speed.

These equations (1) and (2) can be easily solved, thus: Multiply (1) by  $n$  and subtract it from (2),

$$nL=nK+nH+nF;$$

also

$$L_1=nK+nH+n^2F.$$

Hence 
$$L_1 - nL = n^2(n-1)F,$$

or 
$$F = \frac{L_1 - nL}{n(n-1)}.$$

Thus, suppose  $n=2$ , or the second speed is twice the first, then 
$$F = \frac{L_1 - 2L}{2}.$$

Having obtained the value of  $F$ , we can take two fresh determinations of the total loss at two other speeds, and obtain two equations involving  $K$  and  $H$  which can be solved and their values determined.

The non-mathematical reader should note that the practical solution of a pair of linear equations such as

$$\begin{aligned} ax + by &= c, \\ a'x + b'y &= c', \end{aligned}$$

can be obtained in a moment by a graphical construction. Take a sheet of squared paper and set off a pair of rectangular axes. On the vertical axis set off to any scale two lengths equal respectively to  $c/b$  and  $c'/b'$ . On the horizontal axis also set off two lengths  $c/a$  and  $c'/a'$ , reckoning distances from the crossing point of the axes. Then draw straight lines through the extremities of the vertical and horizontal lengths  $c/b$  and  $c/a$ , and also through those corresponding to  $c'/b'$  and  $c'/a'$ . If these two lines intersect, measure off the perpendicular distances of this intersection from the vertical and horizontal axes respectively, and these two distances are the values of the  $x$  and the  $y$  in the two equations.

The method described by Mr. Housman enables us to dispense with any solutions of mathematical equations, and gives the required result graphically by means of a simple geometrical construction. His method is as follows:—The machine under test has current put into its armature and is run as a motor without load, various voltages being applied to the armature, the field magnets being separately excited so as to keep the field constant. A curve is then plotted (*see* Fig. 9) the abscissæ of which represent volts on the armature and the ordinates the current in the armature. It will be found that this curve is practically a straight line, cutting the axis of the current at some distance from the origin. This curve is represented by the line  $AB$  in Fig. 9.

If we take any point B on the curve and draw BC parallel to AO, it is evident that the total loss in the armature is represented by the product  $OC \times CB$ . Next draw AD parallel to OC. This divides the area representing the total loss into two parts, one of which, OADC, is a rectangle, whose area is proportional to OC, and the other part is a triangle ADB; or, if we are considering the product of OC and CB, the other part is a rectangle whose sides are ADDB and its area is proportional to the square OC, since BD is proportional to AD, which is equal to OC. The first of these areas represents the waste of energy per second due to hysteresis and mechanical friction, whilst the second gives the loss due to

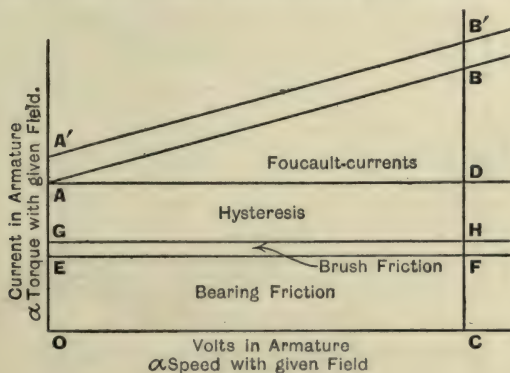


FIG. 9.

the eddy currents. It must be noticed that it is the product of the co-ordinates at any point of AB which represents the waste, and not merely the area of the figure AOCB. In this respect, the diagram resembles the characteristic diagram of a dynamo rather than the indicator diagram of a steam engine.

Having determined the frictional loss by running the armature with the magnets unexcited, we can draw a line GH such that the area OGHC represents the total frictional losses due to bearings and brushes, and the remaining rectangle, GADH, will then represent the loss due to the

magnetic hysteresis, whilst twice the area of the triangle ADB represents the eddy current loss. By running the armature with and without brushes, by means of a belt and motor whose efficiency has been determined, we can separate out the frictional losses at the bearings from those at the brushes. In doing this the belt that is used to drive the armature under test should not be of leather, as this would involve the expenditure of an unknown amount of work in bending the belt over the pulley, but should be a couple of thicknesses of wide silk ribbon, which are sewn together so as to make an endless belt which is fairly strong, and yet

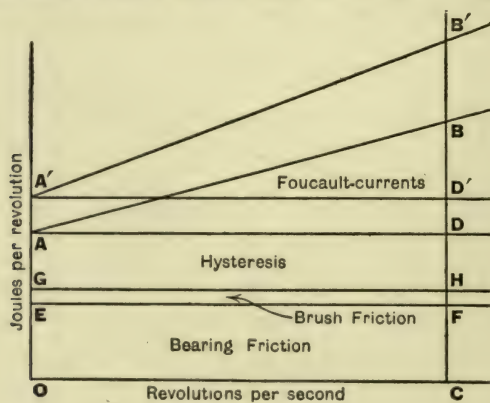


FIG. 10.

requires no sensible expenditure of power in bending it round the pulley.

Returning to the electrical diagram, if a second set of experiments is made with a different strength of field we can obtain a second curve or straight line A'B' which is in general parallel to AB, showing that the eddy currents are constant for a given voltage.

In Fig. 9 all the quantities are electrical, and hence the lines AB, A'B' may be called electrical curves, in order to distinguish them from those in another diagram in which the quantities are all measured in mechanical units (*see* Fig. 10).

The electrical diagram has the disadvantage that for each intensity of field a separate friction line has to be drawn, which makes it difficult to distinguish how much of the whole loss is electrical and how much is frictional when more than one curve is drawn on the same diagram. This difficulty may be obviated by plotting as abscissæ the revolutions of the armature per second instead of the voltage of the armature, and by plotting as ordinates joules per revolution—that is, work per revolution—instead of current flowing through the armature.

It will be noticed that the product of these two quantities in the second diagram is expressed in watts, or work per second, as before, but it only requires a change in scale to make the co-ordinates refer to revolutions per minute and foot-pounds per revolution respectively, in which case the product divided by the number 33,000 gives us horse-power. In this second diagram the friction line has the same place for all field strengths, and hence the frictional and electrical losses are separated by the line GH, which maintains a constant position. It will be seen, then, that the lengths GA and GA' represent the losses by hysteresis in joules per cycle at the two field intensities to which the lines AB and A'B' correspond. Mr. Housman says that the loss per cubic centimetre from this cause is always found to be much greater than that which is generally given.

**§ 15. Continuous-current Motor Testing.**—If a single large continuous-current motor is submitted for test, it may not be convenient to take up the mechanical power directly, and in any case there is some difficulty in measuring it with great accuracy. If two identical machines are available, and if they are shunt machines, then they may be tested by a differential test, as already described in sections 6, 7 and 8 of this Chapter, employing another small dynamo or battery in parallel or in series to provide the internal losses in the two machines. If, however, only one machine is submitted

for test as a motor, then its efficiency must be determined by an independent determination of the core losses, the frictional losses and copper losses, which may be made as described in sections 12 and 13 for a continuous-current dynamo. Having determined these losses, the efficiency at the full-load output can be calculated at once by the expression

$$e = \frac{W}{W + L},$$

where  $W$  is the full-load output in watts on the pulley or the rated value of the motor, and  $L$  is the total internal losses in watts at full load.

The chief objection which can be raised to this method is that the frictional losses under load, if the motor is to be used to drive its load by means of a pulley and belt, cannot be accurately predetermined. In this case there is a side pull on the shaft which necessarily creates a frictional loss in the bearings, which does not exist when a pure torque is applied to the motor shaft by means of another motor, or when the motor is allowed to exert pure torque unaccompanied by any single force. The same remark applies, however, to the determination of the frictional losses made as described in sections 12 and 13 in the case of a dynamo. If, however, for any reason, a brake method cannot be applied, and two identical motors are not available for a differential test, then the resistance of the field and armature circuits, and the motor, should be measured and reduced to their hot value, as described in section 2.

The frictional losses in the machine must be determined as described in sections 12 and 13, and also the hysteresis and eddy current losses, and from the measured value of these internal losses the efficiency must be worked out at various loads by the aid of the rules already given. In the case of motors which are not very large, the efficiency test may be carried out by a direct method. The electrical power delivered to the motor is measured by taking the

product of the current and the terminal voltage, and the mechanical power given up by the motor is measured by putting a load upon it by means of a brake.

A simple form of motor-testing brake has been devised by Mr. Soames (see *The Electrician*, Vol. XL., p. 524). It consists of a strong tripod, which can be erected over any



FIG. 11.

motor to be tested. The tripod carries a steel yard, from which depend a pair of ropes carrying a leather band, passing under the pulley of the motor (see Fig. 11). A

tightening screw enables this band to place any required load on the motor. The sliding weight on the steel yard is then adjusted so as to balance the arm and measure the difference of the tensions of the two vertical ropes, which is the force against which the motor does work. This difference of tensions, reckoned in pounds, multiplied by the revolutions of the pulley per second and its circumference in feet, gives the work done by the motor in foot-pounds per second.

§ 16. **Motor Testing by the Brake Method.**—In the testing of small motors, from 1 H.P. to 10 H.P., in cases in which only the motor itself under test is available, the *brake method* is a convenient method to employ for determining the efficiency. In this case a load is put upon the shaft of the motor, the



FIG. 12.

value of which can be easily determined, and the motor is made to do work against it. For this purpose the motor pulley should be removed, and in its place a disc of iron about 12in. in diameter and 2in. in width should be put, forming a cast-iron pulley. Half-a-dozen pieces of hard wood must be cut into the shape shown in Fig. 12, each having a pair of nicks; the width of these cleats must be such as to fit easily the edge of the cast-iron pulley above mentioned. Two short lengths of well-stretched and plaited cord are then placed in the cleats and fastened in by fine twine or wire, as shown in Fig. 12. To one end of this arrangement is fastened a scale pan in which weights can be placed, and to the other end an iron ring to which a spring balance is connected. To use this brake the motor must be bedded down in a convenient position and the rope brake

adjusted over the pulley, as shown in Fig. 13, the scale pan hanging down on one side and the spring balance fastened to the floor on the other. A certain weight is then placed in the scale pan to apply a tension to the rope brake, the value of which will be determined by the power to be tested. The motor is supplied with electric current through an ammeter, and a voltmeter must be connected across the brushes of the motor. If, then, the motor is started, the product of the readings of these two instruments gives us the power in watts given to the motor electrically. Weights must then be added to the scale pan of the brake until the current taken by the motor

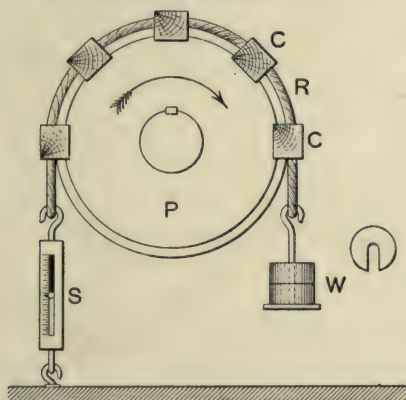


FIG. 13.

reaches any assigned value equal to or less than the full-load current of the motor. When the motor rotates steadily under this load a revolution counter must be employed to take the speed. For this purpose it is better to use a counter which actually counts the number of revolutions, and a good watch or chronometer to give the time, rather than to employ a tachometer giving any speed directly. The operation is most conveniently carried out by two observers; one man with watch in hand notices the position of the seconds hand and gives the word to another observer who has the counter in his hand ready to apply the spindle of it to the spindle of the

motor. At the word he presses the counter against the shaft and lets it run with the motor handle; the observer with the watch then gives word again at the end of one minute, or any other known time, when the first observer withdraws the counter and reckons out the number of revolutions per minute (R.P.M.). If the current does not remain constant during this observation it must be repeated until the current and the volts remain perfectly constant at known values during the time that the speed test is being made.

We have, then, all the materials for calculating the efficiency as follows:—The diameter of the cast-iron pulley and the width of the rope must be carefully measured in inches. Let  $D$  be the diameter of the pulley and  $d$  the diameter of the rope. Then  $D+d$  is the diameter of the mean perimeter of the rope, and is the arm of the torsional couple. Let  $P$  be the weight in the scale pan together with that of the scale pan itself, and let  $P_1$  be the reading of the spring balance in lbs. Then the difference of the tensions of the two sides of the rope brake which hang down vertically is  $P_1 - P$  lbs. If  $D$  is the diameter of the pulley and  $d$  is the diameter of the rope, then  $\frac{1}{2}(D+d)$  is the effective radius or arm of the torque, and the torque ( $F$ ) itself, against which the motor does the work, is given by the expression

$$F = \pi(D+d)(P_1 - P),$$

and the rate at which the work is done by the motor against this torque, or the power in watts given out by it on the flywheel in watts, is equal to

$$W = \frac{746}{550} \frac{N}{60} \frac{\pi}{12} (D+d)(P_1 - P) \text{ watts,}$$

where  $N$  is the number of revolutions of the motor per minute, and  $D$  and  $d$  are measured in inches, and  $P_1$  and  $P$  in lbs. If the current taken by the motor is  $A$  amperes and the potential between the brushes is  $V$  volts, then the power  $W$  taken up by the motor is equal to  $AV$  watts, and hence the

commercial efficiency of the motor, which is the ratio of the power given out to the power taken in, is given by the fraction  $W/AV$ . This fraction expressed as a percentage gives the commercial efficiency of the motor.

These observations should be carefully repeated for each particular load on the motor as determined by the weight put in the scale pan of the brake, and, if possible, should be so arranged as to be taken at loads corresponding to the various decimal fractions of full load. We are able then to set out a curve showing the efficiency of the motor at various fractions of full load (*see* Fig. 1).

The difficulty which is generally experienced in these experiments is that of regulating the speed of the motor so that, while the load is varied as required, the speed is kept constant. If the speed values are different in each experiment at different loads, then the various efficiencies so calculated cannot be considered as points lying upon one curve. The most simple way of evading the difficulty is to take a large number of observations at different loads, and at speeds as nearly the same as possible, and then, having calculated out the efficiency as above explained, to pick out all those values which correspond to one and the same speed, or, at any rate, to values of the speeds not differing by more than  $\frac{1}{2}$  per cent., and to employ these groups of observations in setting out separate efficiency curves. In the complete testing of a motor of this kind the power wasted in the armature and in the fields, due to resistance, should be determined by measuring the resistance of the copper circuits when the motor is warm and has reached a steady temperature after working for some time. The total loss in the motor which, of course, is obtained as the difference of the power put in and that taken out at any load can then be divided into two parts, one part accounted for by the resistances or copper losses and the remainder due to friction, magnetic hysteresis and eddy currents. The frictional losses can be determined in the manner suggested by M. Routin, and the remainder of the

total loss is accounted for by magnetic hysteresis and eddy currents.

In recording the results of a motor efficiency test graphically, the efficiency expressed as a percentage should be set out as the ordinates of a curve, the abscissæ of which represent the fractions of the full load of the motor.

The form of motor brake shown in Fig. 11, p. 555, can be used with advantage when the motor shaft lies very low down near to the bedplate. This form is convenient when, for any reason, it is not possible to replace the ordinary pulley of the motor by a special cast-iron fly-wheel pulley, and also when the axis of the motor is so near the ground that it is not possible to find room for the spring balance and counterpoising weight if these are used in the position already described. The operation, however, of taking the difference in the tensions on the two sides of the belt, which passes round and under the pulley, is exactly the same. The tension on one side of the belt is given by the reading of the spring balance, and on the other can be calculated from the position of the weight on the steelyard arm. In using any form of rope brake it is essential to keep careful watch on the bearings of the motor lest they become overheated. The whole of the work done by the motor is, of course, expended in producing heat in the pulley or cast-iron disc, and this heat is only slowly dissipated by convection and radiation. Hence, generally speaking, it is not possible to test any but rather small power motors by this process, and even then the experiment must be interrupted at intervals to give the pulley time to cool. Generally speaking, a rope brake test of this kind is most convenient for a 1kw. or 2kw. motor, but is not applicable for motors exceeding 5kw. or 6kw., unless a large and special cast-iron pulley is provided.

Another very practical form of brake for the determination of efficiencies of small motors depends upon the production of Foucault or eddy currents in a copper disc when it rotates

between the poles of a magnet. In this case the motor to be tested has a thick copper disc, which may be 10in. or 12in. in diameter and  $\frac{1}{4}$ in. thick, put upon its shaft in place of the pulley. This disc must be made of high conductivity copper and must run very true. Over this disc, and carried by a separate and independent support, a beam is suspended which rests on knife edges like the scale pan of a balance. This beam carries two electromagnets, so placed that the copper disc revolves between their pole pieces without touching them. When the motor is running and these electromagnets are excited eddy currents are induced in the copper disc which tend to retard its motion, and hence cause the electromagnets to be pulled round in the direction of rotation as far as the beam will allow. If, therefore, we apply a weight to this beam so as to keep the magnets in their original position, the torque so applied to the movable part, consisting of the beam and magnets, can be reckoned out in foot-pounds or pound-inches. This, therefore, is the torque against which the motor is doing work, and if its value in foot-pounds is multiplied by the angular velocity of the disc, which is equal to  $2\pi N/60$ , where  $N$  is the revolutions per minute, we have the work done in *foot-pounds* by the motor per second. Hence, if the electrical intake of the motor is measured, we can obtain the efficiency by taking the ratio of the output in mechanical work to the electrical intake, both of them being reckoned in the same units.

**§ 17. Motor Tests by the Cradle Method.**—In the case of very small motors, such as those of  $\frac{1}{4}$  H.P. or  $\frac{1}{16}$  H.P., neither the differential method of testing nor the brake method can be applied; but the following method, due to Prof. Brackett, is applicable to quite small motors, and with care can be made to give excellent results. The motor to be tested is placed upon a cradle consisting of a wooden or iron support, provided with knife edges at its extremities. The cradle is made adjustable, so that the central line of the motor axle,

if produced in both directions, would pass through the knife edges. These knife edges rest on plates carried on vertical pillars, and the cradle itself carries certain rods on which are movable weights. By means of these the centre of gravity of the whole mass can be made to lie in the line adjoining the knife edges. This adjustment is known to be correct when the whole mass, cradle and motor, is just beginning to be top heavy, and it is effected by moving the movable weights upon the rods to various heights. Under these circumstances, if a current is sent into the motor to cause the armature to revolve, and if work is put upon the shaft, so causing the water to exert power, the reaction on the field magnets by the armature will render it necessary to apply a torque to the field magnets to preserve the cradle in zero position. This torque is applied by sliding a weight along an arm attached to the cradle, the arm being graduated in fractions of a foot, and the weight being in pounds or fractions of a pound. Hence the torque in foot-pounds required to preserve the cradle in its zero position is at once found.

It remains to describe the manner of putting the work on the motor. The motor must be provided with a pulley, against which press on either side a pair of curved pieces of hard wood like brake blocks. These are carried on long wooden rods depending from a fixed support, and can be more or less squeezed together by a screw. If, then, these blocks are pinched against the pulley, the motor is made to do more or less work according to the pressure put upon the pulley. The speed of the motor has also to be determined. This is best done by attaching to the motor shaft an endless screw, geared to a toothed wheel of such gear that for 100 revolutions of the motor the wheel makes one revolution. A projecting pin on this wheel is made to displace the hammer of a small gong and strike a blow. Thus the gong sounds once for every 100 revolutions of the motor armature. If, then, by means of a watch the observer takes the time in which 10 gong sounds are made, he has the time of 1,000

revolutions of the motor, and hence the revolutions per minute denoted by  $N$  or the revolutions per second by  $N/60$ . The work done by the motor in one revolution in foot-pounds is therefore equal to  $2\pi$  times the restraining torque reckoned in foot-pounds. Hence, as  $P$  is the weight in pounds or fractions of a pound on the balance arm, and  $L$  is the length in feet or fractions of a foot of the arm or distance of the weight  $P$  from the shaft axis, then  $2\pi PL$  is the work done in foot-pounds by the motor in one revolution, and  $2N\pi PL/60$  is the work done per second. Hence the power  $W$  exerted by the motor in watts is given by

$$W = \frac{746}{550} \frac{N 2\pi PL}{60} \\ = 0.142NPL.$$

If the motor is a continuous-current one, then the electrical power given to the motor is equal to  $AV$  watts, where  $A$  is the current taken up by it in amperes and  $V$  is the terminal potential difference in volts. The efficiency ( $e$ ) is the ratio  $W/AV$ , or is given by the equation

$$e = \frac{746 \cdot N \cdot 2\pi PL}{550 \cdot 60 \cdot AV} \\ = \frac{0.142NPL}{AV},$$

where  $P$  is the weight reckoned in pounds,  $L$  the arm of the couple in feet,  $N$  the revolutions per minute,  $A$  the motor current, and  $V$  the motor terminal voltage.

If the small motor is a polyphase or alternating-current motor of any kind, then the electrical power given to it must be measured in the proper manner by a suitable watt-meter. Let this power be  $W_0$  watts, then in this case the efficiency ( $e$ ) is expressed by the ratio  $W/W_0$ , or

$$e = \frac{0.142NPL}{W_0}$$

The efficiency can be determined for a number of different

values of  $W$ , the power given out and set out in the form of an efficiency curve.

The difficulty which generally arises in making these tests is that of making the motor run at a nearly constant speed while doing variable amounts of work. The simplest plan to adopt is to make a number of experiments, varying  $P$  and  $L$  so as to obtain different values of  $W$ , and then to pick out all those which correspond to values of  $N$  not differing by more than 5 per cent.

**§ 18. Measurement of the Starting and Rotating Torques of Motors.**—In all tests of motors, whether continuous current, alternating, single or polyphase, an important deter-

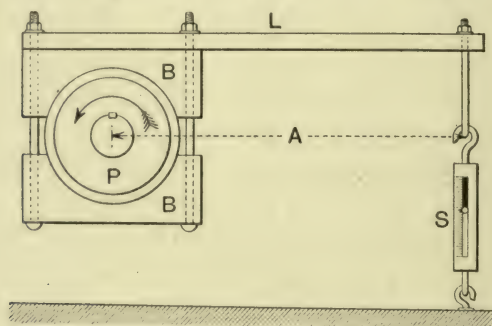


FIG. 14.

mination which must be made in order to form an estimate of the utility of the motor is the measurement of the starting torque. For this purpose, the rotating portion of the motor must be held fixed and the couple required to hold it stationary when the working voltage is put upon the motor gives us the starting torque. If the motor has on its shaft a pulley, the simplest way of determining the torque is to grip on to the pulley a wooden lever in the fashion of a Prony brake (see Fig. 14). The arm of this lever may be 3ft. or 4ft. long, according to the torque of the motor. To the end of the arm is then attached a spring balance, one end of which is fixed to

the ground or to some immovable support. The spring of the balance must be so fixed that the direction in which it stretches is at right angles to the shortest line joining its point of attachment to the lever arm to the centre of the shaft. This distance has then to be measured, and is the arm of the torque. On starting the current into the motor the rotating portion will attempt to move round, but its motion will be resisted by the force applied by the spring balance. The product of the reading of the spring balance and the distance from the point of application of the spring balance to the centre of the motor axis gives us the torque that the motor is exerting. If the spring balance reads in pounds and the length of the arm is measured in feet, we have the torque in foot-pounds, or it may be measured otherwise in kilogramme-metres. It is generally necessary to draw a distinction between the *starting torque* and the *running torque*. The running torque may be ascertained by means of a rope brake put over the pulley, or else over a special cast-iron brake wheel, as already described, and the difference in the tensions of the two sides of the rope brake, multiplied by the radius of the wheel, plus half the diameter of the rope, gives us the running torque which the motor is exerting. If the tensions on the two sides of the rope are so adjusted that the motor is fully loaded, we can in this manner determine the running torque at either full load or any fraction of full load. Generally speaking, this running torque is greater than the starting torque. The determination of the starting torque of an alternating current motor is an important matter, because in some cases there is a very small starting torque. The motor will not exert any power until it has first been brought up to synchronism with the current supplied to it.

### § 19. Alternator Testing, Single phase or Polyphase.—

In testing an alternator, the electrician may be called upon to test either a single machine apart from the engine, or, more

usually, to make a test of a direct-coupled engine and alternator. In either case arrangements have to be made for taking up the electrical power given by the alternator. The machine may be either a single-phase, two-phase, or three-phase machine, and the nature of the power-absorbing circuit will be determined accordingly. It is usual to test single-phase alternators on water resistance alone, as this constitutes a practically inductionless resistance. Such a water resistance may be constructed as described in section 5 of this Chapter. In making the arrangement for testing high-tension alternators of 2,000 volts and upwards, care should be taken that the leads from the machine are well insulated. As the water resistance has generally to be placed at some distance from the alternator, it is necessary to run high-class indiarubber-covered cables, carried on porcelain insulators, which may be fixed to wooden beams suspended from the roof or in any other manner. These cables should be kept well apart, so that there may be no danger of short circuits during the test.

The arrangement for raising and lowering the metal plates into the water should be so well insulated that the attendants cannot possibly receive shocks, and in putting the load on the alternator it should be done very gradually, as the passage of the current sent warms the water and greatly reduces the resistance. Hence, if the plates are lowered too quickly to give the required current, a machine may soon prove to be overloaded. If the water resistance is at some distance from the machine being tested a telephone wire or electric bell wire should be run, so that the attendants in charge of the engine can give signals to those in charge of the water resistance.

In the trial runs carried out in the large engine and dynamo factories to satisfy purchasers, it is usual to bed down the alternators under test and their engines on temporary bedplates in a test-house, and in that case all the arrangements generally exist for conveying the currents safely to the place where the power is to be taken up. Measuring instruments have then to be provided for the determination of the

currents, voltages and powers. If the machine is a single-phase machine being tested on a water resistance, nothing more will be needed than one amperemeter in the power-absorbing circuit and one voltmeter across the terminals of the machine under test. These instruments must be instruments suitable for measuring alternating currents of the frequency given by the machine. In this case the power taken up is given at once by the product of the R.M.S. amperes and the R.M.S. volts. If, then, the machine being tested is a steam alternator, the other measurements required are the indicated horse-power of the engine and the steam consumption in a given time. The arrangements for obtaining these figures, in the case of an alternator, should be as described in section 10 of this Chapter.

It is usual to carry out a six hours' run at full load, varied by a two hours' run at either 25 or 33 per cent. overload, and also certain other shorter runs at three-quarter load, half load and quarter load, to determine four or five points on the efficiency curve.

In making these electrical measurements, the readings of the instruments should not be taken for granted, but the scale reading should be checked by means of the potentiometer. Convenient instruments for measuring the voltage and current, in the case of single-phase alternators, are the hot-wire ammeters and voltmeters of Messrs. Hartmann and Braun, as the approximate values of the current and voltage are then seen by inspection. In the case of polyphase alternators, either two or three-phase, the water resistance has to be modified in nature to suit the case, and when dealing with three-phase machines it is usual to test them, not only on water resistance but also on an inductive resistance constructed with an assigned power-factor of, say, 0.9.

The arrangement for the testing of large three-phase steam alternators may best be understood by the following description of a test made by the Author on a 300kw. steam turbine alternator. The sections of the armature were star-connected.

The power-absorbing resistance consisted in this case of three sets of water resistances, each being joined in series with a variable inductive resistance made as described in section 5 of this Chapter. The arrangements are shown diagrammatically in Fig. 15, where  $W_x$ ,  $W_y$  and  $W_z$  are the three star-connected water resistances, and  $C_x$ ,  $C_y$ ,  $C_z$  are the three variable inductance coils, these latter being each provided with a short-circuiting switch, so that they could be entirely cut out if necessary. In series with two of these arms—viz., the X and Z arms—were placed two Kelvin wattmeters,  $K_x$

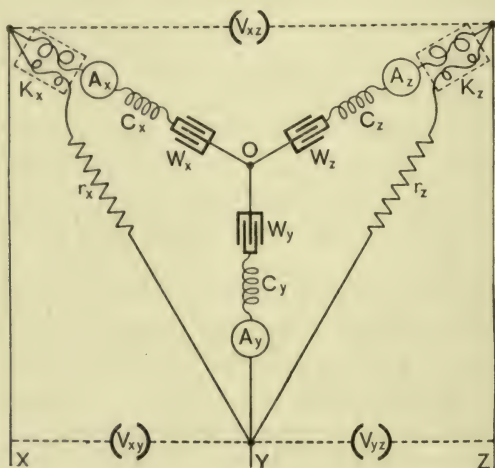


FIG. 15.

and  $K_z$ , the thick wire coils of these balances being in series with the water resistances and coils, and the fine wire or shunt coil of these balances being joined across between the external terminals of the alternator circuits, as shown in the diagram, and variable but known resistances,  $r_x$ ,  $r_z$ , interposed to regulate the currents in the Kelvin balance shunt coils. Three alternating-current ammeters,  $A_x$ ,  $A_y$ ,  $A_z$ , were then placed in series with the three power-absorbing circuits, and from the latter three leads were run back to the terminals of

the alternator circuits. Voltmeters,  $V_{xy}$ ,  $V_{yz}$ ,  $V_{xz}$ , were also provided for measuring the voltage across these places.

Let X, Y, Z represent the three mains proceeding from the terminals of the alternator to the terminals of the power-absorbing resistance. The total voltage,  $V_{xy}$ ,  $V_{yz}$ ,  $V_{xz}$ , was measured between X and Y, Y and Z, and Z and X, and the drop in volts also measured down each water resistance and down each choking coil. The circuits having been adjusted so that the ammeters read as nearly as possible equally, the trial runs were commenced, and the following observations were taken every quarter of an hour :—

Time .. .. .	.....
Boiler pressure .. .. .	lbs. per sq. in.
Stop valve pressure .. .. .	lbs. per sq. in.
Stop valve temperature .. .. .	°C.
Vacuum in cylinder .. .. .	inches of mercury
Speed of engine .. .. .	R.P.M.
Volume of water produced by condensed steam in .. .. . mins.....	.....cubic feet
Temperature of condensed steam.....	.....
Factor to convert volume of water to weight of water at above temperature.....	.....
Pounds of steam used in .. .. . mins.....	.....lbs.
Calculated rate in lbs. per hour at same load.....	lbs.
Main armature current .. .. .	amperes
Voltage across brushes .. .. .	at.....load
$V_{xy} =$ .. .. . $V_{yz} =$ .. .. . $V_{xz} =$ .. .. .	.....
Field current of alternator .. .. .	amperes
Exciter armature current .. .. .	amperes
Exciter field current .. .. .	amperes
Speed of exciter.....	R.P.M.
Reading of wattmeter $K_x = W_x =$ .. .. .	kilowatts
Reading of wattmeter $K_z = W_z =$ .. .. .	kilowatts
Power-factor of load .. .. .	.....
Total power in kilowatts .. .. .	.....

Fraction of full load.....  
 Calculated pounds of steam per kilowatt at .....  
 load= .....  
 Ditto according to Specification .....

Temperature of air.....by observation.....  
 by Specification .....  
 Temperature of alternator armature at end of  
 ..... hours run at ..... load  
 Ditto alternator fields .....  
 Ditto exciter armature .....  
 Ditto exciter fields.....

General remarks on performance during the run :—

There is no need to give the details of the observations made, but the following abstract of the report sent in embodies the results :—

#### *Tests of Three Turbo-Alternators.*

The machines were numbered respectively 843, 844 and 850.

The requirements of the specification fell under eight headings, with the conditions of which the machines had to comply, as below :—

1. General conditions of construction and materials.
2. A running test at normal load.
3. A running test at  $33\frac{1}{3}$  per cent. overload.
4. A steam consumption test at various loads.
5. An insulation test.
6. A temperature test after running at full and overload.
7. A parallel running test.
8. A regulation or voltage test.

1. *General Conditions of Construction.*—Each machine consisted of a steam turbine coupled direct to a three-phase alternator with star-connected armature circuit and four collecting rings, one being the middle point ; also of a direct current exciter on the same shaft. The machines were all

identical, and complied with the general conditions laid down for their construction in the specification for speed and frequency. They appeared to be well built of good materials, and the workmanship and finish were satisfactory and in accordance with the specification. The normal speed was 3,000 revs. per min. and the frequency 50.

2. *Running Test at Normal Load.*—Each machine was run as specified for eight hours continuously at full load of 300kw. on an inductive load with power-factor 0.9. Electrical instruments were provided and subsequently checked for accuracy, which showed that the machine was really carrying its full load all the time. The electrical power was taken up in water resistances outside the test house, and the inductance provided by coils of wire inside. During the run the machines did their work without failure or distress of any part.

3. *Running Test at Overload.*—At the conclusion of the full-load tests each machine was put on to a two hours' overload test, giving 400kw. output, and this run was continued in two cases without stoppage. At the end of the eight hours' load test, the machines all carried this overload with perfect ease. In the case of Nos. 843 and 850 the machines ran for 10 hours on end.

4. *Steam Consumption Tests.*—During the eight hours' run the tests were carried out for steam consumption at full load. The exhaust steam was condensed in a surface condenser and the water measured at known intervals of time. At the same time continuous readings of the electrical output were taken with Kelvin watt balances, subsequently checked and found to be correct. The numerous results are given in detail in the appended sheets. The specification required that the steam consumption at full, three-quarters, half and quarter load should not exceed certain values per electrical horsepower (1 E.H.P.=746 watts), but we have converted the results into pounds of steam per kilowatt (1kw.=1,000 watts).

Each of the machines was tested at full load, but as they were identical it was considered sufficient to take the fractional load tests partly on one machine and partly on another. The results are summarised as follows:—

*Requirements of the Specification.*

Vacuum 25in. at cylinder, rising to 27in. at quarter load.  
Barometer 30in. Steam used in pounds per kilowatt output:—

Full load.	$\frac{3}{4}$ load.	$\frac{1}{2}$ load.	$\frac{1}{4}$ load.
25lbs. ...	26·3lbs. ...	28·7lbs. ...	36·7lbs.

Vacuum 27in. at cylinder, rising to 28in. at quarter load.  
Barometer 30in. :—

Full load.	$\frac{3}{4}$ load.	$\frac{1}{2}$ load.	$\frac{1}{4}$ load.
22·7lbs. ...	24lbs. ...	26·3lbs. ...	33·3lbs.

Actual results of tests made on the machines:—

No. 843. Full load.  $\frac{3}{4}$  load.  $\frac{1}{2}$  load.  $\frac{1}{4}$  load.  
Steam consumption in lbs. per kilowatt 20·06 21·45 23·75 ...

No. 844. Full load.  $33\frac{1}{2}$  per cent. overload.  
Steam consumption in lbs. per kilowatt 21·32 ... 20·88  
Vacuum in inches ..... 26·8 ... 26·3

No. 850. Full load.  $\frac{1}{4}$  load.  
Steam consumption in lbs. per kilowatt 21·6 ... 34·55  
Vacuum in inches ..... 26·8 ... 26·9

All the three machines complied, therefore, with the specification as regards steam consumption in pounds per electrical horse-power, or per kilowatt output, and, in fact, were well under the requirements at full load.

5. *Insulation Tests.*—The insulation was measured as specified by putting an alternating voltage of 2,000 volts on to the armature. In each case the insulation was measured between the armature coils before and after the pressure test. The specification provided that the high voltage should be kept on

for one hour. This was actually done in the case of machine No. 844, but in the case of machines Nos. 843 and 850 it was deemed to be sufficient to keep it on for a quarter of an hour. If the insulation is going to break down it usually does so at once. The machines all stood this test and the armature insulation after the pressure test was as follows:—

No. 844 .....	25 megohms.
No. 850 .....	9 megohms.
No. 843 .....	9 megohms.

6. *Temperature Tests.*—The specification provided that at the end of the eight hours' run on full load with power-factor of 0.9 no electrical part of the machine should rise in temperature more than 40°C. above the air, and after two hours at 33½ per cent. overload no part should rise more than 55°C. above the air, these temperatures being taken with a thermometer.

In the case of the machine No. 844 the following were the temperatures:—

	At end of 8 hours' full load.		At end of 2 hours' overload.
Air temperature.....	23°C.	...	25°C.
Alternator field .....	41°C.	...	41°C.
„ armature. ....	39°C.	...	40°C.
„ rings ...	55°C.	...	50°C.
Exciter field .....	60°C.	...	61°C.
„ armature ...	43°C.	...	45°C.

The other two machines were not stopped at the end of the eight hours' run, but ran right on for two hours more at overload, and then the temperatures were taken:—

No. 850.	At end of 10 hours' run.
Air temperature .....	25°C.
Alternator field .....	43°C.
„ armature .....	39°C.
„ rings .....	46°C.
Exciter field... .	60°C.
„ armature.....	45°C.

No. 843.

At end of 10 hours' run.

Air temperature .....	24.5°C.
Alternator field .....	39°C.
„ armature .....	39.5°C.
„ rings .....	40°C.
Exciter field.....	56°C.
„ armature.....	44°C.

All the machines, therefore, complied with the temperature requirements.

7. *A Parallel Running Test.*—The three machines were run in parallel without difficulty.

8. *A Voltage Drop Test.*—The specification provided that the voltage drop from no load to full load should be 3 per cent. on non-inductive load, and 12 per cent. on an inductive load of power-factor 0.9. The machines were accordingly so tested with the following results:—

No. 843.

Voltage drop on inductive load. Power-factor = 0.89 no load to full load:—

	Voltage at no load.		Voltage at full load.		Drop. Per cent.		Load. Kw.
Experiment 1 .....	500	...	445	...	11	...	340
Experiment 2 .....	487.5	...	452.5	...	7.2	...	297
Experiment 3 .....	497.5	...	452.5	...	9.0	...	321

Voltage drop on non-inductive load:—

Experiment 1 .....	467.5	...	447.5	...	4.3	...	315
Experiment 2 .....	468.5	...	450	...	4.0	...	308
Experiment 3 .....	467.5	...	450	...	3.75	...	298

The difficulty of keeping the load constant on water resistance accounts for the variation. But it is clear that the drop on full inductive load at 0.9 power-factor is less than 12 per cent., whilst the drop on non-inductive load is rather more than 3 per cent.

No. 850.

Voltage drop on inductive load. Power-factor = 0.89. No load to full load :—

	Voltage at no load.		Voltage at full load.		Drop. Per cent.		Load. Kw.
Experiment 1 .....	495	...	435	...	12	...	300
Experiment 2 .....	500	...	443.5	...	11.2	...	305.1
Experiment 3 .....	497.5	...	445	...	10.5	...	302

Voltage drop on non-inductive load :—

Experiment 1 .....	470	...	450	...	4.2	...	298.2
Experiment 2 .....	470	...	450	...	4.2	...	294
Experiment 3 .....	467.5	...	442.5	...	5.1	...	300
Experiment 4 .....	467.5	...	445	...	4.8	...	297.2

The drop in inductive load was therefore 12 per cent. at least, and on non-inductive load from 4 to 5 per cent.

In the case of No. 844, one experiment was made which showed that the drop on non-inductive load from no load to 300kw. full load was just over 5 per cent.

It is, therefore, clear that whilst the machines complied on inductive load with what we assume to be the requirements, they exceeded the stipulated 3 per cent. on non-inductive load by 1 to 2 per cent.

At the conclusion of all the numerous experiments, the electrical instruments were carefully checked against correct standards.

The conclusion is that the three turbo-alternators complied with the specification in all respects, except that they slightly exceeded the specified voltage drop on non-inductive load.

The following theory shows the manner in which the observed quantities are related to one another and to those required to be known.

We assume that the three-phase power is being taken up in three equal and inductive star-connected resistances as shown in Fig. 15. The E.M.F.s acting in these circuits differ, therefore, in phase by 120deg. If OA, OB, OC (*see* Fig. 16) represent the maximum values of the P.D.s between the ends and middle point O of these three

resistances, then the sides AB, AC, BC of the equilateral triangle ABC will represent in relative phase and magnitude the P.D.s between the three mains X, Y, Z. In other words,  $AC = V_{xz}$ ,  $AB = V_{xy}$ , and  $BC = V_{yz}$ . These P.D.s obviously differ 60deg. in phase and are equal to one another. If we call the maximum value of each of these last-named periodic P.D.s  $V_1$ , and if we call the maximum value of the P.D.s in each of the three branches of the power-absorbing resistance, which are represented by OA, OB or OC,  $V$ , then it is obvious that

$$V_1 = \sqrt{3}V.$$

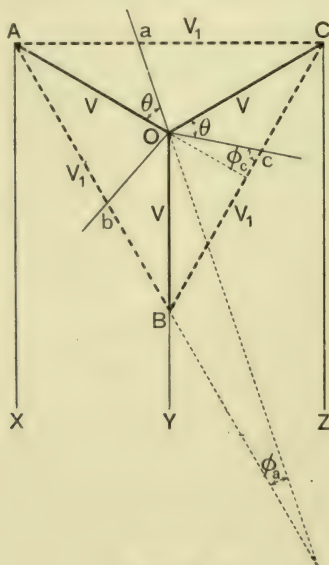


FIG. 16.

If, then, the three branches of the power-absorbing resistances are inductive, the currents in these resistances lag behind the P.D.s between their extremities. Hence, if we desire on the same diagram to draw three lines representing the currents in these three branches, in proper relative phase to the P.D.s, we must represent them by three lines  $Oa$ ,  $Ob$ ,  $Oc$ ,

drawn as in Fig. 16. The equal angles  $AOa$ ,  $BOb$ ,  $COc$ , are the angles of phase difference or lag of current in these inductive circuits behind the voltage. Let this angle be denoted by  $\theta$ . Then if we denote by  $\phi_a$  and  $\phi_c$  the phase angles between the currents represented by  $Oa$  and  $Oc$ , and the P.D.s between the points BC and AB respectively, it is obvious that we have

$$\phi_a = \theta - 30\text{deg.},$$

$$\phi_c = \theta + 30\text{deg.}$$

Hence, if in the circuits OA, OC we insert two wattmeters  $K_x$  and  $K_y$ , the series coils of which convey the currents represented by  $Oa$  and  $Oc$ , and the shunt coils of which are connected between the points AB and BC respectively, it is clear that the scale readings of these wattmeters, which we may represent by  $W_a$  and  $W_c$ , assuming them to read watts directly, are related to the maximum values of the currents, voltages and angles of lag, as follows:—

$$W_a = \frac{\sqrt{3}}{2} AV \cos(\theta - 30),$$

$$W_c = \frac{\sqrt{3}}{2} AV \cos(\theta + 30),$$

where A represents the maximum value of the current in the branches OA or OC, and V represents the maximum value of the P.D.s between the ends of these branches—viz., between O and A or O and C. It is not difficult to show that

$$\tan \theta = \sqrt{3} \frac{W_a - W_c}{W_a + W_c},$$

or that, if we write  $\frac{W_c}{W_a} = a$ ,

$$\text{then} \quad \cos \theta = \frac{a + 1}{2\sqrt{a^2 - a + 1}}.$$

The whole power taken up in the three-phase power-absorbing resistance is, therefore, given by the sum or difference of the two wattmeter readings, according as  $\theta$  is

less than 60deg. or greater than 60deg. When  $\theta=60\text{deg.}$ , then  $\cos \theta=\frac{1}{2}$  and  $\alpha=0$ .

It is easily seen from Fig. 16, that when the angle of lag—viz.,  $\theta$ —is equal to 60deg., one of the wattmeters—viz.,  $K_z$ —will read zero, because the current in its series coil, represented by  $Oc$ , as regards phase, will be 90deg. behind the current in its shunt coil, which in turn is represented in phase by the line  $BC$ . If, then, the angle of lag  $\theta$  increases beyond 60deg. the direction of reading of this wattmeter will reverse, and it will have to be added negatively—that is, subtracted from the reading of the other wattmeter. If we are dealing with a three-phase power-absorbing circuit of unit power-factor—that is, an inductionless circuit—the angle of current lag—viz.,  $\theta$ —is equal to zero. Then  $\cos \theta=1$ , and, therefore,  $\alpha=1$ , or  $W_a=W_c$ . The two wattmeters, therefore, show the same scale reading, and the whole power taken up in the three branches is given by the sum of the readings of the two wattmeters, or by twice that of one wattmeter.

If the currents in the three branches at any instant are denoted respectively by  $a_x, a_y, a_z$ , then we must have

$$a_x + a_y + a_z = 0.$$

Also, if the drops of potential down these branches at any instant are denoted by  $v_x, v_y, v_z$ , respectively, and the instantaneous P.D.s between their ends by  $v_{xy}, v_{yz}, v_{zx}$ , then we have

$$v_{xy} = v_x - v_y,$$

$$v_{yz} = v_y - v_z,$$

$$v_{zx} = v_z - v_x.$$

The power of  $w$  taken up in the three branches at any instant is given by the equation

$$\begin{aligned} w &= a_x v_x + a_y v_y + a_z v_z, \\ &= a_x v_x + (-a_x - a_z) v_y + a_z v_z, \\ &= a_x (v_x - v_y) - a_z (v_y - v_z), \\ &= a_x v_{xy} - a_z v_{yz}. \end{aligned}$$

Accordingly, the *mean power* taken up in the three branches is the mean value of this last expression throughout a

complete period. This, however, is the arithmetic sum or difference of the scale readings of the wattmeters  $K_z$  and  $K_x$ . It is clear from the above equations and from the diagram in Fig. 16 that the total power  $P$  taken up in the three branches is given by the expression

$$P = W_a + W_c,$$

$$\text{or} \quad P = \frac{\sqrt{3}}{2} AV \{ \cos (\theta + 30) + \cos (\theta - 30) \}.$$

If, then,  $\theta = 0$ ,  $\cos \theta = 1$  and  $\alpha = 1$ , we have  $W_a = W_c$ , or the wattmeters read the same, and their scale readings must be added arithmetically. Then

$$P = 3 \frac{A}{\sqrt{2}} \cdot \frac{V}{\sqrt{2}},$$

or  $P$  is equal to three times the product of the R.M.S. value of the current—viz.,  $A/\sqrt{2}$ , and the R.M.S. value of the volt fall  $V/\sqrt{2}$  in each star branch. This must clearly be the case.

Let us, then, take another extreme case—viz., when  $\theta = 90^\circ$ , or the circuits are infinitely inductive. Then

$$\cos \theta = 0, \quad \alpha = -1, \quad \text{and} \quad W_a = -W_c.$$

The wattmeters read the same numerically, but their scale readings must be subtracted from each other to obtain the true power taken up in the whole circuit, which is then obviously zero. Hence we have the following practical rules for measuring the power taken up in the three-phase circuit.

1. Arrange the two wattmeters as described in the A and C star branches, and connect the shunt coils so that each instrument gives a reading.

2. From the arithmetic sum and difference of these readings calculate the value of  $\tan \theta$ , and ascertain whether the angle of lag of current behind voltage in each branch is less or greater than  $60^\circ$ .

3. If  $\theta$  is less than  $60^\circ$ , add the readings of the two wattmeters. If  $\theta$  is greater than  $60^\circ$ , subtract the wattmeter readings to obtain the real power taken up in the whole circuit.

The rule may be stated in another manner not dependent upon an *angle of lag*, as follows:—

Determine first the *power-factor* of each branch. If each branch consists of a water resistance in series with an inductance coil of low resistance, the power-factor is obtained at once by measuring with one and the same electrostatic voltmeter the fall in voltage over the water resistance alone and that over the water and inductive resistance together. The ratio of these two voltages is the power-factor. For if  $R$  is the resistance of the inductionless water resistance, and if  $r$  is the resistance of the inductance coil of which the inductance is  $L$ , then, if the frequency  $n=p/2\pi$ , we have, as the value of the current  $A$  in that branch, either of two expressions

$$A = \frac{V_0}{R} = \frac{V}{\sqrt{(R+r)^2 + p^2 L^2}},$$

where  $V_0$  is the volt drop down the water resistance and  $V$  is the volt drop down the whole branch.

Hence,

$$\frac{V_0}{V} = \frac{R}{\sqrt{(R+r)^2 + p^2 L^2}}.$$

But the right-hand side of this expression is the power-factor of the branch, for it is the ratio of the resistance to the impedance of the circuit, provided that the ohmic resistance  $r$  of the inductance coil is small compared with that of the water or inductionless resistance in series with it. Accordingly the ratio  $V_0/V$  gives us the power-factor of the branch. Then, if this ratio is *greater* than 0.5, the wattmeter scale readings are to be added, and if it is less than 0.5 they are to be subtracted, to obtain the whole power taken up in the inductive three-phase resistance. This last method is, of course, only applicable when the inductive resistance is built up of a water resistance and an iron-cored coil of wire.

Accordingly, if we have to test a three-phase alternator on inductive load, we require two wattmeters, three ammeters, and at least one voltmeter, but preferably three, to obtain all the electrical readings which will enable us to calculate the

true power taken up in kilowatts in the inductive resistance, and therefore the weight of steam consumed per hour per kilowatt output by the steam alternator.

If, instead of being star-connected, the polyphase circuit is delta-connected, then the arrangements required will be the same, except that the series coils of the two wattmeters will be placed in the lines X and Z instead of in the branches. All other connections will, however, be the same, and the same rules will apply to determine whether the wattmeter readings are to be added or subtracted.

**§ 20. Differential Methods of Testing Alternators**—In testing one single alternator the same difficulty arises as to the wasteful consumption of the power as in the case of an efficiency test made with a single continuous-current machine. We have to supply the whole of the power that is dissipated in the resistance. Hence any plan which enables us to avoid the actual power expenditure and yet causes the machine to give its full-load current, is an economical advantage in the testing of large machines. Methods for the differential testing of alternators were suggested by Mr. W. M. Mordey in 1893.\* Mr. Mordey's arrangements were originally suggested with reference to his own alternator, which has a stationary armature divided into sections and a revolving field, but they have been found equally applicable in other cases to machines having a stationary field and a revolving armature. One plan by which an alternator can be made to yield its full-load current with the armature without absorbing the full-load power is as follows:—The diagram in Fig. 17 represents a Mordey alternator in outline, the closed spiral line being the armature circuit. If this armature circuit is opened at a non-symmetrical point and the two unequal sections of the armature joined up so as to oppose one another electrically, an ammeter being inserted, then, on revolving the field

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\* See *Journal of Inst. Elec. Eng.*, Vol. XXII., p. 116, 1893, "On Testing and Working Alternators."

magnet, E.M.F.s. are produced in the two parts which oppose each other, and the external resistance can be so varied that the current which flows through the whole armature is the full-load current of the machine, as shown by the ammeter. When this is the case, the power required to

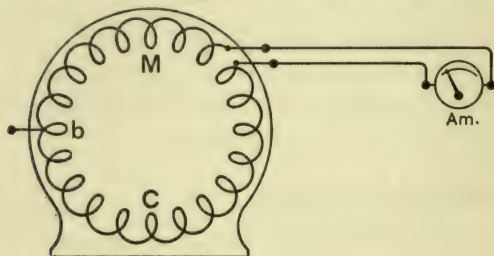


FIG. 17.

drive the field magnets is equal to the power absorbed in friction, eddy currents and  $C^2R$  losses in the armature of the alternator, and gives us, therefore, the means at once of calculating its efficiency. The field magnets can be driven by means of a direct-current motor whose efficiency has previously been determined electrically, and hence, from the

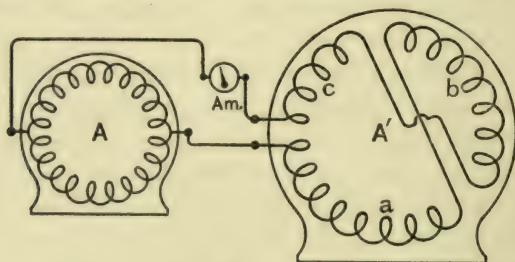


FIG. 18.

electrical power put into this motor, we are enabled to determine the whole power absorbed in the alternator.

Another method, which has the advantage of being a complete electrical method for testing a fixed-armature alternator, is as follows;—The alternator  $A'$  (*see* Fig. 18) has its armature

divided into three parts *a*, *b*, *c*, which are so joined up that the parts *a* and *b* neutralise one another as far as E.M.F. and power are concerned. This machine is then belted to a generator, A, which is made to supply the current to A', and the two machines are run up into synchronism, and without any stop the belt is thrown off. The machine A' then runs as an alternating-current motor in virtue of the power absorbed by the part of the armature marked *c*. If the field of A is suitably excited and the part *c* is properly proportioned, the full working current proper to the machine A' may be forced through the whole armature, and all the internal conditions of full load will be fulfilled, with the exception that, as in the other similar methods, the torque will not be in the same sense round the armature. A single wattmeter reading taken at the terminals of the machine A' will give the whole electrical power transmitted to the machine and absorbed in mechanical and electrical friction, eddy currents and hysteresis, as well as in excitation, if the machine drives its own exciter.

In a discussion which took place on this Paper, Mr. Leslie Miller described a method of the above kind, suggested by Prof. Ayrton and Dr. Sumpner, applicable to machines with a fixed field and rotating armature. This plan, put into practice with a Ferranti alternator, was as follows:—The field-magnet coils of the alternator were connected in parallel, so that the current passed round the upper and lower coils in opposite directions. The Ferranti alternator was driven by a separately excited direct-current motor, and the shafts of the two machines were coupled together. Throughout the whole experiment the currents in the two halves of the Ferranti field were so adjusted, by means of a variable resistance in one of them, as to produce sufficient E.M.F. in the armature (which was short-circuited by an alternating-current ammeter) to send the full-load current through the armature. The alternator was run at different measured speeds, and the electrical power required to drive the direct-current motor

measured. Then, subtracting the power wasted in heating the armatures of the Ferranti alternator and direct-current motor, the power was obtained which was expended on eddy currents, hysteresis and friction of bearings in both dynamo and motor. The power wasted in the motor from these causes was then separately determined by ascertaining the power to drive the motor at the same speed when the spindle was uncoupled from that of the alternator, so that finally the

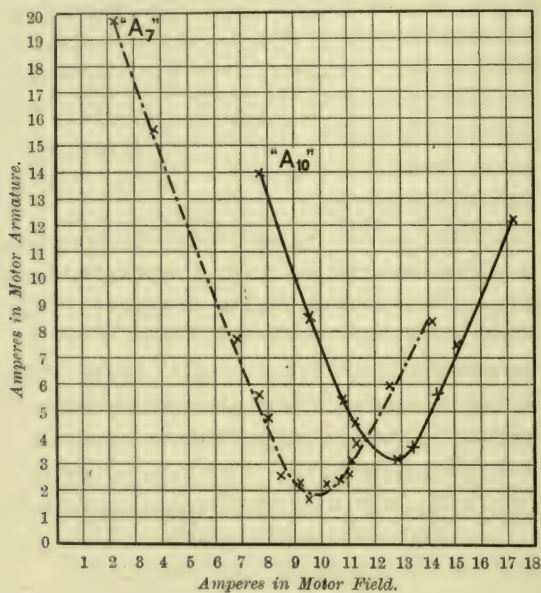


FIG. 19.

power wasted in the Ferranti alternator in eddy currents and in friction was obtained. Knowing this internal loss, the value of the efficiency was obtained as already described.

In connection with this experiment, Mr. Mordey pointed out in his Paper the important fact that, if an alternator is run as a synchronous alternating-current motor, there is a certain field excitation which causes the motor to take in the minimum current into its armature, and that, if the

motor field is strengthened or weakened, the alternating armature will take in more current.

The form of the curve showing the variation of current in the alternating armature with current in the alternator field when the machine is used as a motor is given in Fig. 19. It was found that the same mode of variation of the armature current took place when the alternator, worked as a motor, had load put upon it, and at constant speed for each load there is an excitation that corresponds with the minimum armature current, and the armature current is increased by either strengthening or weakening the machine field. The excitation that gave the lowest armature current, or the highest efficiency and highest power-factor, was found to be the same at all loads with the same impressed P.D.

§ 21. **Alternating-current Transformer Testing.**—In testing an alternating-current transformer of the single-phase or polyphase type, we have to determine the efficiency of the instrument as a transforming device, which involves the determination of the *internal losses*; and also to determine the change in secondary terminal P.D. which takes place as the transformer is loaded up whilst the primary terminal P.D. remains constant, commonly called the *secondary drop*, and also some experiments have to be made to test the *insulation* of the transformer, and finally, some tests, more or less prolonged, to ascertain the *ageing quality* of the iron core.

In making a test of an alternating-current transformer it is desirable to ascertain, first, if possible, the dimensions of the core and the details of the windings of the various circuits, as without this it is impossible to calculate from the observed quantities the magnetic flux and magnetic flux density in the core, or to estimate the hysteresis loss per cubic centimetre. These figures should be obtained and noted, as well as the sizes of the windings and the arrangement of the coils, and also the manner of the insulation,

whether by solid insulating materials or by oil, and also the manner in which the heat produced in the coils is removed, *i.e.*, whether by air cooling or by oil cooling, and whether the air or oil is caused to circulate in any manner. A note should be made of the radiating surface of the transformer, or of the iron case in which it is contained. The next step is to measure the copper resistance of the coils. In the case of the high tension or fine wire coils, this is done by means of Wheatstone's bridge, but the secondary circuits, or thick wire coils, must previously be short-circuited by a thick band of copper connecting the low-tension terminals. In the use of the Wheatstone bridge the same precautions must be taken as in the case of the measurement of the resistance of the field magnets of a dynamo—that is to say, the battery key must be put down first, and be kept down for some time before the galvanometer circuit is closed, to ascertain if the balance is obtained. The resistance of the low-tension or thick wire copper circuit must be obtained by means of a standard low resistance, and the method by fall of potential as described in section 2 of this Chapter, under the head of "Dynamo Testing." In carrying out these measurements, the temperature of the coils should be noted, so that the resistances can be subsequently corrected to the value which they would have at the temperature of the transformer coils when working. The next procedure is to ascertain the core losses in the transformer when working at the frequency for which it is constructed. In the case of single-phase transformers this is carried out with the appliances which are described in Chapter V., Vol. I. of this HANDBOOK, section 5, under the heading "Measurement of the Power taken up in the Case of High-tension Alternating-current Circuits." When dealing with those transformers which are used for the distribution of high-tension current and its reduction to low-tension current—for example, transformers employed for reducing pressure from 2,000 to 200 or 100 volts—it is usual to measure the iron or core loss by measurements taken on

the fine wire or high-tension circuit of the transformer. The arrangements which are found convenient for this apparatus are as follows (see Fig. 20):—

The transformer,  $T$ , to be tested has its primary or fine wire circuit,  $p$ , connected through a switch,  $k_1$ , to a high-tension circuit,  $M_1$ ,  $M_2$ , supplying current at the voltage required by the transformer, say 2,000 volts. In parallel with this primary circuit is placed an inductionless resistance,  $r$ , which should be made by joining together resistance coils of the type devised by the Author, called resistance cages (see Vol. I. of this HANDBOOK, page 80). Thus, for instance, if the circuit on which the transformer is being

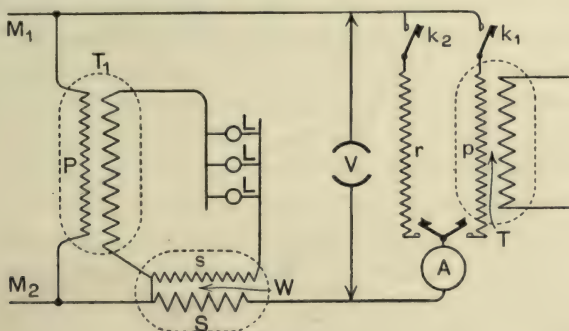


FIG. 20.

tested is a 2,000-volt circuit, then 20 of these cages, assuming each of them to be constructed to stand 100 volts, may be joined in series, and a circuit is thus constructed having a negligible inductance and a resistance of about 2,000 ohms, which, when placed across a 2,000-volt circuit, takes up 1 ampere and dissipates 2,000 watts. If this resistance is found to take up too much power, an inductionless resistance can be constructed of much higher value and absorbing less power, in the manner described in section 6, Chapter IV., Vol. I. of this HANDBOOK. A wattmeter has then to be provided, represented by  $W$  in the diagram, the series coil  $S$ , of which is connected through an ammeter,  $A$ , with the

two switches giving access respectively to the inductionless resistance  $r$  or the primary circuit  $p$  of the transformer under test. A high-tension voltmeter,  $V$ , should then be connected across the terminals of the inductionless resistance,  $r$ . This voltmeter should be of the electrostatic type, and the ammeter,  $A$ , should preferably be a hot-wire ammeter, or one suitable for measuring alternating currents, the readings of which are independent of frequency. These instruments should previously have been carefully calibrated by the methods described in pages 3 and 4, Vol. I. of this HANDBOOK. Another small transformer,  $T_1$ , is then connected, as shown in Fig. 20, in front of the series coil of the wattmeter, and the secondary circuit of this last transformer is joined through a few incandescent lamps,  $L$ , or a suitable inductionless resistance, to the shunt circuit  $s$  of the wattmeter. If we assume that the transformer  $T$  is one stepping the pressure down from 2,000 to 100 volts, then the resistance or lamps put in series with the shunt coil must be so adjusted that, when the current flowing through the series coil of the wattmeter  $S$  is that taken by the inductionless resistance  $r$ , the reading of the wattmeter will be nearly as large as it is possible to obtain on the scale of the wattmeter. The transformer  $T$  is called the auxiliary transformer.

In order to carry out tests with any chance of success or accuracy, the voltage which supplies both the transformer under test and the auxiliary transformer, must be exceedingly constant. It is usually impossible to obtain this on commercial supply circuits, and therefore in the testing of transformers it is desirable, if possible, to have an alternating-current supply furnished from a separate alternator, which is best driven by a direct-coupled electric motor, taking current in turn from secondary cells. In this manner, by regulating the current flowing through the armature of the continuous-current motor, which can be done with great nicety by means of a carbon plate rheostat (*see* page 81, Vol. I. of this HANDBOOK), the observer is able

to control the speed of the alternator so as to preserve the pressure at the high-tension terminals of the auxiliary transformer and the transformer under test perfectly constant. If the experimentalist cannot obtain a separate alternator, but is obliged to make his tests with current taken from a public alternating electric supply, then it may be necessary to insert in the circuit leading to the auxiliary transformer a variable choking coil, by means of which the pressure can be controlled within certain small limits. Thus, suppose, for instance, that the commercial supply circuit is nominally 2,000 volts, but, as a matter of fact, varies from 1,950 to 2,100, then, by the introduction of a variable choking coil, it is possible to keep the pressure at the terminals of the auxiliary transformer always constant at 1,950 volts, and so cover up the irregularities of voltage which are inseparable from the operation of a public electric supply station. A plan of this kind was adopted by the Author some years ago in an extensive series of tests of transformers made for the City of London Electric Lighting Company.

When the observer is satisfied that the primary pressure is constant, and that he has the means for measuring accurately this voltage and the current, the first step is to connect to the circuit of the wattmeter the inductionless resistance  $r$  and to measure with the ammeter  $A$  the current flowing into it and the difference of potential between its extremities with the voltmeter  $V$ . The product of these amperes and volts gives the watts taken up in the inductionless resistance  $r$ .

In order to prevent variations of temperature from affecting the current, this inductionless resistance should be made of some material, like platinoid, with a small temperature coefficient. After testing the current through the shunt coil of the wattmeter, the observer next takes a reading of the wattmeter corresponding to a current,  $A$ , flowing into the inductionless resistance  $r$  and a potential difference at the ends of this resistance equal to  $V$ . Let this scale reading of the wattmeter be denoted by  $w$ . We then know that

corresponding to a wattmeter reading  $w$ , the true power taken up in the inductionless resistance is equal to  $AV$  watts. The next step is to disconnect by the switch the inductionless resistance  $r$  and to connect to the same circuit the primary circuit  $p$  of the transformer under test, the secondary or low-tension circuit of this transformer remaining open. The observer will now obtain a scale reading of the wattmeter which may be denoted by  $W$ . At the same time the current passing into the primary circuit of the transformer must be measured. Generally speaking, this current will be so much smaller than the current  $A$  which flowed into the inductionless resistance in the previous experiment that the same ammeter will not be suitable for measuring the two currents. In the one case the current may approximate to one ampere in value, in the next case it may be a very small fraction, or one-twentieth of an ampere.

In default of a suitable ammeter for measuring accurately small alternating currents the following procedure may be adopted:—In place of the ammeter  $A$  (*see* Fig. 20) is inserted an inductionless resistance having a value of 1,000 or 2,000 ohms. If, then, the current flowing through it is one-twentieth of an ampere, it will create a fall of potential down this resistance of the order of 50 to 100 volts, and this can be measured by a suitable electrostatic voltmeter such as that shown on page 461, Vol. I. of this HANDBOOK. Knowing the ohmic value of this resistance and the volt drop down it, we have at once the value of the small current flowing into the primary circuit of the transformer.

Returning, then, to the wattmeter readings, we have ascertained, by the two experiments corresponding to a power  $AV$  watts, that the wattmeter gives a scale reading  $w$ , and the question then arises, what is the value of the true power corresponding to the scale reading  $W$ ? If previous experiments have shown that the scale reading of the wattmeter is proportional to the power passing through it (a matter which must not be taken for granted), we can then say that the

power corresponding to a scale reading  $W$  is  $\frac{W}{w} AV$  watts. Hence, the whole power taken up in the transformer at the moment of the reading is given by the above expression. This power is made up of two parts: a part due to the loss in the iron core of the transformer, called the iron loss, and a part due to the heating effect of the current in the primary coil, which is called the copper loss at no load. If the current flowing into the primary coil of the transformer at the moment when the wattmeter reading was taken is denoted by  $a$  (in fractions of an ampere), and if the resistance of the primary circuit of the transformer is denoted by  $R_1$ , then the copper loss at no load is  $a^2 R_1$  watts. In general, the heating effect of this small current is so slight that it will not be necessary to correct the value obtained for the resistance of the primary circuit of the transformer at ordinary temperatures.

Deducting this copper loss from the total loss, we then have, as the expression for the iron loss  $I$ , or core loss of the transformer, the equation

$$I = \frac{W}{w} AV - a^2 R_1.$$

As a matter of practical precaution it is always desirable that the inductionless resistance  $r$  should take up about the same power as the transformer  $T$  under test at no load, for under these conditions the reading of the wattmeter  $W$  would be very nearly equal to the reading  $w$ , and there will be the less chance that a want of proportionality between the scale readings of the wattmeter and the true power passing through it, will introduce an error into the calculations. If we are testing a polyphase transformer, either two or three-phase, then it will be necessary to employ two wattmeters, and to connect them up with the transformer as described in section 12, Chapter V., page 515, Vol. I. of this HANDBOOK.

The sum or difference of the readings of these wattmeters will give the true power being taken up in the two or three-phase

transformer, as the case may be. If the transformer is a two-phase transformer, then the power taken up is always the sum of the readings of the wattmeter. If the transformer being tested is a three-phase wattmeter, then the power taken up may be the sum or difference of the wattmeter readings according as the power-factor of each of these circuits is greater or less than 0.5, as already explained in section 19, under the heading of "The Testing of Polyphase Alternators." It has been shown\* that the iron loss or core loss in the transformer is constant at all loads, and hence, when once the core loss has been measured at no load, we have only to add on to this the copper losses at various loads to obtain the total internal loss in the transformer at these loads.

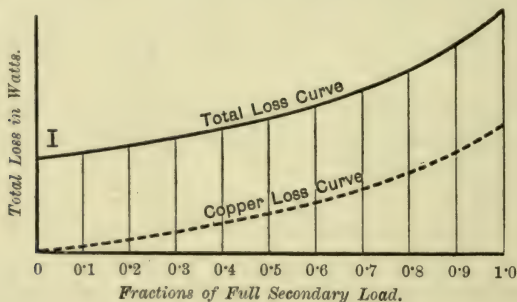


FIG. 21.

Having measured the iron loss, or core loss, in the transformer, and also the resistance of the copper circuits, the best method of setting out an efficiency curve is by means of a total loss curve plotted as follows:—Two rectangular ordinates are taken, along the vertical one being set off distances to represent in watts the internal loss in the transformer, and on the horizontal one distances to represent the total secondary output of the transformer in watts or kilowatts (*see* Fig. 21). It is best to divide the total nominal full output of the transformer on the secondary side into 10 parts,

\* *See* J. A. Fleming, "Experimental Researches on Alternate Current Transformers." *Journal Inst. Elec. Eng.*, Vol. XXI., p. 592, 1892.

and to set off 10 points at equal distances along the horizontal to represent the decimal fractions of the full secondary output. Thus, for instance, in testing a 30kw. transformer, equal distances would be set out to represent 3kw. along the horizontal scale. On the vertical ordinate is then set off a distance representing the measured iron loss in the transformer, and a horizontal line may be drawn parallel to the axis of Y to represent this constant iron loss. We have then to calculate the copper losses in the transformer at various loads. This is done as follows:—The transformer must be set at work on a circuit and be loaded up so that the full nominal secondary current is passing through the secondary coil. For example, suppose the transformer is a 30kw. transformer, and that the secondary terminal P.D. is 200 volts; then the full nominal secondary current is 150 amperes. A low resistance amperemeter, reading up to 150 amperes, should then be joined across the secondary terminals of the transformer, and a voltage applied to the primary terminals which is raised in value until the ammeter on the secondary circuit reads 150 amperes. The current flowing into the primary circuit should then be measured. These two currents will have the same value as when the transformer is at work on full load; but the actual power taken up by the transformer will, of course, merely be the sum of the copper and iron losses under these conditions. Hence not nearly as much power will be required as if the transformer was tested under its usual working circumstances at full load. Nevertheless, the internal losses in the transformer are very nearly the same, and if the transformer is left to itself for some hours its temperature will gradually rise until it reaches a nearly constant amount. In the case of large transformers in iron cases this constant final temperature may not be reached until the transformer has been connected with the circuit for 24 hours, but, in any case, the transformer should be kept working as described until a thermometer, placed somewhere in the interior of the

coils, ceases to show a rise in temperature. When the transformer has thus been brought to its constant temperature it should be switched off the circuit, and the resistance of the primary and secondary coils should be measured as quickly as possible in the manner described at the beginning of this section. These readings give us the value of the *hot resistance* of the two circuits. If this process cannot be carried out, or if the transformer cannot be loaded up in the ordinary manner at full load, then some good guess must be made at the final temperature which the transformer is likely to require, and the copper resistance measured at ordinary temperatures must be corrected, as described in section 2 of this Chapter, to obtain the hot resistance of these circuits. In any case, let  $R_1$  represent the hot resistance of the primary circuit and  $R_2$  represent the hot resistance of the secondary circuit, and let  $A_2$  represent the full-load secondary current of the transformer, which, in the case we are considering—namely, the 30kw. transformers—would be 150 amperes. Then, if  $N_1$  is the number of turns in the primary coil and  $N_2$  the number of turns in the secondary coil, it has been shown\* that the full-load primary current  $A_1$  is nearly equal to  $\frac{N_2}{N_1}A_2$ ; hence the total copper loss,  $C$ , in the transformer at full load is given by the expression

$$C = R_2 A_2^2 + R_1 \left( \frac{N_2}{N_1} \right)^2 A_2^2.$$

Hence we have determined from these measurements the iron loss at no load and the copper loss at full load, and the sum of these two values gives us the total internal loss in the transformer at full load. Let this be denoted by  $L$ , then  $L = C + I$ , signifying that the total loss is equal to the copper loss together with the iron loss, and if the output of the transformer at full load in watts is denoted by  $O$ , and the

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\* See "Experimental Researches on Alternate-current Transformers," *Journ. Inst. Elec. Eng.*, Vol. XXI, 1892.

total loss in watts is denoted by  $L$ , then the efficiency of the transformer at full load is given by the expression

$$e = \frac{O}{O + L}.$$

We can easily obtain the efficiency at intermediate loads, or, what is the same thing, the total loss at intermediate loads, by inserting the proper value in the expression for the total copper loss of any decimal fraction of the full-load secondary current  $A_2$ . Thus, for instance, we may give in the above equation to  $A_2$  successively values of the full-load secondary current, equal to 0.9 of this current, 0.8 of this current, &c., down to 0.1 of this current, and, inserting these values instead of  $A_2$  in the above equation, we obtain the value of the copper loss at the various fractions of full load; and hence we can set out, as in Fig. 21, the total loss curve, where the curved dotted line denotes the variation of copper loss with load, the ordinates of the firm line curve being the sum of the constant iron loss at no load and the copper loss at the load considered, whilst from the last curve we can obtain an efficiency curve as already explained.

For additional information on these experiments, the reader is referred to the Paper by the Author, "Experimental Researches on Alternate-current Transformers," *Journ. Inst. Elec. Eng.*, Vol. XXI., page 594, 1892; and to the Author's treatise on "The Alternate-current Transformer," Vol. I., where details are given of tests made on the total losses of numerous forms of transformers.

**§ 22. Measurement of the Secondary Drop of Transformers.**—An important measurement, in the case of alternating current transformers, is the determination of the amount by which the P.D. of the secondary terminals changes when the transformer is loaded up to its full load, the primary circuit terminal P.D. remaining meanwhile constant. It is usual to express this change in voltage as a percentage of the voltage when the transformer is unloaded. Thus, for instance, if a

transformer is stated to have *2 per cent. secondary drop*, it means to say that, if the secondary P.D. is 100 volts when the transformer is unloaded, it becomes 98 volts when the transformer is fully loaded, the primary voltage meanwhile remaining the same. If a perfectly constant primary P.D. can be secured for the transformer under test, then the measurement of the secondary drop is easily made by connecting a voltmeter, preferably an electrostatic voltmeter, across the secondary terminals, and loading up the transformer on its secondary side until an ammeter placed in the secondary circuit shows that the transformer is giving its full-load current. When this is obtained, a reading of the secondary voltage is made, and also of the primary voltage.

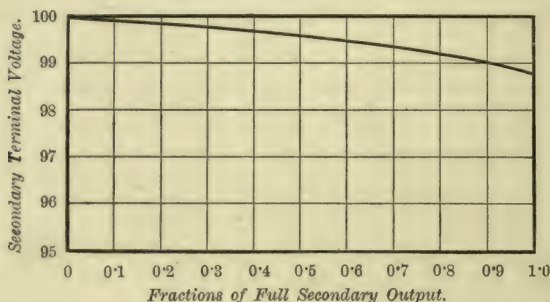


FIG. 22.

The load is then removed from the secondary side, the primary voltage is brought back again to its right value, and a second reading of the secondary voltage is taken. The difference of these two secondary voltages, expressed as a percentage of the voltage at no load, gives the secondary drop. It is usual to set out a secondary drop curve (*see* Fig. 22), the ordinates of which represent the secondary terminal P.D. corresponding to the various decimal fractions of the full load of the transformer.

It is exceedingly difficult to make measurements of secondary drop when the transformer is connected to a commercial circuit, the voltage of which is always changing. It can be

done, however, by introducing into the primary circuit a variable choking coil, so as to cut off all the voltage above a certain amount which is supplied to the primary terminals.

This procedure, however, is impracticable if the primary voltage is changing very rapidly. In that case another method may be adopted, provided that a low-reading alternating-current voltmeter is at hand. To carry out this method we require two identical transformers,  $T_1$ ,  $T_2$ , which are joined up to the same primary circuit,  $M_1$ ,  $M_2$ , and a load, in the form of lamps, LL, must be provided equal to the full secondary load of one of the transformers, which can be switched on by means of switches, S, S, to one of the trans-

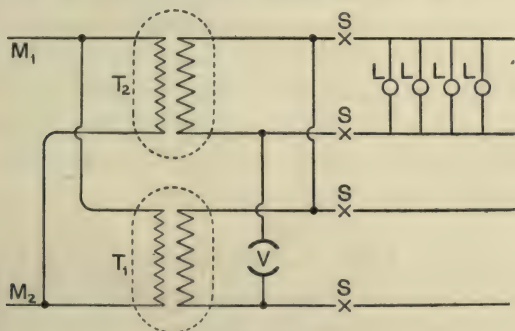


FIG. 23.

formers. The low-reading voltmeter  $V$  is then connected, as shown in Fig. 23, between two corresponding secondary terminals, the other two being joined together by a piece of wire. This voltmeter then reads *the difference* between the P.D.s of the secondary terminals of the two transformers, and when both transformers are unloaded the voltmeter should give no indication. Its indications are not affected by identical changes in P.D. in the two transformers, and it is not affected therefore by any variation in the primary voltage. If, however, one of the transformers is loaded up to full load, then the voltmeter reads the difference between the P.D.s of the secondary circuits at no load and at full load—in other

words, it gives directly the drop. For this purpose, if possible, an electrostatic voltmeter should be employed, but instruments of this type for reading 1 or 2 volts are not found in every laboratory.

In the absence of an electrostatic voltmeter, we can use a hot-wire voltmeter of the Hartmann and Braun type, or an electromagnetic instrument may be used, provided that it is suitable for the frequency of the transformers employed. It must, however, be a voltmeter which gives a visible scale-reading for 1 or 2 volts, and must be capable of indicating accurately one-tenth of a volt in the neighbourhood of 1 volt. The secondary drops which have to be measured in practice generally lie between 1 and 5 volts. The accurate determination of the secondary drop is important in the case of transformers which have to work together in parallel, because, if these transformers are not constructed for equal secondary drop, the load between the various transformers in a bank will not be equally distributed, and some of the transformers will become overheated.

**§ 23. Insulation Tests of Transformers.**—An important test in connection with alternating-current transformers is an insulation test, to determine whether the insulation between the primary and secondary coils, and these coils and the core, and also the iron case, is sufficient to prevent disruptive discharges, which would destroy the transformer and cause, perhaps, great injury to person. It is usual to specify that the transformer must stand a voltage equal to double its high-tension voltage, applied for one hour, between the high-tension terminals and the secondary terminals, or between the high-tension terminals and the core or case of the transformer. Thus, in transformers intended for transforming from 2,000 volts to 200, it should be tested as described for an hour with 4,000, or, better still, with 5,000 volts. In addition to this, it is usual to flash on for a moment a still higher voltage between the primary circuit and the secondary

or primary and the core or case. Transformers in practical work are subject to constant but very brief strains or rises in electrical pressure, due to sudden disconnections in the circuit, and the transformer may, therefore, be in actual work momentarily subject to pressures five or ten times as great as its normal working pressure. It is always safe, therefore, to apply for one moment, or flash as it is termed, a transformer with five times its maximum working high-tension potential. Thus, if a transformer is intended to work with 2,000 volts, a pressure of 10,000 volts should be applied momentarily as described, and, if the transformer passes this flash test, as well as the test for one hour with the pressure double the working pressure, it may be assumed to be safe for ordinary purposes, provided it does not become damp or is improperly used.

**§ 24. Temperature Tests.**—A temperature test should always be applied to the transformer by connecting it to a primary circuit at its normal primary voltage, and allowing it to run under full load for at least two hours, and then remain for at least 24 hours connected to the circuit, but unloaded on its secondary side. During this time temperature measurements should be taken by thermometers placed inside the case and against the coils, in order to ascertain what is the final constant high temperature reached by any part of the transformer. Fig. 24 shows a temperature curve of a transformer put through such a 24-hour test, indicating the manner in which the temperature varied during part of that time.

Although it is customary to measure temperatures in transformers by means of thermometers, as above described, this method is not nearly so satisfactory as that of determining the average temperature by the resistance of the copper circuits. If we know the resistance of the primary circuit of the transformer when it is cool—that is, at ordinary temperature, say  $16^{\circ}\text{C}$ .—and if at any moment we measure the resistance of the same circuit when the transformer is warm, we can tell

the temperature of the copper, knowing that this resistance increases 0.1 per cent. per degree Centigrade. Hence, the temperature measurement is best made by such a resistance measurement. Rise in temperature reduces the iron loss in a transformer and increases the copper loss, but no part of the transformer, and when at its final highest constant temperature, could exceed  $70^{\circ}\text{C}$ ., because beyond that point the shellaced cotton coverings which are employed as insulation

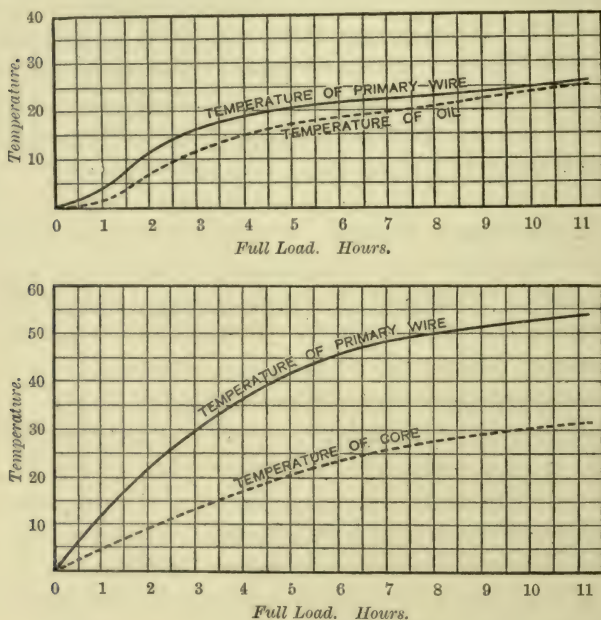


FIG. 24.

to the wire begin to be very brittle, and would soon become carbonised and conductive in large transformers.

The arrangements provided for getting rid of the heat dissipated in the core and coils are most important. In some cases a stream of air is driven through the transformer by means of a fan; in other cases the transformers are immersed in oil, and cold water is circulated through pipes immersed in

the oil. In some cases the iron box in which the transformers are placed are fluted, or have ridges cast on their exterior surface, to increase the radiating surface and so keep down the temperature of the oil. At least 4 square inches of radiating surface should be provided for every watt wasted in the transformer, unless some artificial means of cooling and removing the heat are provided.

The curves in Fig. 24, taken from a Paper on "Transformer Tests" by Prof. W. E. Goldsborough,\* show the very slow rise of a transformer up to its maximum or steady temperature, even when fully loaded. It appears that, for the usual iron-cased transformer, a constant temperature is not reached under 12 to 24 hours at full load.

**§ 25. Ageing Tests of Transformers.**—Another important test, which, however, cannot be carried out in a short time, is the ageing test of the iron. It has already been pointed out (*see* section 17, Chapter IV.) that the result of moderate heating, especially above 60°C. or 70°C., is to increase the hysteresis loss in iron, and, accordingly, that part of the core loss in the transformer which depends upon hysteresis will go on progressively increasing if the core becomes heated in work and if the transformer is kept constantly connected to the circuit. This increase in core loss, in some cases, is very serious. Many instances have occurred in which transformers have had their core loss increased to three times the original core loss in the course of a few months' usage. If the core loss of very large transformers increases in this manner, it involves a very serious increase in the expenditure for keeping the core magnetised. For instance, if a 30kw. transformer has a core loss which, as far as regards the iron core, amounts to 1 per cent. of its full secondary output, this involves an expenditure of 300 watts in the iron core, from one-half to two-thirds of which may be hysteresis loss. If

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\* Read before the National Electric Light Association, U.S.A., May, 1899. See also *The Electrician*, Vol. XLIII., p. 871.

this transformer is kept connected to the surface all day long, then the hysteretic core loss will amount to nearly five Board of Trade units per diem, and if we reckon the value of the Board of Trade unit delivered to the transformer as only worth a penny, an increase in the core loss of 100 per cent. involves an increase of 5d. per diem in the cost of preserving the core magnetised and the transformer ready for delivery of electric energy on its secondary side. Hence, no test of any large number of transformers can be considered as complete which does not involve an ageing test. This, however, is a matter requiring time, and it cannot be hurried. In some specifications for transformers the price is fixed on the basis that the contractor should be paid a certain sum of money on delivery for the transformer, and that the remainder shall be reserved for three months to be handed to the contractor, either with a bonus or with a deduction, according to the result of tests made at the end of three months, to show whether the core loss has remained constant or has increased.

Brands of mild or low carbon sheet steel are now, however, made for transformer cores in which the ageing is extremely small, and there is, therefore, no longer any excuse for employing rapidly-ageing iron for this purpose.

**§ 26. Differential Testing of Transformers.**—In the testing of large transformers the same difficulty as to the expenditure of power occurs as in the case of large dynamo and motor testing. It is necessary to have a mode of testing transformers in which, whilst the circuits are traversed by the full-load currents, the actual expenditure of power is not equal to the normal full-load output of the transformer. This can be achieved by the employment of a method of testing transformers suggested by Dr. W. E. Sumpner, which is similar in principle to the differential method of testing dynamos and motors due to Dr. Hopkinson.

If it is desired to test two large transformers without any greater power expenditure than that due to the internal

losses in the transformers, it can be done if two identical instruments are available, together with a third smaller transformer. In this case the two large and equal transformers have their high-tension circuits joined up in opposition to one another, as shown in Fig. 25, where  $T_1$  and  $T_2$  are the two transformers and  $P_1$  and  $P_2$  are the two primary circuits which are joined together. The two secondary circuits  $S_1$  and  $S_2$  are joined up, as shown in the diagram, to a pair of low-tension secondary mains,  $M_1$ ,  $M_2$ , from which alternating current of the right frequency, and at a pressure equal to the

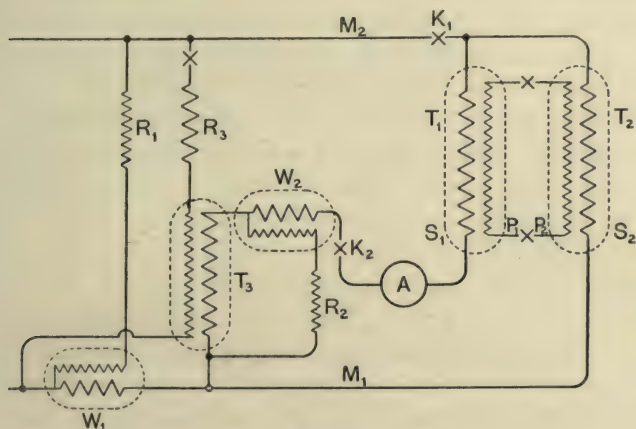


FIG. 25.

normal secondary voltage of the transformers, can be supplied. In the secondary circuit of one of the transformers,  $T_1$ , is placed an ammeter,  $A$ , to measure the current passing through the secondary circuit  $S_1$ . In series with the secondary circuit  $S_2$  is placed the secondary circuit of a small auxiliary transformer,  $T_3$ , the purpose of which is to supply part of the power dissipated in the two transformers  $T_1$  and  $T_2$ , and it corresponds, therefore, with the battery or the third dynamo which is employed in the series differential method of testing dynamos and motors. The primary circuit of this third

transformer  $T_3$  must be connected through a variable resistance,  $R_3$ , with the mains  $M_1$  and  $M_2$  supplying the current to the two transformers. Two wattmeters,  $W_1$  and  $W_2$ , must then be provided, one of which,  $W_1$ , is placed in the main line  $M_1$ , its series coil taking the current supplied to the two transformers  $T_1$  and  $T_2$ , and its shunt coil being connected through a variable resistance,  $R_1$ , between the two mains  $M_1$  and  $M_2$ . The other wattmeter,  $W_2$ , is placed in front of the small auxiliary transformer  $T_3$  and its shunt coil is connected through a variable resistance,  $R_2$ , across the low-tension terminals of the auxiliary transformer  $T_3$ . Two switches,  $K_1$  and  $K_2$ , are also inserted in the circuits as shown.

To measure the efficiency of the two transformers  $T_1$  and  $T_2$  we proceed as follows:—The switches  $K_1$  and  $K_2$  are both closed, and the resistance  $R_3$  in the primary circuit of the auxiliary transformer  $T_3$  is regulated until the current passing through the ammeter  $A$  is equal to the normal full-load secondary current of the transformer  $T_1$ . In that case we know that the primary circuits  $P_1$  and  $P_2$  are being traversed by the full-load primary current proper to the transformers  $T_1$  and  $T_2$ . The auxiliary transformer  $T_3$  is then supplying a part of the power which is dissipated in the transformers  $T_1$  and  $T_2$  in iron and copper losses, whilst some part of this power is being supplied through the mains  $M_1$  and  $M_2$ . If, therefore, we take a reading of the wattmeter  $W_2$  and also of the wattmeter  $W_1$  the sum of these readings is equal to the total iron and copper losses in the two transformers  $T_1$  and  $T_2$ , together with any power lost in the coils of the ammeter  $A$  and of the cables used in connecting up the transformers. Let this last power loss, which may be made as small as we please, be called  $w$ . Then, if  $V$  is the P.D. of the two mains  $M_1$  and  $M_2$ , which, by arrangement, is equal to the normal secondary voltage of either of the transformers  $T_1$  or  $T_2$ , then the efficiency  $e$  of each of the transformers  $T_1$  and  $T_2$  is the square root of the overall efficiency of the two taken together, and, hence, the efficiency ( $e$ ) of either

of the transformers  $T_1$  and  $T_2$  is given by the expression

$$e = \sqrt{\frac{AV - (W_1 + W_2 - w)}{AV}}.$$

The small amount of power,  $w$ , taken up in the ammeter and leads can be obtained as follows:—The switch  $K_1$  is left open, and the low-tension coils of the transformers  $T_1$  and  $T_2$  are short-circuited. The reading of the wattmeter  $W_2$  then gives us the value of the power  $w$ . The iron core losses of the transformers  $T_1$  and  $T_2$  can also be determined by leaving the switch  $K_2$  open and closing the switch  $K_1$  and then observing the reading of the wattmeter  $W_1$ . This last wattmeter should be suitable for measuring about one-tenth of the full load of each transformer. The auxiliary transformer  $T_3$  should be wound so that its low-tension coils can safely pass the full-load secondary current of the transformers  $T_1$  or  $T_2$ , and under these circumstances it should have a voltage of about one-tenth of the normal voltage of the low-tension coils of  $T_1$  or  $T_2$ , and the wattmeter  $W_1$  must have a scale range suitable for measuring this power.

The above-described method of differential testing is well adapted for prolonged trials of transformers in the workshop, because both circuits of the transformers subjected to the test are then traversed by the full-load secondary current, and therefore the transformers themselves are under the same conditions as they would be in working on full load, except that the full-load power is not being dissipated on resistance. In this manner two transformers can be tested with less than one-tenth of the power otherwise needed for a single transformer, and the method has the double advantage that it not only enables large transformers to be tested economically, but also that it can be done on a small alternator. Thus, for instance, if two 100kw. transformers have to be tested, the transformation ratio of which is from 2,000 to 200 volts, if these transformers had to be tested in the ordinary way on

resistances the energy consumption during a six-hours' full-load run would be equal to 1,200kw. hours, a very considerable expenditure. If, however, they are tested differentially, as above described, by Dr. Sumpner's method, then the expenditure of power for the full-load test would be not more than 10kw., and it could be supplied at 200 volts; hence, quite a small alternator would enable the test to be carried out, and the actual expenditure during a six-hours' full-load trial would be only 60kw. hours instead of 1,200.

For further details of the differential method of testing transformers the reader is referred to *The Electrician*, Vol. XXIX., 1892, pages 223, 615.

In those cases in which a single very large transformer has to be tested, it is best achieved by measuring first the iron core loss on open secondary circuit and then measuring the resistance of the copper circuits and calculating the copper loss for a certain assumed working temperature. Thus, suppose that a 150kw. transformer is found to have an iron core loss of 1,000 watts on open secondary and that the copper resistances are 10.0 ohms and 0.4 ohm respectively at 15°C. We increase these by 25 per cent. and call them 12.5 ohms and 0.5 ohm to agree with a maximum working temperature of 60°C. Then, if the voltages are 10,000 and 2,000 on the primary and secondary sides, the copper losses at full load are  $(15)^2 \times 12.5 = 2,812$  watts and  $(75)^2 \times 0.5 = 2,812$  watts. The total loss is then 6,624 watts at full load and the efficiency 96 per cent. The copper loss may also be measured directly by the wattmeter by short-circuiting the secondary coil through an ammeter and applying such a voltage to the primary terminals that the full-load currents circulate through the copper coils. A wattmeter inserted in the primary circuit may then be employed to take two readings, first, of the total power absorption at the impressed voltage, and, second, of the small iron loss at this voltage. The difference of these readings is the copper loss at full load.

§ 27. **Alternate-current Motor Testing. Single-phase Synchronous Motors.**—In testing a single-phase synchronous alternating-current motor we have to determine, in the first place, the efficiency at various loads, which involves measuring the electrical power taken up and the mechanical power given out on the pulley. Secondly, we have to measure the power-factor, *i.e.*, the ratio of the true power taken up by the motor at no-load to the volt amperes or product of the terminal P.D. in volts and the current flowing into the motor. Thirdly, we have to measure the internal losses, dividing them into copper losses, iron losses and frictional losses. Fourthly, a measurement must be made of the starting torque and running torque at various loads; and, lastly, of the overload which will cause the motor to pull up or break up step.

The measurement of the power taken up must be effected by means of a properly constructed alternating-current wattmeter (*see* Vol. I. of this HANDBOOK, Chapter V., page 508). This wattmeter must be carefully calibrated on an inductionless resistance, as described in the section on Alternating-current Transformer Testing, and an examination should be made of the instrument to ascertain whether it reads alternating-current power correctly when the power-factor of the power-absorbing circuit is much less than unity. This, however, will be the case if the instrument is properly constructed, and if there are no metal parts near the movable coil. The power given out by the motor must be taken up by means of a rope brake, or a magnetic or hydraulic brake, as described in the sections on continuous-current motor testing. At the same time that the measurement of the true power taken up by the motor is made by the wattmeter, the current flowing into the motor should be measured by an alternating-current ammeter of the hot-wire type, and the P.D. should be measured by a voltmeter of the electrostatic type. The electrical arrangements should be as shown in Fig. 26, where W is the wattmeter, A the ammeter, V the voltmeter, S the standardising resistance, and M the motor under test. The

motor having been started, and the rope brake carefully applied to the pulley, work is put upon the motor by gradually increasing the weights put into the scale pan of the rope brake, and the tensions on the two sides of the belt are then read, whilst simultaneously the speed  $N$ , or number of revolutions per minute, of the motor is taken, and another observer takes the readings of the amperemeter, the voltmeter and the wattmeter. The wattmeter is then calibrated by switching off the motor circuit and switching on the inductionless standardising resistance  $S$ . This standardising resistance should be of such a size that the power taken up by it is approximately the same as that taken by the motor under test. Supposing that the reading of the wattmeter

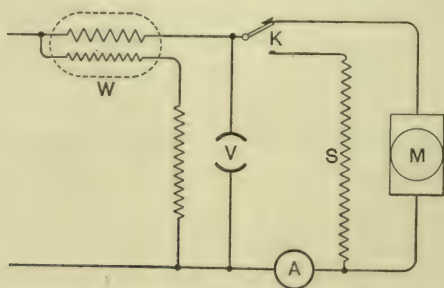


FIG. 26.

when it is connected to the alternating-current motor is  $W$ , the reading of the wattmeter when it is connected to the inductionless resistance is  $w$ , the current flowing into the inductionless resistance at that moment is  $a$  amperes, and the difference of potential between its ends  $v$  volts; then the reading of the wattmeter  $W_1$ , when connected to the alternating-current motor, corresponds to

$$\frac{W_1}{w} \cdot av \text{ watts.}$$

Let this power be called  $W$ . Then also let the current flowing into the motor be denoted by  $A$  and the terminal P.D. in volts by  $V$ ; then the volt amperes or apparent power

supplied to the motor is equal to the product  $AV$ . The ratio of  $W$  to  $AV$  is called the *power-factor* of the motor corresponding to its particular state of load. This power-factor may be anything between 0.5, or even less, and unity, but it will generally be a number not far from 0.7 or 0.8 when the motor is fully loaded.

Having taken readings of the power absorbed, the power given out, and the volt amperes corresponding to various fractions of full load of the motor, we can set out a curve of efficiency; a curve showing the variation of the power-factor with load; and also a curve the ordinates of which are the difference between the power taken in and the power given out, corresponding to various loads, which is called the total loss curve (*see* Fig. 21). A measurement has then to be made of the starting torque and the running torque at various loads, as described in section 18, and finally a test must be made to ascertain the percentage of overload which will cause the motor to break out of step, and to be pulled up by dropping out of synchronism with the working current. This is easily done, by means of a power-absorbing brake, by simply loading up the brake until the motor stops and noticing the reading of the spring balance at that instant.

If it is desired to separate out the internal losses into the frictional losses and remaining losses, this must be done by applying Routin's method, for which *see* section 13 of this Chapter.

**§ 28. Polyphase Motor Testing. Two and Three-Phase.—**Tests of polyphase motors involve the measurement of polyphase power, as described in Vol. I. of this HANDBOOK, Chapter V., section 11. To carry out this measurement with a two or three-phase alternating current or induction motor, two wattmeters are necessary, each suitable for the measurement of alternating-current power. If, for example, the motor under test is a two-phase motor, one with two

separate circuits, the two wattmeters must be connected, as shown in Fig. 27, with the two circuits of the alternating-current motor, and each of these wattmeters must be calibrated on its own independent inductionless standardising resistance, as described for a single-phase alternating-current test in the previous section. The true power taken up by the motor is, then, in every case, the sum of the readings of these wattmeters taken separately, and all the other tests applied to the motor for power-factor, total loss, starting and running torque are carried out exactly in the same manner as for a single-phase motor.

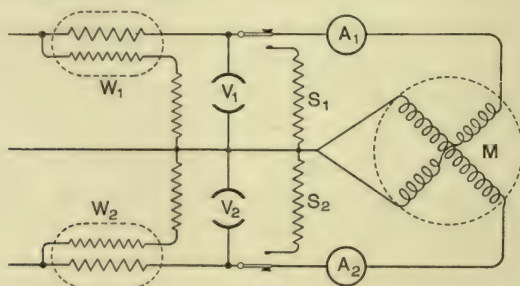


FIG. 27.

The testing of a three-phase induction motor involves rather more care, because it is desirable to ascertain first the power-factor of each branch of the circuit, in order that we may know whether the wattmeter readings are to be added or subtracted. The arrangements for testing a three-phase induction motor are as shown in Fig. 28. They involve the possession of two wattmeters suitable for the measurement of alternating-current power, which must be separately standardised on inductionless power-absorbing resistances, as described in the section on the Testing of Single-phase Alternating-current Motors. These wattmeters are connected up with their series coils in two of the main leads conveying the power to the alternating-current motor, and their shunt coils joined up between these two leads and the third lead, as

shown in the diagram. Ammeters are then placed in each of the three leads to read the currents, and a voltmeter is employed to read the P.D. between the motor terminals taken pair and pair. This can be effected quickly by means of one voltmeter and a three-throw switch, which connects the terminals of the voltmeter in succession between any two of the three leads. At the same time that these electrical readings are being taken the speed of the motor must be taken by means of a speed indicator or speed counter, and load must be put upon the motor by means of a friction or magnetic brake from the readings of which the mechanical power exerted by the motor can be obtained, as already described.

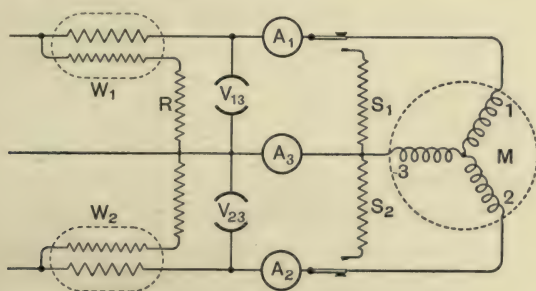


FIG. 28.

Let  $A_1$ ,  $A_2$ ,  $A_3$  be the three currents in the three lines, the root-mean-square values being understood, and let the motor terminals respectively be indicated by the figures 1, 2 and 3. Then let  $V_{12}$  be the P.D. between the terminals 1, 2,  $V_{23}$  that between the terminals 2, 3, and  $V_{31}$  that between the terminals 3, 1; and let  $W_1$  be the reading of the wattmeter the series coil of which is placed in lead 1, and  $W_2$  the reading of the wattmeter which is placed in lead 2. Then, to determine the power absorption of the motor, we have first to ascertain the power-factor of the motor. In general this can best be done by taking the readings of the two wattmeters. Suppose, in the first place, that the motor circuits are star-connected, and

that  $W_1$  and  $W_2$  are the readings of the two wattmeters placed with their series coils in the lines 1 and 2, and their shunt coils connected between lines 1 and 3, and 3 and 2. When the motor is fully loaded one of these wattmeters will read less than the other. Let the smaller scale reading be  $W_2$  and that of the other wattmeter  $W_1$ . Then the power-factor, P.F., of the motor is given by the expression

$$\text{P.F.} = \frac{a+1}{2\sqrt{a^2-a+1}},$$

where  $a = W_2/W_1$  or the ratio of the smaller to the larger scale reading. Having, then, settled which wattmeter reading shall be called  $W_2$ , we can load the motor to various loads and take the wattmeter readings.

If the power-factor for any load is greater than 0.5, we add the wattmeter readings to obtain the whole power taken up in the motor; but if the power-factor is less than 0.5, we subtract the readings. In taking the wattmeter readings it will be found that, as the load on the motor decreases, the readings of both wattmeters decrease, and at a certain stage one of them falls to zero. At lower loads it will be necessary to reverse the shunt coil connections of this wattmeter, and it will then give a scale reading as usual, but at and below the load for which one wattmeter thus reverses its direction of deflection the two wattmeter readings must be subtracted to obtain the power-absorption of the motor. This reversal of deflection of one wattmeter corresponds, as shown in section 9 of this Chapter, to a lag of current of 60deg., or to a power-factor of 0.5. If the centre point of a three-phase star-connected motor, or other balanced three-phase star-connected inductive circuit, is accessible, then the power-factor may be measured by a method due to Frankenfield (see *Electrical World and Engineer*, Vol. XL., p. 245, August, 1902; or *Science Abstracts*, Vol. VI., p. 32, Abstract 92). Two wattmeters are connected as shown in Fig. 29. Then the reading of the first wattmeter,  $W_1$ , is proportional to

$AV\sqrt{3}\sin\theta$ , and that of the second,  $W_2$ , in tandem with it to  $AV\cos\theta$ , where  $A$  is the current in each power arm and  $V$  the voltage between the centre point 0 and each terminal, 1, 2 or 3 (see Fig. 29). Hence the ratio of the wattmeter readings—viz.,  $W_1/W_2=\sqrt{3}\tan\theta$ , from which the power-factor, or  $\cos\theta$ , can be obtained at once.

Having measured the electrical power taken up, we can by a rope brake, measure the mechanical power given out on the pulley, as described in the previous section.

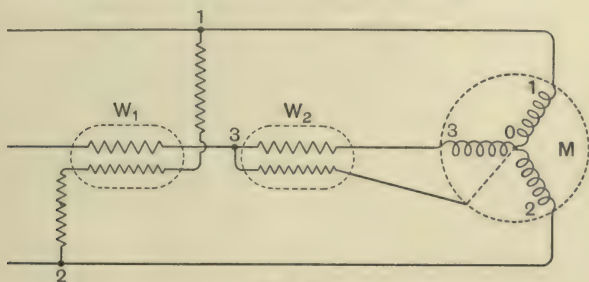


FIG. 29.

For additional information on the subject of dynamo, motor and transformer testing the reader may be referred to the following Papers:—

G. CLAUDE. "Dynamo Efficiency." *L'Électricien*, Vol. XV., p. 42, 1898; also *Science Abstracts*, Vol. I., p. 99, 1899.

The author describes Routin's method of measuring dynamo efficiency.

See also *The Electrician*, Vol. XL., p. 409, for a letter by Mr. I. R. Ashworth (*ibid.*, p. 493), who claims to have described the method in *The Electrician* of Feb. 17, 1893, with an important modification. Mr. Ashworth discards the brake as unreliable, and increases the moment of inertia,  $I$ , of the armature by a known amount,  $I'$ , by attaching to the shaft an iron disc. In Routin's notation we have, then,

$$I = \frac{I' \tan \alpha'}{\tan \alpha - \tan \alpha'}.$$

The moment of inertia of the armature having been found, the product of this and the rate of decrease of the angular velocity  $\omega$  of the armature gives the retarding torque acting at that instant on the armature and the value of  $I\omega \frac{d\omega}{dt}$ , the power taken up by the various sources of loss, *e.g.*, friction, hysteresis and eddy currents acting at that moment.

For an example of the use of the Routin method the reader may consult a Paper by

C. LIEBENOW. "Dynamo Testing." *Elek. Zeits.*, Vol. XX., p. 274, 1899 ; or *Science Abstracts*, Vol. II., p. 641, 1899.

The results of a Routin test on a 150kw. generator giving 2,300 amperes at 65 volts at 254 revs. per min. are given, as follows :—

Bearing friction .....	0·41kw.
Brush     ,,       .....	0·69   ,,
Hysteresis loss .....	0·43   ,,
Eddy current loss .....	2·00   ,,
C <sup>2</sup> R loss in armature .....	8·80   ,,
,,     ,,    fields.....	2·27   ,,
Total loss .... .	<u>14·6kw.</u>

The efficiency is, therefore, 91·0 per cent.

See also G. DETTMAR. "Analysis of Dynamo Losses." *Elek. Zeits.*, Vol. XX., p. 203, 1899 ; or *Science Abstracts*, Vol. II., Abstract 1,430, 1899.

E. STRYMEERSCH. "Determining Hysteresis and Eddy Current Losses in Dynamos." *Science Abstracts*, Vol. IV., 1901.

The methods of Housman and Swinburne for separating out the core losses are described.

An example is given of a test of a 4·5kw. dynamo = 45 amperes × 100 volts, resistance of the armature being 0·074 ohm and of the fields 57·1 ohms. Hence, the back E.M.F. in the armature at full voltage and normal speed is

$$100 - \left( 45 - \frac{100}{57.1} \right) 0.074 = 96.8 \text{ volts.}$$

When this dynamo was driven as a motor with an applied P.D. of 96·8 volts, it took 4·9 amperes. Hence, the total losses are  $4.9 \times 96.8 = 475$  watts. By running at various speeds at constant excitation, and drawing a Housman diagram, the hysteresis loss was found to be 194 watts, and the eddy-current loss 281 watts.

W. PEUKERT. "On Dynamo Losses." *Elek. Zeits.*, Vol. XXII., p. 393, 1901 ; or *Science Abstracts*, Vol. IV., Abstract 1,795, 1901.

A method is given for determining the frictional, hysteresis and eddy-current losses separately. The dynamo is brought up to normal speed by being belted to a small motor, and when the desired speed is obtained the belt is knocked off and the rate of decay of the angular velocity noted. Frictional loss is said to increase at a greater rate than the speed.

On the same subject see also

G. DETTMAR. "Iron Losses in Generators." *Elek. Zeits.*, Vol. XXI., p. 944, 1900 ; also *Science Abstracts*, Vol. IV., Abstract 448, 1901.

The author draws attention to the fact that eddy-current losses in the pole-pieces are a source of energy loss in some forms of continuous-current dynamo.

§ 29. **Temperature Test of Dynamos and Motors.**—It is necessary to apply in all cases a temperature test to a motor or dynamo when under examination. The durability of a motor or dynamo and its power to run for considerable periods of time without injurious heating are qualities of greater practical value than a very high efficiency. The final temperature reached by the machine when running for long periods under load is, too, a matter of great importance. The general rule is to stipulate for a run of six hours under full load, sometimes followed immediately by a two hours' run at 33 per cent. overload or three hours at 25 per cent. overload. At the end of this time the temperature of certain parts of the apparatus is taken and should not exceed that of the surrounding air by amounts which have been variously defined by different authorities. Thus, the American Institute of Electrical Engineers, in a report on "Standardisation," named the following temperature rise above the surrounding air, taken at 25°C., which should not be exceeded :—

Field and armature, taken by resistance.....	50°C.
Commutator and collectors, taken by thermometer ...	55°C.
Bearings and other parts of machine .....	40°C.
Electric circuits, taken by resistance .....	50°C.
Transformer circuits, taken by resistance .....	50°C.
Ditto                      ditto                      thermometer.....	40°C.

the rise in temperature to be referred to a standard temperature of 25°C. and normal conditions of ventilation ; but where the surrounding temperature is higher, for every degree Centigrade above 25°C. the above-named limiting temperature is to be decreased by 0·5 per cent.

The British Admiralty specification provides that, in the case of dynamos, the temperature of the armature one minute after stopping a six hours' continuous run at full load must not exceed  $30^{\circ}\text{F.}$  above the surrounding atmosphere. This is, however, a severe test, and it is usual to name  $40^{\circ}\text{C.}$  or  $45^{\circ}\text{C.}$  as the maximum rise of temperature to be permitted. In the case of large transformers, there is no doubt that a limit of  $30^{\circ}\text{C.}$  rise, as measured by copper resistance, is desirable, since the copper loss forms the largest portion of the full load loss. As high conductivity copper increases in resistance 0.4 per cent. per degree Centigrade, the difference between the loss at  $40^{\circ}\text{C.}$  or  $45^{\circ}\text{C.}$  and that at  $30^{\circ}\text{C.}$  above the atmospheric temperature is by no means negligible.

In all cases, where possible, the temperature of electric circuits should be ascertained by taking their electric resistance, as the mere contact reading of a thermometer bulb with the outer layers of coils of cotton-covered shellaced wire by no means gives the true temperature of the metal or insulation. In the case of transformer circuits it is quite easy to measure quickly the resistance of the fine wire copper coil, the other being short-circuited; but in the case of armatures a bridge measurement cannot generally be made. We are obliged, therefore, to have recourse to the mercury-in-glass or platinum resistance thermometer, but in all specifications the mode in which the temperature is to be taken should be carefully set out.

FINIS.

# INDEX.

- Absorbing Powers of Various Surfaces for Light, 349, 350  
Absorption and Reflection, Measurement of, 341  
Aerial Wire, Capacity of, 122  
Ageing Tests of Transformers, 601  
Air Condenser, 124  
Alternating Current Electric Meter, Chamberlain and Hookham, 84  
Alternating Current Electric Meter, Mordey-Fricker, 72  
Alternating Current Electric Meter, Shallenberger, 74  
Alternating Current Electric Meter, Westinghouse, 80  
Alternating Current Electric Meter, Wright-Ferranti, 75  
Alternating Current Motor Testing, 607  
Alternating Current Transformer Testing, 585  
Alternating Meters for Polyphase Circuits, 85  
Alternator, Differential Method of Testing, 581  
Alternator Testing, 565  
Alternator, Three-Phase, Method of Testing, 575  
Ammeters, Graphical, 38  
Ampere-Hour Meter, 2  
Ampere-Hour Meter, Ferranti, 41  
Ampere-Hour Meters, 23  
Ampere-Hour Meters, Self-Integrating, 39  
Ampere-Hour, The, 2  
Ampere-Second, The, 2  
Anderson's Method of Measuring Inductance, 192  
Arc Lamp Photometer, 323  
Armature Resistance, Measurement of, 498  
Aron Meter, The, 53  
Aron Meter, Theory of the, 53  
Ayrton and Perry Meter, 53  
Ayrton and Perry Secohmmeter, 197-200  
Baily's Experiments on Magnetic Hysteresis, 466  
Ballistic, Constant of Galvanometer, Measurement of, 18  
Ballistic Galvanometer, Adjustment of, 13  
Ballistic Galvanometer, Practical Forms of, 11  
Ballistic Galvanometer, The, 3  
Ballistic Galvanometer, Theory of, 4  
Ballistic Galvanometer, Use of, for Measuring Magnetic Flux Density, 397  
Bastian Electrolytic House Meter, 35  
Battery Test Form, 107  
Battery Testing, 99  
Bidwell's Experiments on Magnetic Tractive Force, 445  
Bismuth, Resistance of, Variation with Temperature, 403  
Bismuth Spiral for Measuring Magnetic Fields, 402  
Bismuth, Variation of Resistance in Magnetic Fields, 401  
Bunsen Disc Photometer, 277  
Bunsen Disc Photometer, Theory of, 274  
Cable Capacity, Measurement of, 161  
Cable Capacity Measurements, 151-153  
Candle Balance, Sugg's, 242  
Candle Power, 230  
Candle Standard, 236  
Capacities, Rules for Adding, 153  
Capacity of Cables, Influence of Resonance on, 150  
Capacity, Definition of, 111  
Capacity, Determination of, in Absolute Measure, 127  
Capacity, Determination of, by Alternating Currents, 154  
Capacity Determination by Comparison Methods, 145  
Capacity Determination, De Sauty's Method, 145

- Capacity Determination, Method of Mixtures, 147
- Capacity Determination, Thomson and Gott's Method, 147
- Capacity in Electrostatic Units, 113
- Capacity, Measurement of, by Ballistic Galvanometer, 128
- Capacity, Measurement of, by Charge and Discharge through Galvanometer, 130
- Capacity, Measurement of, by Commutator and Wheatstone's Bridge, 140
- Capacity, Measurement of Electric, 111
- Capacity, Measurement of, by Rotating Commutator, 133, 135
- Capacity of Polycore Cables, 164
- Capacity, Small, Determination of, 136-138
- Capacity, Standards of Electric, 112
- Capacity of Telegraph Wires, 121
- Capacity of Three-Core Cables, 164
- Capacity of Two-Core Cables, 168
- Carbon Filament Electric Lamp Standard, 258
- Chamberlain and Hookham Alternating Current Meter, 84
- Chamberlain and Hookham Meter, Patents for, 66
- Chamberlain and Hookham Meters, 68
- Chamberlain and Hookham's Ampere-Hour Meter, Theory of, 40
- Coaxial Cylinders, Electric Capacity of, 117
- Commercial Permeameters, 453
- Comparison Methods for Capacity Determination, 145
- Concentric Spheres, Electric Capacity of, 115
- Condenser, 112
- Condenser, Glass Plate, 126
- Condenser, Mica, 125
- Contact Breaker for Inductance Measurement, 202
- Coulomb, The, 2
- Coulombmeters, 23
- Coulombmeters, Classification of 24
- Curves of Illumination, Determination of, 338
- Cyclical Magnetisation Curves, Delineation of, 427
- Cyclical Magnetisation Curves of Iron, 428, 429
- Cylindrical Condenser, Electric Capacity of a, 118
- Damping, Correction for, 17
- Damping Factor, 16
- Dead-Beat Displacement, 6
- Demagnetisation Factor, 387
- Demagnetising Force, 387
- Dielectric, Constants Table of, 225-227
- Differential Measurement of Dynamo Efficiency by Parallel Method, 527
- Differential Measurement of Efficiency of Dynamos and Motor by the Series Method, 522
- Differential Method of Testing Dynamo Efficiency, 517
- Differential Testing of Dynamos by Kapp's Method, 521
- Differential Testing of Transformers, 602
- Diffusing Globes, Absorption of, 342
- Dimension-Ratio of Magnetic Test-Pieces, 388
- Disc, Electric Capacity of a, 113
- Discrimination Photometer, Fleming, 335
- Drysdale Permeameter, 456
- Du Bois Magnetic Balance, 448
- Dynamo Efficiencies and Losses, 499
- Dynamo Efficiency, Measurement of, 502
- Dynamo Efficiency, Routin's Method of Determination of, 542
- Dynamo Efficiency, Various Definitions of, 500
- Dynamo Losses, 502
- Dynamo and Motor Testing, Example of, 530
- Dynamos and Motors, References to Papers on Testing, 613
- Dynamo Testing, 496
- Edison Electrolytic Meter, 26, 27
- Efficiency Curve, 495
- Efficiency, Definition of, 495
- Efficiency of Dynamo, Measurement of, 502
- Efficiency Testing of Direct-Coupled Engines and Dynamos, 535
- Efficiency Tests of Dynamos by Measurement of Stray Power, 540
- Electric Arc as a Standard of Light, 261
- Electric Current, Time Integral of, 1
- Electric Energy, Graphic Representation of, 47
- Electric Energy, Measurement of, 45
- Electric Energy Meters, 46
- Electric Glow Lamps, Life Curves of, 318
- Electric Glow Lamps, Target Diagrams in Photometry of, 316
- Electric Glow Lamps, Variation of Candle-Power with Age, 314
- Electric House Meters, Classification of and References to Papers on, 55
- Electric Lamps, Photometry of, 308
- Electric Meter, Aron, 53
- Electric Meter, Bastian, 35

- Electric Meter, Chamberlain and Hookham, 68  
 Electric Meter, Davis-Conrad, 80  
 Electric Meter, Edison, 26  
 Electric Meter; Evershed, 69  
 Electric Meter, Ferranti, 42  
 Electric Meter, Frager, 60  
 Electric Meter, Johnson-Phillips, 62  
 Electric Meter, Long-Shattner, 28  
 Electric Meter, Mengarini, 50  
 Electric Meter, Mordey-Fricker, 72  
 Electric Meter, Shallenberger, 74  
 Electric Meter, Westinghouse-Shallenberger, 77  
 Electric Meter, Wright, 37  
 Electric Meter, Wright-Ferranti, 75  
 Electric Meters, Essential Qualities of, 94  
 Electric Meters for Inductive Circuits, 81  
 Electric Meters, Patent Specifications for, 57  
 Electric Quantity, Graphic Measurement of, 2  
 Electric Quantity, Measurement of, 1  
 Electric Quantity, Measurement of by Ballistic Galvanometer, 21  
 Electric Quantity, Measurement of, by Voltmeter, 31  
 Electrical Power, Absorption of, 504  
 Electricity Meters, References to Papers on, 44  
 Electrolytic Meters, 24  
 Elihu Thomson Watthour Meter, 51  
 Ellipsoid, Electric Capacity of an, 114  
 Ergmeters, 46  
 Evershed Motor Meter, 69  
 Evershed Watthour Meter, 51  
 Ewing's Bar and Yoke Permeameter, 439  
 Ewing's Hysteresis Tester, 481  
 Ewing's Magnetic Bridge, 451  
 Ewing's Traction Permeameter, 447  
 Fechner's Law, 329  
 Ferranti Ampere-Hour Meter, 41  
 Ferranti Meter, 65  
 Ferranti Meters, Patents for, 65  
 Ferranti Meter, Theory of, 42  
 Flame Standards, 235  
 Flame Standards, Influence of Surrounding Atmosphere on, 252  
 Fleming and Clinton Contact Breaker for Inductance Measurement, 203  
 Fleming and Clinton Rotating Commutator for Capacity Measurements, 135  
 Fleming-Ediswan Standard Photometric Glow Lamp, 261  
 Flicker Photometer, Rood, 287  
 Flicker Photometer, Whitman, 288  
 Form Factor, Influence of, on Capacity Determination, 155  
 Fox-Talbot-Abney Variable Aperture Disc, 284  
 Frager Meter, 60  
 Galvanometer, The Ballistic, 3  
 Galvanometer, Ballistic, Practical Forms of, 11  
 Glass Plate Condenser, 126  
 Graphical Ammeters, 38  
 Guard Ring, 115  
 Harcourt Pentane Lamp, 236, 246  
 Hefner Lamp, Reichsanstalt Pattern, 250  
 Hefner Unit, Value of in Standard Candles, 251  
 Henry, The, 171  
 Heterochromatic Photometry, 328  
 Hoffman's Voltmeter, 31  
 Hopkinson's Bar and Yoke Apparatus, 437  
 Hopkinson's Method of Testing Dynamo Efficiency, 518  
 House Meters, Electric, Classification and References, 55  
 House Meters, Testing of, 89  
 Housman's Diagram for Separation of Hysteresis and Eddy Current Losses in Armatures, 550  
 Hughes' Bridge for Measurement of Inductance, 221-223  
 Hysteresis, Alternating, 465  
 Hysteresis and Eddy Current Loss, Separation of, 548  
 Hysteresis of Iron, Variation of, in a Rotating Magnetic Field, 468  
 Hysteresis Loss in Good Sheet Iron, 474  
 Hysteresis Loss and Maximum Magnetic Flux Density, Relation of, 471  
 Hysteresis Loss in Soft Iron Wire, 473  
 Hysteresis Loss in Transformer Plate, 473  
 Hysteresis Loss, Variation of, with Time, 485  
 Hysteresis Losses for Various Flux Densities, 470  
 Hysteresis, Measurement of, 460  
 Hysteresis, Measurement of, by Wattmeter, 475  
 Hysteresis, Rotating, 465  
 Hysteresis Tester, Ewing's, 481  
 Hysteresis, Variation of with Magnetisation, 467  
 Hysteresis, Workshop Methods for Measuring, 478  
 Hysteretic Constants for Different Materials, 470

- Illumination, 231  
 Illumination in Candle-Feet of Various Places, 341  
 Illumination Photometer, Weber, 300  
 Illumination Photometry, 337  
 Inductance of Circular Conductor of One Turn, 173  
 Inductance Coil, 514  
 Inductances, Comparison of, by the Secohmmeter, 201  
 Inductance, Determination of by Periodic Currents, 205  
 Inductance of an Endless Solenoid, 175  
 Inductance, Influence of Form Factor upon, 206  
 Inductance, Measurement of, 111, 170, 180  
 Inductance, Method of Measuring by the Secohmmeter, 197  
 Inductance, Mutual, 207  
 Inductance, Mutual, Standards of, 210  
 Inductance, Standards of, 177  
 Inductance of Straight Circular-Sectioned Wire, 171  
 Inductance, Units of, 171  
 Inductances, Measurement of Small, 220  
 Insulation Tests of Transformers, 598  
 Intrinsic Brilliancy, 232  
 Iron, Magnetic Ageing of, 485  
  
 Johnson-Phillips Ampere-Hour Meter, 62  
 Joule Meters, 45  
  
 Kapp's Method of Testing Dynamos, 521  
 Kiloline, Definition of the, 372  
  
 Lamb and Walker Permeameter, 454  
 Lambert's Law, 232  
 Logarithmic Decrement, Measurement of, 10, 15  
 Logarithmic Decrement, Napierian, 9  
 Logarithmic Decrement, Variation with Resistance of Galvanometer Circuit, 20  
 Long-Schattner Meter, 28  
 Long-Schattner Prepayment Meter, 29  
 Luminous Efficiency, Measurement of, 301  
 Luminous Flux, 230  
  
 Magnetic Balance, Du Bois, 448  
 Magnetic Bridge, 449  
 Magnetic Circuit, 365  
 Magnetic Circuit, Equations for the, 374  
 Magnetic Definitions, 365  
 Magnetic Field, 367  
 Magnetic Field, Measurement of, 391  
 Magnetic Fields, Measurement of, by Gauss' Method, 406  
 Magnetic Flux, Definition of, 372  
 Magnetic Flux Density, Measurement of, by Variation of Bismuth Resistance, 400  
 Magnetic Force in Interior of Long Solenoid, 394  
 Magnetic Force, Line Integral of, 368  
 Magnetic Force, Measurement of, 391  
 Magnetic Force, Measurement of, by Ballistic Galvanometer, 391  
 Magnetic Force, Terrestrial, Measurement of, 407  
 Magnetic and Iron Testing, 365  
 Magnetic Permeability of Various Samples of Iron and Steel, 425  
 Magnetic Potential, 368  
 Magnetic Quantities, 366  
 Magnetic Reluctance, 369  
 Magnetic Reluctance of Joints, Correction for the, 440  
 Magnetic Susceptibility, Measurement of, 490  
 Magnetic Test-Pieces, Standard Forms of, 378  
 Magnetic Test-Pieces, Typical Forms of, 390  
 Magnetisation Curve of Iron, Determination, 417  
 Magnetisation Curves, 376  
 Magnetisation Curves, Bar and Yoke Method for Determining, 437  
 Magnetisation Curves for Iron and Steel, Samples of, 379-381  
 Magnetisation Curves, Meaning of Cyclical, 376  
 Magnetometer, Construction of, 409-411  
 Magnetomotive Force, 368  
 Magnetomotive Force and Ampere-turns, Law Connecting, 370  
 Matthew's Integrating Photometer, 325-327  
 Maxwell's Method of Measuring Inductance, 190  
 Mean Spherical Candle-Power, Determination of, 320  
 Measurement of Cable Capacity, 161  
 Measurement of Electric Energy, 45  
 Measurement of Electric Quantity, 1  
 Measurement of Inductance, 170, 180  
 Measurement of Inductance, Anderson's Method for, 192  
 Measurement of Inductance, Lord Rayleigh's Method for, 182  
 Measurement of Inductance, Maxwell's Method for, 190  
 Measurement<sup>\*</sup> of Inductance by the Secohmmeter, 197  
 Measurement of Inductance, Stroud's Method for, 195

- Measurement of Luminous Efficiency, 301  
 Mengarini Watthour Meter, 49  
 Meter Test Form, 98  
 Mica Condenser, 124  
 Microcoulomb Meter, 2  
 Microcoulomb, The, 2  
 Moment of Inertia, 409  
 Mordey and Fricker Meter, 71  
 Mordey's Method of Testing Alternators, 581  
 Motor Brake, 555  
 Motor Testing, 553  
 Motor Testing, Alternate Current, 607  
 Motor Testing, Polyphase, 609  
 Motor Tests by Cradle Method, 561  
 Muirhead Standard Air Condenser, 124  
 Muirhead's Investigations on Capacity Measurement, 150  
 Mutual Inductance, 207  
 Mutual Inductance, Carey Foster's Method for Determination, 213  
 Mutual Inductance, Comparison of, by Maxwell's Method, 217  
 Mutual Inductance, Measurement of, 213  
 Mutual Inductance, Standards of, 210  
  
 Napierian Logarithmic Decrement, 9  
 Nichol's Researches on Photometry, 293  
 Nichol's Spectro-Photometer, 292  
  
 Oscillatory Motion, 6  
 Oscillatory Motion, Theory of, 7  
 Oxygen, Liquid, Magnetic Susceptibility of, 494  
  
 Parallel Plate Condenser, Electric Capacity of a, 116  
 Pentane Lamp, Harcourt, 236  
 Permeability Curves, 430, 431  
 Permeability of Iron, Variation with Annealing, 433  
 Permeability of Iron, Variation of, with Magnetic Force, 432  
 Permeability, Tests of Iron Sheet and Wire for, 435  
 Permeameter, Drysdale, 456  
 Permeameter, Ewing's, 447  
 Permeameter, Lamb and Walker, 454  
 Permeance, Definition of, 371  
 Photometer, Arc Lamp, 323  
 Photometer Bunsen Disc, 273  
 Photometer Diffusion, 286  
 Photometer, Discrimination, 335  
 Photometer Dispersion, 271  
 Photometer, Fleming, 283  
 Photometer, Flicker, 287  
 Photometer, Gas Referees', 269  
 Photometer, Illumination, 297  
 Photometer, Lummer-Brodhun, 280  
 Photometer, Matthews' Integrating, 325-327  
 Photometer, Ritchie Wedge, 266  
 Photometer Rooms, 305  
 Photometer Rooms, Arrangements of, 305  
 Photometer, Rotating Disc, 283  
 Photometer, Selected Ray, 290  
 Photometer Spectro, 292  
 Photometer, Total Reflection, 279  
 Photometer, Wedge, 278  
 Photometers, 264  
 Photometric Measurements, 264  
 Photometric Processes, 264  
 Photometric Units, 229  
 Photometry, 229  
 Photometry of Electric Lamps, Arrangements for, 306-308  
 Photometry, Heterochromatic, 328  
 Photometry, Illumination, 337  
 Photometry, References to Papers on, 354  
 Polyphase Alternator Testing, 565  
 Polyphase Electric Meters, 85  
 Polyphase Electric Meters, Testing of, 97  
 Polyphase Motor Testing, 609  
 Power Absorbing Rheostats, 506  
 Power Factors of Motors, Measurement of, 612  
 Preece and Trotter Illumination Photometer, 298  
 Primary Batteries, 103  
 Primary Batteries, Testing of, 104  
  
 Quadrant, The, 171  
  
 Rayleigh's Method of Measuring Inductance, 182  
 Reflecting Power, Measurement of, 346  
 Reflecting Power of Various Surfaces, 351  
 Reichsanstalt Pattern of Photometer Bench, 311  
 Reluctance, Definition of, 371  
 Reluctance, Magnetic, 369  
 Reluctivity, 370  
 Reluctivity, Definition of, 370  
 Residual Discharge, 112  
 Retentivity, 374  
 Roget's Experiments on Magnetic Ageing of Iron, 486  
 Rood's Flicker Photometer, 287  
 Rope Brake for Testing Motors, 556  
 Rotating Commutator, 133  
 Rousseau Diagram for Determination of Mean Spherical Candle-Power, 321  
 Routin's Method of Determining Dynamo Efficiency, 542

- Rowland's Methods of Measuring Inductance, 207
- Secohm, The, 171
- Secohmmeter, Measurement of Inductance by the, 197
- Secondary Batteries, References to and Papers on, 108
- Secondary Batteries, Testing of, 107
- Secondary Drop of Transformers, Measurement of, 595
- Self-Integrating Ampere-Hour Meters, 39
- Simmance Pentane Lamp, 249
- Soames' Motor Brake, 555
- Solenoid, Inductance of a, 175
- Sphere, Electric Capacity of a, 113
- Standard Candles, 238
- Standard Sperm Candle, 236
- Standards of Inductance, 177
- Standards of Light, 235, 262
- Starting and Rotating Torques, Measurement of, 564
- Steinmetz Curve, 469
- Stroud's Method of Measuring Inductance, 195
- Sumpner's Experiments on Measurement of Reflecting and Absorbing Power, 346
- Sumpner's Method of Testing Transformers, 602
- Telegraph Wires, Electric Capacity of, 121
- Temperature Tests of Dynamos and Motors, 615
- Temperature Tests of Transformers, 599
- Terrestrial Magnetic Force, Measurement of, 412
- Testing Batteries, 99
- Testing of House Meters, 89
- Testing of Secondary Batteries, 107
- The Henry, 171
- Theorem concerning Time-Constants of Bridge Arms, 185
- Three-Core Cables, Capacity of, 164
- Three-Phase Alternator, Method of Testing, 575
- Three-Phase Alternator, Test of, 567
- Time-Constant, Definition of Term, 181
- Time-Integral of Electric Current, 1
- Traction Methods for Determining Magnetisation Curves, 443
- Transformer Losses, Measurement of, 589-592
- Transformer Testing, 585
- Transformers, Ageing Tests of, 601
- Transformers, Insulation Tests of, 598
- Transformers, Measurement of Secondary Drop of, 595
- Transformers, Temperature Tests of, 599
- Tuning Fork Commutator, 131
- Typical Magnetic Circuits, 388
- Violle Light Unit, 256
- Violle Platinum Standard, 258
- Voltameters, 23
- Water Rheostat, 515
- Water Voltmeter, 31
- Watthour Meters, 45
- Watthour Meters, Classification of, 46
- Watthour Meters, Theory of, 51
- Weber, Definition of the, 372
- Weber Illumination Photometer, 299
- Westinghouse - Shallenberger Electric Meter, Theory of, 78
- Whitman's Flicker Photometer, 288
- Wire, Best Size of, for Power Absorbing Rheostats, 510
- Wright's Electrolytic House Meter, 37



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NEW VOLUMES in preparation.—See page 14.

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"The Electrician"	32	Electricity and Magnetism	16
Newspapers and Periodicals	2	Electric Lighting and Transmission of Power	19
International Wire Tables	2	Wiring and Electric Light Fitting	23
International Telegraph Convention and Service Regulations	2	Telegraphy and Telephony	23
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		Miscellaneous	28

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**CHAPTER III.—The Theory of Simple Periodic Currents.**—Variable and Steady Flow—Current and Electromotive Force Curves—Fourier's Theorem—Mechanical Harmonographs—Mathematical Sketch of Fourier's Theorem—Practical Application of Fourier's Theorem to the Harmonic Analysis of a Periodic Curve—Simple Periodic Currents and Electromotive Forces—Description of a Simple Periodic Curve—The Mean Value of the Ordinate of a Sine Curve—The Square Root of the Mean of the Squares of the Ordinates of a Simple Periodic Curve—Derived Curves—Inductance and Inductive Circuits; Inductance, Resistance and Capacity of Circuits; Inductive and Non-Inductive Circuits—Faraday's and Henry's Experiments on Self-Induction—Edlund's and Maxwell's Arrangement for Exhibiting Inductance of Circuits—Electro-Magnetic Momentum—Electrodynamic State and Electromagnetic Energy—Co-efficient of Mutual Induction; Energy of Two Circuits—The Unit of Inductance—Value of the Self-Induction in Henry's for Various Instruments—Current Growth in Inductive Circuits—Analogy of Current and Velocity Change; Fundamental Equations for Current Growth in Inductive Circuits—Equation for the Establishment of a Steady Current in Inductive Circuits—Time Constant of an Inductive Circuit—Logarithmic Curves—Instantaneous Value of Simple Periodic Current—Solution of Current Equation—Impedance of Inductive Circuit—Relations of Impressed Electromotive Force, Current and Impedance—Geometrical Illustrations—Impressed and Effective Electromotive Forces—Clock Diagram of Electromotive Forces in Inductive Circuit—Triangle Representing Resistance, Impedance and Reactance of Circuit—The Mean Value of the Power of a Periodic Current—Geometrical Theorem—Power Curves for Inductive and Non-Inductive Circuits—Experimental Measurement of Periodic Currents and Electromotive Forces—Mean Square Value—Method of Measuring the True Mean Power Given to an Inductive Circuit—Theory of the Wattmeter—Divided Circuits—Important Trigonometrical Lemma—Impedance of Branched Circuits—Wattmeter Measurement of Periodic Power—Mutual Induction of Two Circuits of Constant Inductance—The Flow of Simple Periodic Currents into a Condenser—Time Constant of a Condenser—Charging a Condenser through a Resistance—Condenser equation—Annulment of Inductance by Capacity—Representation of Periodic Currents by Polar Diagrams—Initial Conditions on Starting Current Flow in Inductive Circuits—Complex Periodic Functions—Apparent and True Power given to Inductive Circuits—Power Factor.

**CHAPTER IV.—Mutual and Self-Induction.**—The Researches of Joseph Henry—Experiments with Coils and Bobbins; Discovery of Self-Induction—Mutual Induction—Induction at a Distance—Induction between Telephone Circuits—Induction over Great Distances—Induced Currents of Higher Orders—Inductive Effects by Transient Electric Currents—Magnetic Screening—Direction of Induced Currents—Various Qualities of an Induced Current—Elementary Theory of Mutual Induction of Two Circuits—Theory of Induction Coil with Non-Magnetic Core—Comparison of Theory and Experiment—Duration of Induced Currents—Magnetic Screening Action of Good-Conducting Masses—Faraday's and Henry's Experiments—Willoughby Smith's Investigations on Magnetic Screening—Dove's Experiments and Henry's Views on same—The Reaction of the Secondary Currents on the Primary Circuit in the Case of an Induction Coil—Induction Balance and Sonometer—Transmission of Alternating Currents through Conductors—Prof. Hughes' Experiments—Lord Rayleigh's Researches—Flow of Current through Conductor Surface Flow of Alternating Currents—Increased Resistance of Conductors for Alternating Currents of High Frequency—Limiting Size of Conductors for Conveyance of Alternating Currents—Stephan's Analogies—Electro-Magnetic Repulsion—Elihu Thomson's Experiments—Electro-Magnetic Rotations.

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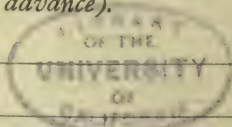
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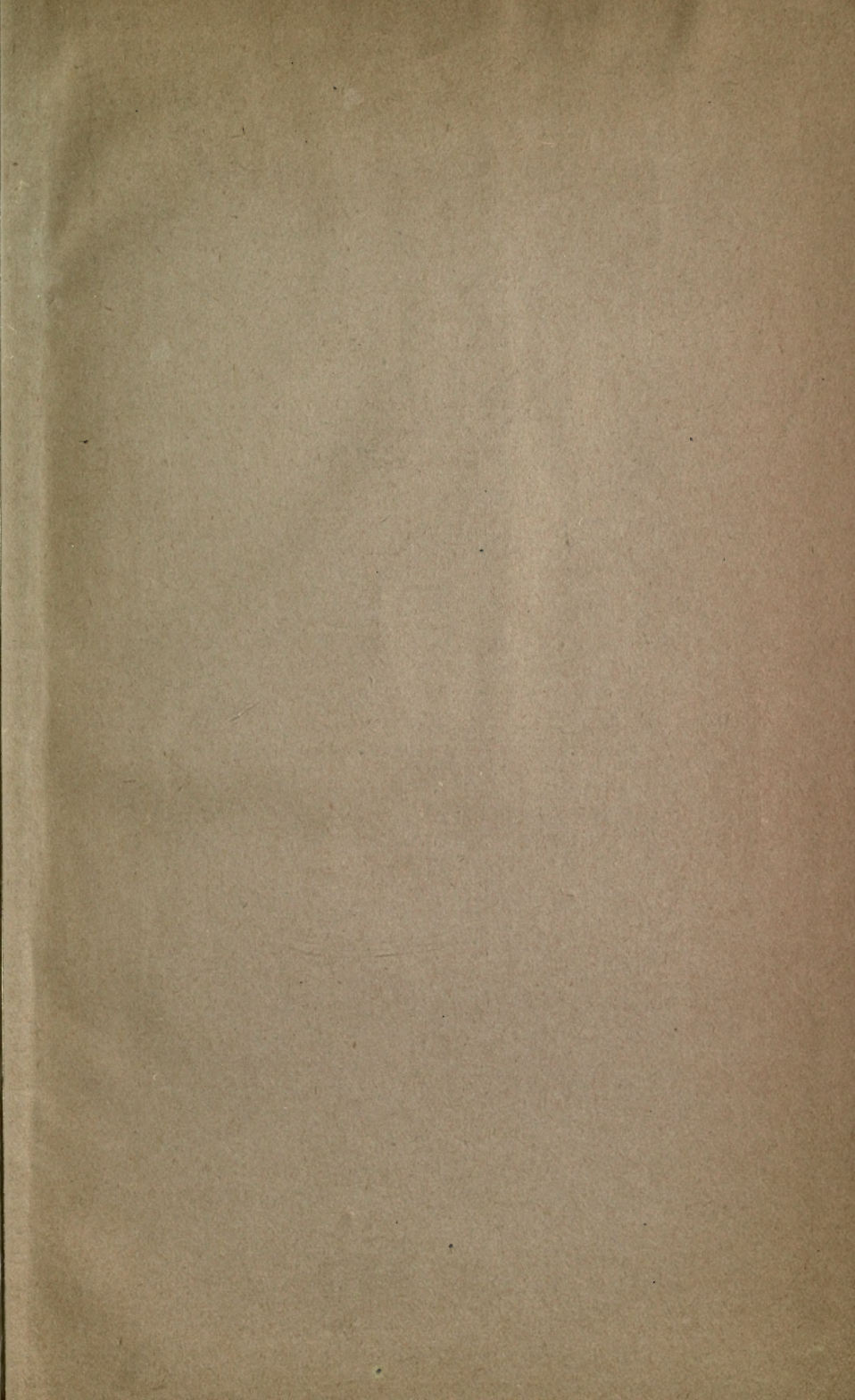


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